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Competing Mechanisms: Communication under Exclusivity Clauses

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Abstract

In the present note, we show that a weak restriction on out of equilibrium beliefs allows to extend the Revelation Principle to exclusive agency games, even if we consider mixed strategy equilibria. Next, we argue that this result does not extend to games with several agents, even if we restrict the analysis to pure strategy equilibria.

Keywords: Competing Mechanisms, Exclusive Contracts, Incomplete Information, Participation decision
JEL Classification: D82.

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1 Introduction

This paper contributes to the analysis of strategic competition in the presence of exclusive negotiation. Exclusivity is a relevant feature in several economic frameworks. In labour markets typically a worker can at most enter into one contractual relationship. Similar legal restrictions hold in franchising and in procurement contracts, as well as in regulatory and financial settings. Most of these situations feature strategic competition among principals, who offer contracts in the presence of incomplete information over the relevant market characteristics. Yet, the quest for a full theoretical understanding of these models raises a number of relevant issues.

On the one hand, it is an established finding that whenever principals compete over contracts in the presence of privately informed agents, the restriction to direct revelation mechanisms may involve a loss of generality. The result has been documented in a number of game-theoretic examples, which crucially exploit the fact that an agent participates with many principals at a time (see Peters (2001) and Martimort and Stole (2002)). By using indirect communication mechanisms, any principal can control an agent's out-of-equilibrium behavior so to create threats that prevent unilateral deviations by his opponents. Direct revelation mechanisms might not flexible enough to reproduce the same threats.[1] This suggests that if negotiations are not exclusive, there exist equilibrium outcomes that can be supported by indirect communication mechanism and not by direct ones.

On the other hand, most economic applications of competition with exclusive negotiations restrict principals to use direct revelation mechanisms from the outset. That is, principals commit to offer some incentive schemes contingent on the private information revealed by agents, these offers are such that each agent finds it incentive compatible to truthfully report her private information to the principal she negotiates with. This restriction is made in canonical models of exclusive negotiation under incomplete information such as Rothschild and Stiglitz (1976) for the single-agent case and (McAfee (1993) for the multi-agent one, among others.

Hence, a natural question is: is there any loss of generality in restricting principals to use direct revelation mechanisms when exclusive negotiation is the rule? A positive answer would provide a rationale to the approach taken by economic applications. The present paper investigates this issue. Specifically, we prove two results. First, we consider the single-agent case and identify a set of equilibrium outcomes that can always be supported by direct revelation mechanisms. Any equilibrium in such set is strongly robust in the sense of Peters (2001) and Han (2007), i.e. agents' beliefs at equilibrium are such that any principal can break agents' indifferences in the most favorable way for him both on and off the equilibrium path. Although demanding this condition is how-

[1] An example along the same lines is provided by Epstein and Peters (1999), who consider a multiple agent setting and crucially exploit the fact that every agent sends a message to all pr

ever tight. We show by means of an example that equilibria that fail to be strongly robust induce outcomes that cannot be supported by means of direct revelation mechanisms.

Second, we consider the multi-agent case and show that the restriction to strongly robust equilibria does not guarantee that one can safely rely on direct revelation mechanisms. To make this point we construct an example with two principals and two agents, that exhibits an equilibrium outcome in the game with general mechanisms that cannot be reproduced by direct revelation ones. A well-known example proposed by Peck (1997) and related to our result, shows that in a context of exclusive contracts and several agents, the restriction to type-revelation mechanisms is problematic when principals randomize over mechanisms. Being unrelated with principals' randomization over mechanisms, our example generalizes Peck's (1997) result. We provide a very simple example, in which there is no issue of untruthful report of the agents, because types are irrelevant. Agents only take participation decisions. In such a framework, communication by means of (private) messages allows principal 1 to specify his out-of-equilibrium behavior, as a function of the agents' participation decisions and messages. Hence, for principal 1, communication (even if types are not relevant) is an instrument to trigger his opponent's deviations and to control the agents' participation decisions.

Related literature This research is related to the literature on competing principals and exclusive negotiations. When dealing with a single agent, recent approaches to oligopolistic screening, building on the classical Rothschild and Stiglitz (1976) model of insurance with competing identical firms, restrict the unique agent to participate with at most one principal.[2]

In games with several agents, theoretical works postulate competition with exclusive negotiations to analyze multiple auctions (see McAfee (1993), Peters and Severinov (1997), Moldovanu, Sela, and Shi (2008)) and competing hierarchies frameworks (e.g. Caillaud, Jullien, and Picard (1995), Martimort and Piccolo (2010)), among others.

2 The model

This section introduces a multiple-principal multiple-agent model of incomplete information in which exclusive clauses are imposed from the outset. We refer to a scenario in which several principals (indexed by $j \in \mathcal{J} = \{1,...,J\}$) contract with several agents (indexed by $i \in I = \{1,...,I\}$). Each agent has private information about her preferences. The information available to agent i is represented by a type $\omega^i \in \Omega^i$. We denote

[2]Some examples are Biglaiser and Mezzetti (1993), who deal with differentiated principals and a single agent, Rochet and Stole (2002) who study an oligopolistic framework with uncertain (and private) participation choices of the agent, and Piaser (2007) among others. Gal-Or (1991) presents a sequential game in which firms decide whether or not to contract with the same agent, some sort of exclusive negotiation is an endogenous outcome of such a decision.

$\omega \in \Omega = \underset{i \in I}{\times} \Omega^i$ a state of the world. Principals have common beliefs on the probability distribution of the states of the world, and F is the corresponding distribution function.

Each principal j may take an action $x_j \in X_j$. Agents only take participation decisions: every agent i can participate with at most one principal.[3]

Specifically, let $a^i_j \in \{Y,N\}$ represent the participation decision of agent i with principal j. We denote a^i the vector of participation decisions of agent i with all principals, which is simply a sequence of acceptance and rejection decisions. Exclusivity implies that a given agent i cannot accept more than one contract. [4]

Definition 1 *[***Exclusivity***] Let A^i be the set of possible participation choices for each agent i, an exclusive set-up is such that, for every agent i*

$$A^i = \left\{ a^i = (a^i_1, a^i_2, ..., a^i_J) \in \{Y,N\}^{\sharp J} \ : a^i_j = \{Y\} \textit{ for at most one } j \right\}$$

Each principal fully observes the set of agents who participate with him. Thus, an incentive scheme for principal j is a measurable mapping $c_j : \{Y,N\}^{\sharp I} \to \Delta(X_j)$, with $\Delta\left(X_j\right)$ denoting the set of probability distributions over X_j, and C_j is the set of such mappings for principal j, i.e. $c_j \in C_j$. A single principal communicates with agents by means of the (private) messages that he receives from those agents who participate with him. To make the relationship between the agents' participation choices and the structure of communication explicit, we assume that every message space M^i_j for every i and j is sufficiently rich to include the element $\{\emptyset\}$ corresponding to the information "agent i does not communicate with principal j". In addition, each space M^i_j is taken to satisfy the standard size restriction $\sharp M^i_j > \sharp \Omega^i$ for all i and j.

Principal j can make his decisions contingent on the array of messages m_j he receives, with $m_j = \left(m^1_j, m^2_j, ..., m^I_j\right) \in M_j = \underset{i \in I}{\times} M^i_j$. Final allocations are determined by the multilateral contracts that principals independently sign with agents. More formally, we say that a mechanism proposed by principal j is the measurable mapping $\gamma_j : M_j \to \Delta\left(C_j\right)$. We take Γ_j to be the set of mechanisms available to principal j and $\Gamma = \underset{j \in \mathcal{J}}{\times} \Gamma_j$ to be the set of mechanisms available to all principals. All relevant sets are taken to be compact and measurable. Observe that a (pure) strategy for principal j is any mechanism $\gamma_j \in \Gamma_j$.

The competing mechanism game relative to Γ begins when each principal publicly and simultaneously commits to a mechanism. Having observed the array of offered mechanisms $(\gamma_1, \gamma_2, ..., \gamma_J) \in \Gamma$ and given their own type, agents simultaneously send a message to each of the principals. In the final step, payoffs realize.

[3] The model can straightforwardly be extended to any arbitrary set of agents' actions.

[4] This model is in line with economic applications of multiple-principal multiple-agent games postulating that each of the agents can at most deal with one principal.

In this incomplete information game, a strategy for each type of any agent i associates to every profile of offered mechanisms a joint decision in terms of participation and communication. More precisely, take $m_j^i \in M_j^i$ to be the message that agent i sends to principal j and denote $M^i = \underset{j\in\mathcal{J}}{\times} M_j^i$, then let $S^i = \left\{ s^i \in M^i \times A^i : m_j^i = \emptyset \text{ if } a_j^i = \{N\} \right\}$ be the set of joint participation and communication choices for agent i. Agent i's strategy is then the measurable mapping $\lambda^i : \Gamma \times \Omega^i \to \Delta\left(S^i\right)$, with $\lambda^i = (m^i, a^i)$.

Every profile of agents' strategies $\lambda = \left(\lambda^1, \lambda^2, ..., \lambda^I\right)$ induces a probability distribution over principals' decisions. Given the array of mechanisms $\gamma = (\gamma_1, \gamma_2, ..., \gamma_J)$ and the state of the world ω, we take $\beta_j^\lambda \in \Delta(X_j)$ to be the probability distribution over principal j's decisions induced by the strategy profile λ and take $\beta^\lambda \in \Delta(X)$ to be the joint distribution over all principals.

Now, let principal j's payoff be given by $v_j : X \times A \times \Omega \to \mathbb{R}_+$, while let $u^i : X \times A \times \Omega \to \mathbb{R}_+$ represent the payoff to agent i, with $X = \underset{j\in\mathcal{J}}{\times} X_j$ and $A = \underset{i\in I}{\times} A^i$. For a given array of agents' actions $a = (a^1, a^2, ..., a^I)$ and of principals' decisions $x = (x_1, x_2, ..., x_J)$, the state contingent utilities of agent i and principal j are $u^i(x, a; \omega)$, and $v_j(x, a; \omega)$, respectively. Then, the expected payoff to each agent i of type ω^i can be written as:

$$U^i(\gamma, \lambda, \omega^i) = \int_{\Omega^{-i}} \int_X u^i(x, a; \omega^i, \omega^{-i})\, F(\omega^{-i}|\omega^i)\, d\beta^\lambda\, d\omega^{-i}, \tag{1}$$

with $F(\omega^{-i}|\omega^i)$ representing the conditional probability of ω^{-i} given ω^i.

Similarly, the payoff to principal j is given by:

$$V_j(\gamma, \lambda) = \int_\Omega \int_X v_j(x, a; \omega)\; F(\omega)\; d\beta^\lambda\, d\omega \tag{2}$$

with $F(\omega)$ being the probability of the state of the world ω.

We typically say that the strategy profile $\lambda = \left(\lambda^1, \lambda^2, ..., \lambda^I\right)$ constitutes a continuation equilibrium relative to Γ if for every $\omega^i \in \Omega^i$, for every $i \in I$ and for every $\gamma \in \Gamma$:

$$U^i\left(\gamma, \lambda, \omega^i\right) \geq U^i\left(\gamma, \lambda^{i\prime}, \lambda^{-i}, \omega^i\right) \tag{3}$$

for all $\lambda^{i\prime}$.

We take Perfect Bayesian Equilibrium (PBE) to be the relevant solution concept.

Consider now a situation in which principals are restricted to offer direct incentive compatible mechanisms. A mechanism available to principal j is said to be *direct* if agents can only communicate their types to principal j, i.e. if $M_j^i = \Omega^i \cup \{\emptyset\}$ for every i. Observe that direct mechanisms represent agent i's decision to reject principal j's contract by means of using the direct message $\{\emptyset\}$. We take $\tilde{\gamma}_j : \underset{i\in I}{\times} \left(\Omega^i \cup \{\emptyset\}\right) \to \Delta\left(C_j\right)$ to be the corresponding direct mechanism for principal j, and we let Γ_j^D be the set of such direct mechanisms, i.e. $\tilde{\gamma}_j \in \Gamma_j^D \subseteq \Gamma_J$. A direct mechanism is *incentive compatible*

from the point of view of principal j if, given the mechanisms offered by the other principals, it induces a continuation equilibrium in which agents truthfully reveal their types to him. More formally, given $\gamma_{-j} \in \Gamma_{-j}$, $\tilde{\gamma}_j \in \Gamma_j^D$ is a direct incentive compatible mechanism for principal j if the array $\left(\tilde{\gamma}_j, \gamma_{-j}\right)$ induces a continuation equilibrium in which each type ω^i of agent i reports her true type to principal j.

Observe that a mechanism which is incentive compatible from the viewpoint of principal j may induce agents to lie to some principal other than j.

If $\Gamma = (\Gamma_1, \Gamma_2, \ldots, \Gamma_J)$ is the set of mechanisms available to the principals, we take G^Γ to be the corresponding multiple-principal multiple-agent game, and we denote G^D the multiple-principal multiple-agent game in which principals are restricted to direct mechanisms.

Finally, we introduce the definition of strongly robust equilibrium in the sense of Peters (2001) and Han (2007). A *strongly robust equilibrium* is a Perfect Bayesian Equilibrium in which, regardless of the continuation equilibrium selected by the agents, no principal j can improve on his own payoff by deviating to some other mechanism.[5] More formally, in a given game G^Γ the array of principals' mechanisms $\gamma = (\gamma_1, \cdots, \gamma_J)$ and of agents' strategies $\lambda = (\lambda^1, \cdots, \lambda^I)$ constitute a strongly robust equilibrium relative to Γ if for every $j \in \mathcal{J}$, every mechanism $\gamma_j \in \Gamma_j$ and every continuation equilibrium λ:

$$V_j(\gamma_j, \gamma_{-j}, \lambda) \geq V_j(\gamma_j', \gamma_{-j}, \lambda') \qquad \forall \gamma_j' \in \Gamma_j, \forall \lambda' \neq \lambda \tag{4}$$

that is

$$\int_\Omega \int_X v_j\left(x_j, x_{-j}, a; \omega\right) \, F(\omega) \, d\beta^\lambda \, d\omega \geq \int_\Omega \int_X v_j\left(x_j', x_{-j}, a'; \omega\right) \, F(\omega) \, d\beta^{\lambda'} \, d\omega \tag{5}$$

for every $x_j' \in X_j$, $\lambda' = (m', a')$.

3 Principals competing for a single agent

Consider a situation in which there is only one agent $I = 1$ with relevant types $\omega \in \Omega$. When a principal proposes a mechanism to an agent, he anticipates that the agent (if she accepts the offer) will send him a truthful message, even if she is indifferent between telling the truth and lying. By definition, incentive compatible mechanisms put restrictions on out-of-equilibrium beliefs. Therefore, if one restricts attention to incentive compatible mechanisms by construction, one cannot characterize equilibria

[5] Observe that a mechanism γ_j is a pure strategy for principal j. A mixed strategy for principal j would be a probability distribution over mechanisms. For convenience, we define strong robustness only of equilibria in which principals play pure strategies. The extension to mixed strategies is straightforward.

in which the agent lies. Hence, for an equilibrium allocation to be sustained by direct mechanisms it is crucial to determine how agent's indifferences should be broken.

Having observed that, we rely on the notion of strongly robust equilibrium to tackle these problems and we show that in a single-agent framework strongly robust equilibria in general mechanism games G^{Γ} with exclusive negotiation are also equilibria in direct mechanism games G^{D}.

Let $a_j(\gamma_j,\gamma_{-j};\omega) \in \{Y,N\}$ be the participation choice of type ω when principals offer the mechanisms (γ_j,γ_{-j}). Exclusivity implies that for every principal $j \in \mathcal{J}$ and for every $\omega \in \Omega$, we can define the set $\hat{M}_j(\gamma_j,\omega)$ as follows:

$$\begin{aligned} \hat{M}_j(\gamma_j,\omega) &= \underset{m_j \in M_j}{\arg\max}\, U\left[\gamma_j(m_j),\gamma_{-j},\lambda,\omega\right] \\ &\quad s.t.\ a_j(\gamma_j,\gamma_{-j};\omega) = Y. \end{aligned} \tag{6}$$

This is the set of optimal messages that type ω would send to principal j, once she has chosen to accept j's contractual offer. At this stage, since we assume exclusive contracting, the set of (non-trivial) messages that type ω may send will not depend on the offers of those principals whose contracts have not been accepted, i.e. $\hat{M}_j(\gamma_j,\omega)$ for all $j \in \mathcal{J}$.[6] In general, the set $\hat{M}_j(\gamma_j,\omega)$ will not be a singleton and it will be true that: $\forall m_j, m_j' \in \hat{M}_j(\gamma_j,\omega)$

$$U\left(\gamma_j(m_j),\gamma_{-j}(\emptyset),\omega\right) = U\left(\gamma_j(m_j'),\gamma_{-j}'(\emptyset),\omega\right) \quad \forall \gamma_{-j}' \in \Gamma_{-j}, \gamma_{-j}' \neq \gamma_{-j}. \tag{7}$$

with $U\left(\gamma_j(m_j),\gamma_{-j}(\emptyset),\omega\right)$ being the indirect utility for type ω when contracting with principal j.

We are now ready to show that strongly robust equilibria in general mechanism games G^{Γ} with exclusive negotiation are also equilibria in direct mechanism games G^{D}.

Theorem 1 *Take any game G^{Γ} such that $J \geq 2$ and $I = 1$. Then, every strongly robust equilibrium outcome of G^{Γ} is an equilibrium outcome of G^{D} in which principals play an incentive compatible mechanism.*

Proof.

In a given game G^{Γ} consider a strongly-robust equilibrium outcome supported by the array of principals' mechanisms $\gamma = (\gamma_j,\gamma_{-j})$ and by the agent's continuation strategy $\lambda = (m,a)$. Denote V_j and U the equilibrium payoffs for every principal $j \in \mathcal{J}$ and for the agent, respectively.

Let us now consider the game G^{D} and construct the following direct mechanisms. For every principal j, let the direct mechanism be $\tilde{\gamma}_j(\omega)$ such that:

[6] In common agency games, in which typically there is only one agent and contracts are non-exclusive, this does not hold in general.

$$\tilde{\gamma}_j(\omega) \;=\; \begin{cases} \gamma_j\left(m_j\left(\gamma_j,\gamma_{-j};\omega\right)\right) & \text{if} \quad a_j(\gamma_j,\gamma_{-j};\omega)=Y \\ \gamma_j\left(\hat{m}_j(\gamma_j,\omega)\right) & \text{if} \quad a_j(\gamma_j,\gamma_{-j};\omega)=N \end{cases} \tag{8}$$

with $\hat{m}_j\left(\gamma_j,\omega\right) \in \hat{M}_j\left(\gamma_j,\omega\right)$, as defined in (6). Finally, we let $\tilde{\gamma}_j(\emptyset)=\gamma_j(\emptyset)$.

In G^D, let us construct the agent's strategy $\tilde{\lambda}=(\tilde{m},\tilde{a})$. On the equilibrium path, $\tilde{\lambda}$ prescribes the following:

$$\forall \omega \in \Omega,\ \forall j \in \mathcal{J} \qquad \tilde{a}_j\left(\tilde{\gamma}_j,\tilde{\gamma}_{-j};\omega\right)=a_j\left(\gamma_j,\gamma_{-j};\omega\right) \tag{9}$$

and

$$\begin{cases} \tilde{m}_j(\tilde{\gamma}_j,\tilde{\gamma}_{-j};\omega)=\omega & \text{whenever} \quad \tilde{a}_j\left(\tilde{\gamma}_j,\tilde{\gamma}_{-j};\omega\right)=Y \\ \tilde{m}_j(\tilde{\gamma}_j,\tilde{\gamma}_{-j};\omega)=\emptyset & \text{whenever} \quad \tilde{a}_j\left(\tilde{\gamma}_j,\tilde{\gamma}_{-j};\omega\right)=N. \end{cases} \tag{10}$$

Equation (9) states that for every type and principal, the agent takes the same participation choice she was taking in the game G^{Γ} when confronted with $\left(\gamma_j,\gamma_{-j}\right)$. Equation (10) states that every type truthfully reveals information when $(\tilde{\gamma}_j,\tilde{\gamma}_{-j})$ are offered. Given (8) this is an optimal behavior for the agent, hence the proposed direct mechanisms are incentive compatible.

For every array of offers $\left(\tilde{\gamma}'_j,\tilde{\gamma}_{-j}\right) \in \Gamma^D$ and for every j, let $\tilde{\lambda}\left(\tilde{\gamma}'_j,\tilde{\gamma}_{-j},\omega\right)$ be any best reply of type ω such that: ω communicates truthfully with principal j whenever she is indifferent between several messages, and if the offers $\left(\tilde{\gamma}'_j,\tilde{\gamma}_{-j}\right)$ are such that ω is indifferent over her participation choices, she chooses to contract with j.

The proof is now developed by contradiction. Suppose that $(\tilde{\gamma},\tilde{\lambda})$ is not an equilibrium in G^D. There hence exists a principal j and an incentive compatible mechanism for that principal $\tilde{\tilde{\gamma}}'_j$ such that:

$$V_j\left(\tilde{\tilde{\gamma}}'_j,\tilde{\gamma}_{-j},\tilde{\lambda}\left(\tilde{\tilde{\gamma}}'_j,\tilde{\gamma}_{-j}\right)\right) > V_j\left(\tilde{\gamma}_j,\tilde{\gamma}_{-j},\tilde{\lambda}\left(\tilde{\gamma}_j,\tilde{\gamma}_{-j}\right)\right)=V_j. \tag{11}$$

Observe that given the definition of $\tilde{\lambda}$, any payoff available to principal j following a deviation to some direct mechanism can be attained through the incentive compatible mechanism $\tilde{\tilde{\gamma}}'_j$. As a consequence, we restrict attention to incentive compatible mechanisms in the inequality (11).

Let us now look back at the game G^{Γ}. For every principal j, let $\phi_j : M_j \to \Omega$ be a surjective mapping that maps any message $m_j \in M_j$ into a type ω. Since $\sharp M_j \geq \sharp\Omega$ for each j, it is always possible to construct such a mapping. Then, we construct the indirect mechanism γ'_j as follows:

$$\gamma'_j(m_j)=\tilde{\tilde{\gamma}}'_j\left(\phi_j(m_j)\right) \qquad \forall m_j \in M_j \setminus \{\emptyset\} \tag{12}$$

and

$$\gamma'_j(\emptyset) = \tilde{\tilde{\gamma}}'_j(\emptyset). \tag{13}$$

We first prove that for every $\omega \in \Omega$, if $\tilde{a}_j\left(\tilde{\tilde{\gamma}}'_j, \tilde{\gamma}_{-j}; \omega\right) = Y$ and the agent communicates truthfully with principal j in G^D, then there exists a strategy $\lambda' = (m', a')$ such that it is a best reply to choose:

$$\begin{cases} a'_j\left(\gamma'_j, \gamma_{-j}; \omega\right) &= Y \\ m'_j\left(\gamma'_j, \gamma_{-j}; \omega\right) &= \phi_j^{-1}(\omega). \end{cases} \tag{14}$$

with respect to principal j's proposal.

In other words, we want to prove that if:

$$U\left(\tilde{\tilde{\gamma}}'_j(\omega), \tilde{\gamma}_{-j}(\emptyset), \omega\right) \geq U\left(\tilde{\tilde{\gamma}}'_j(\emptyset), \tilde{\gamma}_{-j}(\bar{\omega}), \omega\right) \quad \text{with } \bar{\omega} \neq \omega \tag{15}$$

then, it is also true that:

$$U\left(\gamma'_j(m_j), \gamma_{-j}(\emptyset), \omega\right) \geq U\left(\gamma'_j(\emptyset), \gamma_{-j}(m'_{-j}(\gamma'_j, \gamma_{-j}, \omega)), \omega\right) \tag{16}$$

Assume not. Hence, assume that in the game G^Γ type ω strictly prefers to reject principal j's proposal and accept any of the contracts other than j, i.e.

$$U\left(\gamma'_j(m_j), \gamma_{-j}(\emptyset), \omega\right) < U\left(\gamma'_j(\emptyset), \gamma_{-j}(m'_{-j}(\gamma'_j, \gamma_{-j}, \omega)), \omega\right) \tag{17}$$

By construction,

$$U\left(\gamma'_j(m_j), \gamma_{-j}(\emptyset), \omega\right) = U\left(\tilde{\tilde{\gamma}}'_j(\omega), \tilde{\gamma}_{-j}(\emptyset), \omega\right) \tag{18}$$

and

$$U\left(\gamma'_j(\emptyset), \gamma_{-j}(m'_{-j}), \omega\right) = U\left(\tilde{\tilde{\gamma}}'_j(\emptyset), \gamma_{-j}(m'_{-j}), \omega\right) \tag{19}$$

with $m'_{-j} \equiv m'_{-j}(\gamma'_j, \gamma_{-j}, \omega) \in \hat{M}_{-j}(\gamma_{-j}, \omega)$ being any (array of) message that type ω may optimally send to principals other than j having accepted one of their offers.

Substituting in equation (17) the expressions obtained in (18) and (19), we get:

$$U\left(\tilde{\tilde{\gamma}}'_j(\omega), \tilde{\gamma}_{-j}(\emptyset), \omega\right) < U\left(\tilde{\tilde{\gamma}}'_j(\emptyset), \tilde{\gamma}_{-j}(\phi'^{-1}_{-j}(\omega)), \omega\right) \tag{20}$$

i.e. accepting the proposal of principal j and being truthful is not a best reply in G^D, either. A contradiction.

For those types who do not participate with principal j in G^D, if $\tilde{a}\left(\tilde{\tilde{\gamma}}'_j, \tilde{\gamma}_{-j}; \omega\right) = N$ then $a'\left(\gamma'_j, \gamma_{-j}; \omega\right) = a\left(\gamma'_j, \gamma_{-j}(m_{-j}); \omega\right) = N$.

Hence, λ' is a best reply to (γ'_j, γ_{-j}) in G^Γ, which implies that:

$$V_j\left(\gamma_j', \gamma_{-j}, \lambda'\left(\gamma_j', \gamma_{-j}\right)\right) = V_j\left(\tilde{\tilde{\gamma}}_j', \tilde{\gamma}_{-j}, \tilde{\lambda}\left(\tilde{\tilde{\gamma}}_j', \tilde{\gamma}_{-j}\right)\right) > V_j. \tag{21}$$

A contradiction. ■

Theorem 1 shows that an equilibrium outcome of a general mechanism game can be supported as an equilibrium in game with direct (incentive compatible) mechanisms. A crucial requirement for such result, is that the equilibrium of the general mechanism game be strongly robust, that is independent on a specific system of players' beliefs. When we try to extend Theorem 1 to every equilibrium in G^{Γ} relaxing strong robustness, difficulties arise. Let us qualify our statement by means of a very simple game with two principals and one agent, and two states of nature.

Example 1 *Let $I = 1$, $J = 2$ and $\Omega = \{\omega^1, \omega^2\}$. Let the decision set of Principal 1 ($P1$, henceforth) be given by $X = \{x_{11}, x_{12}, x_{13}\}$ and the decision set of Principal 2 ($P2$, henceforth) be given by $Y = \{y_1, y_2\}$.*

Let the agents' types be equally likely. We model a game with exclusive contracting, hence every type of agent decides on acceptance/rejection of a single one out of all contractual proposals and sends messages to the principal she has chosen to negotiate with. In this game, the set of participation decisions of the agent is given by $\{YN, NY, NN\}$, with for example YN denoting the decision to accept $P1$'s offer and decline $P2$'s one, and so on.

If the agent's type is ω^1, let the payoff matrices be given below.

- *If the participation decision is YN*

	y_1	y_2
x_{11}	$(-5,0,4)$	$(-5,0,4)$
x_{12}	$(6,0,4)$	$(6,0,4)$
x_{13}	$(5,0,4)$	$(5,0,4)$

- *if the participation decision is NY*

	y_1	y_2
x_{11}	$(0,1,3)$	$(0,2,3)$
x_{12}	$(0,1,3)$	$(0,2,3)$
x_{13}	$(0,1,3)$	$(0,2,3)$

in which every array represents the payoff of $P1$, that of $P2$ and the utility of the agent, respectively.

Similarly, if the agent's type is ω^2, let the payoff matrices be given by:

- *if the participation decision is YN*

	y_1	y_2
x_{11}	$(5,0,4)$	$(5,0,4)$
x_{12}	$(1,0,4)$	$(1,0,4)$
x_{13}	$(-5,0,4)$	$(-5,0,4)$

- *if the participation decision is NY*

	y_1	y_2
x_{11}	$(0,1,4)$	$(0,2,4)$
x_{12}	$(0,1,4)$	$(0,2,4)$
x_{13}	$(0,1,4)$	$(0,2,4)$

If the agent rejects all offers, whatever her type is, every player gets a payoff of 0*. Observe that in the example, type* ω^2 *is always indifferent between participating with* $P1$ *or with* $P2$*, since she always gets a payoff of 4. In addition, when both types accept the offer of* $P1$*, also type* ω^1 *gets a payoff of 4.*

In this configuration, if the principals offer (direct) incentive compatible mechanisms and the agent reports truthfully and $P1$ *is able to reach a payoff of 5. To get to this outcome, it is enough for P1 to offer the following direct mechanism* $\tilde{\gamma}_1(\omega)$*:*

$$\tilde{\gamma}_1(\omega) = \begin{cases} x_{11} & \text{if} \quad YN \text{ and } \omega = \omega^2 \\ x_{13} & \text{if} \quad YN \text{ and } \omega = \omega^1 \\ \text{any allocation in } X & \text{if} \quad NY \end{cases}$$

Given $\tilde{\gamma}_1(\omega)$*, no matter what* $P2$ *will offer, it is optimal for the agent to report her type truthfully and accept the offer of* $P1$ *whatever her type is.* $P1$ *will reach a payoff of 5.*

Now, consider the indirect mechanism game G^Γ*, in which the message spaces for the agent are given by* $M_1 = M = \{m_1, m_2\}$ *and* $M_2 = \{m\}$*. It is always an optimal behavior for the agent to send the same message whatever her type. If the principal anticipates such a best reply, then his maximal payoff is 3. Hence, in the indirect mechanism game* G^Γ *there exists an equilibrium in which the principal gets a payoff of 3 and the agent may misrepresent her type. If the principal is restricted to offer direct incentive compatible mechanisms, it is impossible to support that as an equilibrium outcome in the game* G^D*.*

The example clarifies the implications of the restriction on beliefs that the principals put on agent's behavior, both in term of participation and/or truthful communication.[7] Imposing strong robustness at equilibrium allows to put restrictions on players' beliefs that avoid such kind of problems. Example 1 indeed violates Theorem 1, since the equilibrium outcome of the game G^Γ in which the principal gets a payoff of 3 is not strongly robust.

[7] Observe that we could make the example generic, weakening the agent's indifferences without altering the result. We chose a simple formulation to provide the intuition.

Theorem 1 has been written for the case in which principals play pure strategy and the agent responds with a pure strategy in participation and communication, the results directly extend to the case in which the agent mixes over participation and/or communication.

Corollary 1 *Theorem 1 remains valid if we consider mixed strategy equilibria.*

Proof. See Appendix. ■

Assume that the initial equilibrium in G^{Γ} is such that the principals offer an array of mechanisms $(\gamma_1, \ldots, \gamma_J)$ and the agent replies with some mixed strategy over participation and messages, λ. Provided the exclusive framework, every principal j can replicate the agent's randomizations over messages and participation, by means of probability distributions over his decision rules, and can offer to the agent a menu of probability distributions among which to choose. By doing so, the initial uncertainty over the agent's actions is incorporated in the principals' menu, among which each type of agent chooses deterministically her payoff-maximizing element.

4 The Multi-Agent Case

In this section, we argue that in competing mechanism models in which exclusivity clauses are imposed from the outset the restriction to direct truthful mechanisms involves a loss of generality. We present here an example of a two-principals, two-agents case exhibiting an equilibrium outcome in the game in which indirect communication mechanisms are allowed that cannot be sustained in the simpler situation in which principals are restricted to direct truthful mechanisms.

The first example in this spirit is due to Peck (1997), whose result *"relies on firm 2 randomizing over which mechanism it selects. Indeed, if all firms are deterministically selecting mechanisms, then for every given equilibrium there is a direct revelation mechanism, one for each firm, which yields an equilibrium in which agents' arrival probabilities are unaffected, outcomes are unaffected, and all mechanisms are incentive compatible"(p.—, Peck (1997)).*

Without relying on stochastic mechanisms, we propose an example in which the principals offer deterministic mechanisms and the revelation principle fails to apply. The intuition is very neat: communication by means of (private) messages allows principal 1 to specify his out-of-equilibrium behavior as a function of participation decisions and messages, by doing so Principal 1 triggers his opponent in case of deviations and controls the agents' participation decisions. Hence, he can profitably use messages to increase his payoff.

Example 2 *Let $I = J = 2$ and $\Omega^1 = \Omega^2 = \{\omega\}$. In addition, take $X_1 = \{x_{11}, x_{12}\}$ and $X_2 = \{x_{21}, x_{22}\}$ be the decision sets of principal 1 and 2, respectively. Each of the agents decides on accepting or rejecting the allocations proposed by each principal, denoted by $a = \{Y, N\}$. The game is exclusive, hence each agent can accept at most one of the proposals. The payoffs in the following tables represent the utilities of the two principals and those of the two agents.*

The example is constructed in such a way that if both agents do not participate in the game, then every player gets a payoff of 0.

Let the payoff matrix be given by:

	YN			*NY*			*NN*		
YN		x_{21}	x_{22}		x_{21}	x_{22}		x_{21}	x_{22}
	x_{11}	$(4,0,1,4)$	$(4,0,1,4)$	x_{11}	$(10,5,5,4)$	$(5,10,2,3)$	x_{11}	$(4,0,2,0)$	$(4,0,2,0)$
	x_{12}	$(0,0,8,1)$	$(0,0,8,1)$	x_{12}	$(0,-10,0,0)$	$(0,0,0,0)$	x_{12}	$(0,0,-2,0)$	$(0,0,-2,0)$
NY		x_{21}	x_{22}		x_{21}	x_{22}		x_{21}	x_{22}
	x_{11}	$(1,0,3,0)$	$(1,6,2,6)$	x_{11}	$(0,1,2,1)$	$(0,1,0,0)$	x_{11}	$(0,1,-2,0)$	$(0,1,1,0)$
	x_{12}	$(0,0,3,0)$	$(0,5,6,0)$	x_{12}	$(0,1,2,1)$	$(0,1,0,0)$	x_{12}	$(0,1,-2,0)$	$(0,1,1,0)$
NN		x_{21}	x_{22}		x_{21}	x_{22}		x_{21}	x_{22}
	x_{11}	$(1,0,0,1)$	$(1,0,0,1)$	x_{11}	$(0,1,0,0)$	$(0,1,0,0)$	x_{11}	$(0,0,0,0)$	$(0,0,0,0)$
	x_{12}	$(0,0,0,1)$	$(0,0,0,1)$	x_{12}	$(0,1,0,0)$	$(0,1,0,0)$	x_{12}	$(0,0,0,0)$	$(0,0,0,0)$

in which agents decide on participation (in the external box) and principals choose actions in the internal 2x2 cells. For every agent, the possible choices are simply given by $\{YN, NY, NN\}$, with YN denoting the decision to accept the offer of $P1$ and reject the offer of $P2$ and so on.[8]

The payoff matrix complies with some typical economic features:

1. *If an agent does not participate with any of the principals, she gets a payoff of 0, independently of what the other players do.*

2. *If no agent participates with a given principal, that principal gets a payoff of 0.*

3. *The decisions of a principal whose proposals are never accepted by any agent, neither affect the payoff of the other principal nor of the agents.*

We first consider the situation in which principals are allowed to offer arbitrary communication mechanisms. For the sake of the example, let $M_1^i = M = \{m_1, m_2\}$ for $i = 1,2$ be the set of messages that agents can use with $P1$ and let $P2$ only use a trivial message space, i.e. $M_2^i = \{m\}$ for $i = 1,2$. A mechanism proposed by a principal associates to any array of messages and participation choices a single decision.

Specifically, we argue that the profile of decisions:

- *$P1$ offers x_{11} and $P2$ offers x_{21} with probability one,*

[8] Exclusivity implies that YY is not a feasible choice for any agent.

• *A*1 *accepts the offer of P*1 *and A*2 *accepts the offer of P*2

and the associated payoff array $(10,5,5,4)$ *can be supported at equilibrium in our multiple-principal multiple-agent game.*

Let P1 play the mechanism:

$$\gamma_1(m_1^1, m_1^2, a) = \begin{cases} x_{11} & \text{if } m_1 \text{ from either } A1, A2 \text{ or both, and if } A1 \text{ accepts his offer} \\ x_{12} & \text{otherwise} \end{cases}$$

*and let P*2 *play the mechanism* γ_2*: "for every profile of participation decisions and messages,* x_{21} *is selected".*

Given these mechanisms, agents play a continuation game over messages and participation decisions whose payoffs are represented in the following matrix

	YNm_1	YNm_2	NYm	NN
YNm_1	$(1,4)$	$(1,4)$	$(5,4)$	$(2,0)$
YNm_2	$(1,4)$	$(8,1)$	$(0,0)$	$(-2,0)$
NYm	$(3,0)$	$(3,0)$	$(2,1)$	$(-2,0)$
NN	$(0,1)$	$(0,1)$	$(0,0)$	$(0,0)$

*where the first payoff is that of A*1*, the second one is that of A*2*, and the action* YNm_1 *stands for the choice of participating with P*1 *sending him the message* m_1 *and so on. It is straightforward to check that the unique equilibrium in the game among agents prescribes to A*1 *to participate with P*1 *sending him the message* m_1 *and to A*2 *to participate with P*2*, to reach the payoff* $(5,4)$*. Given the agents' choices, the principals will offer* (x_{11}, x_{21}) *reaching the payoffs* $(10,5)$*, respectively.*

*To show that there are no unilaterally profitable deviations for principals, it is enough to look at P*2*, since P*1 *is earning his maximal payoff of* 10*. For P*2 *to have a profitable deviation, he must offer a mechanism such that* x_{22} *and* x_{11} *are implemented, A*1 *accepts the offer of P*1*, and A*2 *the offer of P*2*. First, let us assume that P*2 *deviates toward the single offer* x_{22}*. However, following such a deviation, the continuation game among agents induces the following payoff matrix:*

	YNm_1	YNm_2	NY	NN
YNm_1	$(1,4)$	$(1,4)$	$(2,3)$	$(2,0)$
YNm_2	$(1,4)$	$(8,1)$	$(0,0)$	$(-2,0)$
NY	$(6,0)$	$(6,0)$	$(0,0)$	$(1,0)$
NN	$(0,1)$	$(0,1)$	$(0,0)$	$(0,0)$

in which the outcome $(2,3)$ *can never implemented at equilibrium. Hence, the considered deviation is not profitable for P*2*. Our argument can be generalized to any*

stochastic mechanism. If A1 participates with P1 and sends him the message m_1, then A2 has no reason to participate with P2 and send him message such that x_{22} would be implemented. Considering the game we built, there is no possibility for P2 to reach, by means of randomization over X_2, a payoff which is higher than 5. In order to do that, P2 should propose to A2 a random mechanism such that when A1 participates with P1 sending m_1, she gets (at least) a payoff of 4. This is not possible. Hence, lotteries are not helpful.

Let us now consider the situation in which principals are restricted to direct truthful mechanisms. In this simple setting, such mechanisms allow every principal to make his decision contingent to any array of participation choices that agents are taking with him.

We claim that the same profile of decisions cannot be supported at equilibrium in this direct mechanism game. Let us consider P1's mechanism: in this example, x_{11} is a strictly dominant strategy for P1 whenever A1 participates with him (no matter what will be the decision of A2) and whenever A2 participates with him (whatever the choice of A1), otherwise is weakly dominant. Hence, we can suppose that P1 will offer x_{11} at equilibrium for every possible participation decision of the agents. Given this offer, it is not a best reply for P2 to offer x_{21}, P2 can do strictly better offering x_{22}. In that case, the continuation game among agents would be:

	YN	*NY*	*NN*
YN	$(1,4)$	$(2,3)$	$(2,0)$
NY	$(2,6)$	$(0,0)$	$(1,0)$
NN	$(0,1)$	$(0,0)$	$(0,0)$

which exhibits only one Nash equilibrium, in which A1 participates with P2 and A2 with P1. In this case, P2 can reach a payoff of 6, which makes the deviation profitable.

5 Conclusion

This paper contributes to the analysis of multi-principal multi-agent games of incomplete information, when every agent can exclusively negotiate with one principal only. When considering competing mechanism games with only one agent, the strong robustness property provides a rationale for restricting to truth-telling direct mechanisms in economic applications.

Games with several agents remain problematic even if we restrict the analysis to pure strategy strongly robust equilibria. With several agents a principal can create coordination between agents by appropriately choosing his mechanism, and deter treats that would be credible otherwise. Hence, it may be profitable for a principal to propose indirect mechanisms rather than direct ones.

6 Appendix

Proof of Lemma 1 We extend the result presented in Theorem 1 to mixed strategy equilibria. Let type ω participate with principal j with probability $p\left(a_j = Y \left| \gamma_j, \gamma_{-j}, \omega \right.\right) \in (0,1)$, and once principal j's offer has been accepted, send the message m_j with probability $p\left(m_j \left| a_j = Y, \gamma_j, \gamma_{-j}, \omega \right.\right) \in (0,1)$, with $m_j \neq \emptyset$.

Let $\lambda_j : \Gamma \times \Omega \to \left\{\hat{M}_j \times Y\right\} \times \{\emptyset \times N\}$ be the joint probability of the pair (m_j, a_j) relative to a given principal j. The participation and the communication choices are not independent, hence the joint probability can be expressed as the product of the conditional probability of sending the message m_j given the action a_j, times the marginal on the participation decision a_j. Let $\delta_j\left(m_j | a_j\right)$ be the conditional probability that type ω sends the message m_j to principal j, having received the array of offers (γ_j, γ_{-j}) and having chosen a_j, i.e. $\delta_j\left(m_j \left| a_j \right.\right) \equiv pr\left(m_j \left| a_j, \gamma_j, \gamma_{-j}, \omega \right.\right)$, and let $\delta(a_j) \equiv pr\left(a_j \left| \gamma_j, \gamma_{-j}, \omega \right.\right)$ be the marginal probability over the participation decision, then

$$\lambda_j\left(m_j, a_j \left| \gamma, \omega \right.\right) = \delta_j\left(m_j | a_j\right) \delta(a_j) \tag{22}$$

Let Λ_j be the set of all such probability distributions over messages and participation decisions.

Given any mechanisms γ_j, we can define $\gamma_j\left(c_j \left| m_j, a_j \right.\right)$ the (conditional) probability that principal j implements the incentive scheme c_j given he receives the message m_j and observes the choice a_j. Then, taking λ_j as given, we can construct the unconditional probability of the incentive scheme c_j for principal j as follows: $\forall \lambda_j \in \Lambda_j$,

$$\gamma_j\left(c_j, \lambda_j\right) = \int_{\omega \in \Omega} \int_{m_j \in M_j} \int_{a_j \in A_j} \gamma_j\left(c_j \left| m_j, a_j \right.\right) \lambda_j\left(m_j, a_j \left| \gamma, \omega \right.\right) f(\omega)\, da_j\, dm_j\, d\omega,$$

Finally, let $\gamma_j(\lambda_j)$ be the probability distribution over all incentive schemes of principal j in C_j induced by the mixed strategy λ_j:

$$\gamma_j(\lambda_j) = \left(\gamma_j\left(c_j, \lambda_j\right)\right)_{c_j \in C_j} \tag{23}$$

and let γ_j be the set of such probability distributions, so that $\gamma_j : \Lambda_j \to \Delta(C_j)$:

$$\gamma_j = \left(\gamma_j\left(\lambda_j\right)\right)_{\lambda_j \in \Lambda_j}. \tag{24}$$

Now, if type ω plays according to some $\lambda_j \in \Lambda_j$ at equilibrium, principal j can offer the set of probability distributions γ_j in (24) and commit to such decisions according to the agent's choices. Then, it is optimal for the agent to choose with probability one the probability distribution $\gamma_j\left(\lambda_j\right)$ within the set (or menu) γ_j and then play accordingly. Thus, every principal $j \in \mathcal{J}$ can equivalently offer the menus γ_j as in (24), and let the agent choose deterministically the relevant probability distribution in each menu.

Given the definition of λ_j in (22), we can write:

$$\gamma_j\left(c_j,\delta_j\right)=\int_{\omega\in\Omega}\int_{m_j\in M_j}\gamma_j\left(c_j\left|m_j,a_j\right.\right)\delta_j\left(m_j\left|a_j\right.\right)\delta_j\left(a_j\right)f(\omega)\;\;dm_j\,d\omega, \tag{25}$$

that is the probability of implementing the incentive scheme c_j conditional on the participation choice a_j, given the distribution over messages δ_j.[9]

Let us denote $\gamma_j(\delta_j)$ the probability over all incentive schemes in C_j given the conditional distribution δ_j, *i.e.*

$$\gamma_j(\delta_j)=\left(\gamma_j\left(c_j,\delta_j\right)\right)_{c_j\in C_j} \tag{26}$$

In this new formulation of the game, the agent's message choices are deterministic and we can construct direct mechanisms along the same lines of Theorem 1 in the text. Whenever $a_j\left(\gamma_j,\gamma_{-j},\omega\right)=Y$, we set $\tilde{\gamma}_j\left(\omega\right)=\gamma_j\left(\delta_j(m_j|Y)\right)$. If $a\left(\gamma_j,\gamma_{-j},\omega\right)=N$, we set

$$\tilde{\gamma}_j\left(\omega\right)=\gamma_j\left(\hat{\delta}_j\left(\gamma_j,\omega\right)\right),$$

with $\hat{\delta}_j\left(\gamma_j,\omega\right)$ being any probability distribution over the set $\hat{M}_j\left(\gamma_j,\omega\right)$, which is the set of optimal messages for type ω when she accepts principal j's proposal as defined in (6).

With such a construction and dealing with a strongly robust equilibrium in G^{Γ}, we can reconstruct the strategy of the agent $\tilde{\lambda}$ in the direct mechanism game G^D as an incentive compatible strategy. Furthermore, we can without loss of generality state that the agent plays a pure strategy at the deviation stage, too. Then, the arguments presented in Theorem 1 in the text apply and we can conclude that there do not exist profitable deviations in G^D.

References

BIGLAISER, G., AND C. MEZZETTI (1993): "Principals competing for an agent in the presence of adverse selection and moral hazard," *Journal of Economic Theory*, 61(2), 302–330.

CAILLAUD, B., B. JULLIEN, AND P. PICARD (1995): "Competing vertical structures: precommitment and renegotiation," *Econometrica*, pp. 621–646.

EPSTEIN, L. G., AND M. PETERS (1999): "A revelation principle for competing mechanisms," *Journal of Economic Theory*, 88(1), 119–160.

[9]Given our construction $\gamma_j\left(c_j,\delta_j\right)$ is a non-trivial measure only when the agent accepts principals j's contract, i.e. $a_j(\gamma_j,\gamma_{-j},\omega)=Y$, otherwise $\delta_j\left(m_j\left|a_j\right.\right)=0$ for every $m_j\neq\emptyset$.

GAL-OR, E. (1991): "A common agency with incomplete information," *RAND Journal of Economics*, 22(2), 274–286.

HAN, S. (2007): "Strongly robust equilibrium and competing-mechanism games," *Journal of Economic Theory*, 137(1), 610–626.

MARTIMORT, D., AND S. PICCOLO (2010): "The Strategic Value of Quantity Forcing Contracts," *American Economic Journal: Microeconomics*, 2(1), 204–229.

MARTIMORT, D., AND L. A. STOLE (2002): "The revelation and delegation principles in common agency games," *Econometrica*, 70(4), 1659–1673.

MCAFEE, P. (1993): "Mechanism design by competing sellers," *Econometrica*, 61, 1281–1312.

MOLDOVANU, B., A. SELA, AND X. SHI (2008): "Competing auctions with endogenous quantities," *Journal of Economic Theory*, 141(1), 1–27.

PECK, J. (1997): "A note on competing mechanisms and the revelation principle," mimeo, Ohio State University.

PETERS, M. (2001): "Common Agency and the Revelation Principle," *Econometrica*, 69(5), 1349–1372.

PETERS, M., AND S. SEVERINOV (1997): "Competition among sellers who offer auctions instead of prices," *Journal of Economic Theory*, 75(1), 141–179.

PIASER, G. (2007): *International Taxation Handbook: Policy, Practice, Standards and Regulation*chap. Labor mobility and income tax competition. CIMA Publishing, Oxford.

ROCHET, J.-C., AND L. A. STOLE (2002): "Nonlinear pricing with random participation," *Review of Economic Studies*, 69(1), 277–311.

ROTHSCHILD, M., AND J. STIGLITZ (1976): "Equilibrium in Competitive Insurance Markets," *Quarterly Journal of Economics*, 90, 629–649.