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Imitation and Efficient Contagion^{*}

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Abstract

This paper is about the diffusion of cooperation in an infinite population of networked individuals repeatedly playing a Prisoner's Dilemma. We formulate conditions on payoffs and network structure such that, starting from an initial seed group, imitative learning results in the overall adoption of cooperation — efficient contagion. Key to this result is the pattern of interaction among players who are at the same distance from the initial seed group. We find that the more these agents interact among themselves rather than with players who are closer to or further away from the initial seed group, the easier it is for efficient contagion to take place. We highlight the importance of cycles for efficient contagion, and show that the presence of critical edges prevents it. We also find that networks organized as dense clusters sparsely connected to one another tend to resist efficient contagion. Finally, we find that the likelihood of efficient contagion in a network increases when information neighborhoods extend beyond interaction neighborhoods.

JEL classification: C7; D8

Keywords: networks; imitation; contagion

1 Introduction

Most economic and social interactions take place locally, within restricted subsets of a larger population. Interactions occur among relatives, friends, colleagues, business partners, neighbors in geographic or cultural space, etc. The set of pairwise relationships between agents constitutes a network, within which knowledge, information and other resources

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can be shared, exchanged and produced, new behaviors can be learned and tested, and economic value in general is created.¹

When it comes to the diffusion of new behavior or technology, the existence of a fixed local interaction structure implies that diffusion takes place gradually. Starting from an initial group of individuals, diffusion progresses through the population like an epidemic, as contacts occur between infected and susceptible neighbors at increasing distance from the first infected individuals. There is however a considerable difference with the spread of epidemics. When exposure is all that is required, the speed and extent of contagion increase when edges are as “global” as possible, connecting players randomly across the network with as few local cycles as possible. By contrast, when the decision to adopt is based on payoff comparisons, the extent and speed of contagion depend on much finer structural properties of the iterated neighborhoods of the initial seed group.

The focus of the paper is on the Prisoner’s Dilemma. In the setting considered, in any period, all agents play simultaneously the same, one-shot, two-by-two symmetric game with each of their neighbors in the network. In addition to playing, agents are learning. This they do through imitation, adopting the strategy of their best neighbor. Oftentimes in the literature agents imitate the average-payoff-maximizing strategy — instead, we simply assume that agents imitate the strategy with largest payoff, a behavior which research in psychology has found to be common. We formulate results on imitative contagion by efficient behavior in arbitrary regular networks.² These results exploit the structure of the iterated neighborhoods of a finite seed group of efficient players. The intuition is as follows. Cooperation generates higher payoffs than defection if players are segregated, i.e., when players interact mostly with players of their kind at any stage of contagion (and thus the cooperators are relatively well protected from exploitation by defectors). This takes the form of a constraint on the number of interactions with players at the same distance from the initial seed group. We show that this constraint more generally relates to the presence of cycles in the network. Trees, i.e., networks without cycles, do not permit efficient contagion. Likewise, the existence of critical edges prevents contagion — so not only should there be cycles, but each agent should be part of at least one of them. We also find that a set of individuals which is only sparsely connected to the rest of the network will resist efficient contagion. This could be bad for social networks, as they often display the characteristic features of a small world — sparse networks consisting of dense clusters of individuals connected by a few clique-spanning ties. Regarding the likelihood of contagion in a given network, we contend that longer cycles are more sensitive to the choice of the initial seed group than shorter ones. Therefore, even if short cycles are by no means necessary, it is likely that they are more robust to the choice of the seed group. As a consequence, although clustering (neighbor commonality) is also not necessary, it can

¹Different neighborhoods can serve different purposes: typically, information could be gathered from a wider neighborhood than the one over which interaction takes place. We come back to this in Section 3.2.2.

²Networks in which all nodes have the same degree, i.e., the same neighborhood size.

support contagion. Lastly, distinguishing the interaction and information neighborhoods in the vein of recent research (see the discussion below), we find that iterated neighborhoods should grow fast for efficient behavior to spread, a situation which is perfectly compatible with the absence of cycles.

This paper is obviously related to a large set of references on imitation and local interaction in two-by-two symmetric games. We only review this literature very selectively, emphasizing differences from or similarities with our findings as these are discussed. The next section presents the model and introduces the notion of shell-wise contagion. We then move on to the results, discussing first the condition on iterated neighborhoods, and then examining its more general network implications, before finally distinguishing the interaction and information neighborhoods.

2 The model

2.1 Network definitions

We consider a countably infinite set S of nodes (players) and a set g of edges (links) between unordered pairs in S . The pair (S, g) defines a *network*. Letting ij denote the link between i and j , $ij \in g$ indicates that i and j are linked in the network (S, g) . The *neighborhood* of i consists of the players to whom i is directly linked, and is denoted $N_i = \{j \neq i : ij \in g\}$. The size of the neighborhood of i is the number of links held by i , also called its *degree*, $n_i = \#N_i$. Our focus is on regular networks, i.e., networks in which degree is identical across nodes: $n_i = n, \forall i \in S$. The neighborhood of a set of players is the union of neighborhoods of the players in that set.

A *path* is a sequence of links $i_1i_2, \dots, i_pi_{p+1}$ between unordered pairs in S such that $i_li_{l+1} \in g$ and $i_l \neq i_{l+1}$ for all $l = 1, \dots, p$. The length of a path is the number of edges it contains. A *cycle* of length p is a path of length p such that the first and last nodes coincide: $i_1 = i_{p+1}$.

The number of links in the shortest path between i and j , denoted $d_{i,j}$, is the (geodesic) *distance* between i and j in the network (S, g) . We have $d_{i,j} = 1, \forall j \in N_i$. We suppose that $d_{i,j} < \infty, \forall i, j \in S$, i.e., the network is *connected*.

Consider now an arbitrary finite group $X \subset S$ of players. The set X will play the role of the seed group, i.e., the set of players starting the epidemic. The distance between X and player $i \notin X$ is $d_{X,i} = \min_{j \in X} \{d_{i,j}\}$. We write N_X^d for the d -iterated neighborhood of X , $N_X^d = \{i \in S : d_{X,i} = d\}$, with $d \geq 0$ and $N_X^0 = X$. We define the d -*shell* of X as the set of players who are *within* distance d of X , augmented with X , as

$$X_d = \bigcup_{0 \leq l \leq d} N_X^l. \quad (1)$$

The d -*shell* of X is the union of X and its iterated neighborhoods up to distance d .

We finally introduce a notation specific to the problem we consider. Let

$$N_{i,X}^k = \{j \in N_i : d_{X,j} = d_{X,i} + k\}, \quad (2)$$

with $k \in \{-1, 0, +1\}$, be the neighbors of i whose distance to X equals i 's distance to X augmented with k . We can now define, for a given finite set X and any $i \in S$, the *in-degree* of i as $n_{i,X}^{-1} = \#N_{i,X}^{-1}$, i.e., the number of neighbors of i who belong to the previous iterated neighborhood of X . Similarly we define the *out-degree* of i as $n_{i,X}^{+1} = \#N_{i,X}^{+1}$, and the *intra-degree* of i as $n_{i,X}^0 = n - n_{i,X}^{-1} - n_{i,X}^{+1} = \#N_{i,X}^0$. In-, and out- and intra-degree are all with respect to X , which this notation makes clear; they play a particularly important role in the discussion of contagion below.

Figure 1: Neighborhood structure: in-, out- and intra-degree of i with respect to X ; in this example, we have $(n_{i,X}^{-1}, n_{i,X}^0, n_{i,X}^{+1}) = (1, 1, 2)$ and $n = 4$.

Figure 1 provides an illustration for the notations just introduced. Assuming that $i \in N_X^d$, we see that i 's neighborhood overlaps with three distinct iterated neighborhoods of X , yielding in-, out- and intra-degree of 1, 2, and 1. By construction, each player in a given iterated neighborhood of X always has at least one link to the previous iterated neighborhood, i.e., $n_{i,X}^{-1} \geq 1, \forall i \in S - X$. It is however possible that i has no link to the next iterated neighborhood of X .

2.2 The game and learning

Assume players repeatedly engage in the symmetric two-by-two Prisoner's Dilemma depicted in Table 1. We use a common parametrization: the cooperator pays a cost c for his opponent to receive a benefit $b > c > 0$. The defector pays no cost and distributes a small positive mutual penalty payoff $p < b$ with the additional assumption that $b > c + p$. Therefore, playing D is a dominant strategy, while C is the only symmetric and efficient strategy (in the sense that both players choosing C dominates any symmetric mix of play).

	C	D
C	$b - c$	$p - c$
D	b	p

Table 1: Payoffs to the row player in the Prisoner's Dilemma.

The *action* of player $i \in S$ is $\omega_i \in \{C, D\}$. A *configuration* is $\omega = (\omega_i)_{i \in S}$, the profile of actions adopted by all players in S . The *aggregate payoff* for player i is

$$P(i, \omega) = \sum_{j \in N_i} p(\omega_i, \omega_j) = \gamma_i(b - p) + np - nc \cdot \mathbf{1}_{\{\omega_i = C\}} \quad (3)$$

where $p(\omega_i, \omega_j)$ is the payoff from playing ω_i against ω_j , $\gamma_i = \#\{j \in N_i : \omega_j = C\}$ is the number of cooperators in i 's neighborhood and $\mathbf{1}_{\{\omega_i = C\}}$ equals 1 if $\omega_i = C$ and 0 otherwise. Aggregate payoff is the sum of payoffs collected by an agent from interacting with his n neighbors in the same two-by-two symmetric game, with the same action. At the end of each period, each agent chooses an action for the next period by imitating others whom he observes to be earning higher payoffs. Specifically, in each period, each agent observes his own payoff, and the payoff and action of each every other agent in his neighborhood. Agent i sticks or switches to C whenever the highest payoff in his augmented neighborhood, $N_i \cup \{i\}$, is achieved by a C -player. Otherwise, i sticks or switches to D . In case of a tie it is assumed C is chosen. So there is always a unique imitative best response for any player i to any configuration ω .

We assume that players conform strictly to this learning principle, without trembles or mutations, and thus a player whose action is also played by all his neighbors does not change behavior. This means the adoption of a new behavior requires direct exposure to this behavior, an assumption that is relaxed in Section 3.2.2 where we distinguish interaction and information neighborhoods. Formally, let $c_i \in \arg \max_{\{j \in N_i \cup \{i\} : \omega_j = C\}} P(j, \omega)$ denote a C -player collecting the largest payoff among the C -players in $N_i \cup \{i\}$ and let $d_i \in \arg \max_{\{j \in N_i \cup \{i\} : \omega_j = D\}} P(j, \omega)$ be a D -player getting the largest payoff among the D -players in $N_i \cup \{i\}$. When i observes only one type of behavior the current action is maintained. By contrast, when both behaviors are observable, the unique imitative best response of i to configuration ω is C if $P(d_i, \omega) \leq P(c_i, \omega)$, and it is D otherwise. A configuration ω' is said to be an imitative best response to configuration ω when each player in ω' plays a best response to ω .

It is worth commenting briefly on the assumption that agents imitate the most successful neighbor. In the Prisoner's Dilemma, if agents are myopic expected payoff maximizers, then the strictly dominant action D is chosen systematically. Payoff maximization however assumes that agents know a great deal about the game they play and how to play it, which is neither reasonable nor necessary. Economic agents are often unfamiliar with the set of actions they can choose from, and with the details of the strategic environment into which they are immersed. When information is incomplete but the performances and actions of others are observable, imitation is a natural behavior that economizes on

time and cognitive resources, and as put by Alós-Ferrer and Weidenholzer (2008:256), imitating the best player “...nicely captures some experimentally observed features of actual human behavior, as reported by the psychology literature.”. Mimetism on financial markets, conspicuous consumption and related forms of status-seeking behavior by individuals, best practice identification and benchmarking by firms, reference groups and role models in sociology all involve imitation. There is ample and strong experimental evidence that agents use imitation in decision making (see Pingle and Day 1996; Selten and Apesteguia 2005; Apesteguia et al. 2007; Offerman and Schotter 2009; Duersch et al. 2010) except in the case of complex situations with little information available (Bosch-Domènech and Vriend, 2003).

2.3 Shell-wise contagion

Under what conditions on network structure can an initial group of cooperative players start an epidemic which, through local imitation, eventually leads to cooperation being played by the entire population? To answer this question, we will consider a specific form of contagion, which we refer to as shell-wise contagion. In Section 2.1 we have defined the d -shell of a set of players X as the union of that set and all its iterated neighborhoods up to distance d . As we assume that in each time period, each agent adapts his behavior by imitating the best agent in his neighborhood without mutations (deterministic synchronous learning), the only agents likely to change behavior are those who have neighbors playing different actions. Assume that the agents in X do not change behavior (see the second remark below), the only susceptible players are thus in the neighborhood of X . We will focus on the conditions under which they all change from D to C . Once this has happened, the only susceptible players are the ones we just considered and their D -playing neighbors in the neighborhood of the neighborhood of X . Again we focus on the conversion of those agents as an entire group, making sure that the agents who changed from D to C in the first period do not switch back, and so on.

Formally, we say that a finite-sized set of players X seeds efficient shell-wise contagion in a network (S, g) when, starting with cooperators in X and defectors elsewhere in period 0, all and only agents in the d -shell of X cooperate at the end of period d , for any $d \geq 0$. Shell-wise contagion is thus a particular form of contagion in which successive iterated-neighborhoods convert until eventually the whole population. Put in a more precise way, X seeds efficient shell-wise contagion when the imitative best response to the configuration in which all and only players in X_d play C is the configuration in which all and only players in X_{d+1} play C , for all $d \geq 0$. Provided we can find a seed group X , once its members play C , in the next period the neighbors of X switch to C , in the next period their neighbors switch, and so on, one iterated neighborhood of X after the previous one and no player ever switching back to D , until the whole population eventually cooperates.

A few remarks are in order. First, shell-wise contagion is contingent upon the particular choice of X . Different starting sets will generate different sequences of iterated neighborhoods with different outcomes in terms of contagion. Ideally we would like to be able to

characterize networks in which cooperation spreads for any starting set X . A more detailed discussion of this is provided in Section 3.1.2.

A second remark is about X itself. It might be reasonable to conceive of X as fairly small, but not so small that it penalizes cooperation. Specifically, X should not be smaller than the union of a player and his neighborhood to ensure that any cooperative player in X sees another cooperative player in X generating the largest possible cooperative payoff of $n(b - c)$. In the next iterated neighborhoods of X this will happen by construction. So we simply assume that in the starting set X , each player is connected to (at least) one player whose entire neighborhood is included in X .

A final remark is about the type of contagion we consider. We consider a situation in which contagion of behavior is monotone. Once an agent voluntarily switches to cooperation, this agent will not switch back at a later point in time. Another, less demanding form of contagion would be the continuing growth of the initial C -cluster by the addition of at least one player in each period. Unlike shell-wise contagion, which ensures conversion of the entire population, such a form of contagion only ensures that an infinite-sized cluster forms, which could possibly coexist with an infinite-sized D -cluster. Such a general form of contagion is considered by Morris (2000), in the case of a Coordination Game played under myopic best reply. At each period, the entire neighborhood of the current C -cluster could convert to C , but it is also possible that only one player in that neighborhood converts to C . The sole requirement made by Morris is that this process goes on until only a finite-sized set of D -players remains. The difficulty with this type of contagion is that the spread of efficient behavior cannot be described anymore in terms of iterated neighborhoods of the initial seed group.

The present paper relies on synchronous learning and shell-wise contagion, acknowledging that conditions for other (more general) forms of contagion (as in Morris 2000) are much harder to characterize. It is worth mentioning a recent, alternative approach which uses mean field theory. Lopez-Pintado (2008) and Jackson and Yariv (2007) focus on asymptotic forms of (monotone) contagion and ask whether a small proportion of adopters of a new behavior can contaminate an entire population. Lopez-Pintado considers the Coordination Game, and identifies a risk-dominance threshold, which depends on the degree distribution, below which contagion occurs — the paper is very close in spirit but technically different from Morris (2000). Jackson and Yariv’s focus, by contrast, is on the smallest proportion of initial adopters (the seed group) able to tip the whole population (their game is a different, more general one). Due to the mean field approximation, network structure in these papers boils down to a degree distribution, thus leaving aside many structural properties likely to influence contagion dynamics in a network.

Synchronous deterministic updating is the focus of cellular automaton research. Probably the earliest relevant contribution is Nowak and May’s (1992) computational study of imitation and local interaction. The (fairly crude) study identifies a radical change: cooperation is massively present when the cooperation cost is low, but disappears above a threshold value. This early paper has spurred a considerable amount of (mostly computa-

tional) research in which the effects on cooperation of the payoff structure of the game, the specifics of the learning mechanism (synchronicity or asynchronicity) and the properties of the network of edges between players are studied in great detail (see for instance the (121 pages!) survey by Szabó and Fáth 2007). More recently, Durieu et al. (2007) use results on dominating sets to provide a complete characterization of step-by-step contagion in finite regular networks, and point at the possibility of non-homogeneous equilibria as well as cycles. Their study is representative of studies on threshold cellular automata, for which it is known that in the limit either a fixed point or a cycle of length two is reached (see for instance Goles and Olivos 1980). Finally, it is worth mentioning two recent papers by Montanari and Saberi (2010) and Young (2011) which have a different focus from ours (on the Coordination Game) but try to characterize finely the impact of network structure on new behavior adoption, using notions of contagion that focus on the speed of contagion in relation to network topology, and specifically providing upper bounds on contagion time.

3 Results

3.1 Efficient shell-wise contagion

3.1.1 Contagion seeded by a finite set of players

We have seen in Subsection 2.2 that the unique imitative best response of player i to configuration ω is C if $P(d_i, \omega) \leq P(c_i, \omega)$, and it is D otherwise, where c_i or d_i is the most successful cooperator or defector in $N_i \cup \{i\}$. The inequality $P(d_i, \omega) \leq P(c_i, \omega)$ is rewritten in terms of the number of cooperators in the neighborhoods of c_i and d_i as

$$\gamma_{c_i} \geq \gamma_{d_i} + \ell, \text{ with } \ell = cn/(b - p). \quad (4)$$

Given our payoff structure, payoff differences translate into differences in the number of C -players involved. Equation 4 has the interpretation that i needs to observe a cooperative player who plays with at least ℓ more cooperative players than any player he observes defecting. The key aspect is that the numbers of cooperators and defectors in an agent's neighborhood does not matter, only their difference, $\gamma_{c_i} - \gamma_{d_i}$, does. The threshold ℓ is in general not an integer number, and so the behavior of the system can be expected to change at the breakpoints at which ℓ does become integer. For any (c, b, p, n) such that $0 \leq \ell < 1$, a difference of one cooperative player is sufficient to make C more attractive than D ; this is the most favorable context for cooperation. As ℓ increases, the attractiveness of cooperation declines. Intuitively, efficient contagion will require that cooperators remain relatively unexposed to defectors and so keep generating large payoffs (while defectors keep generating low payoffs) as contagion unfolds. Without loss of generality we assume that (c, b, p, n) are such that $\ell < n - 1$.

In agreement with the findings in the literature (for a survey, see Section 6 in Szabó and Fáth 2007), in our model, the attractiveness of cooperation (and thus its likelihood),

which is encapsulated in the value of ℓ , decreases with the cooperation cost, c , increases with the cooperation benefit b , falls with the mutual penalty payoff p , and decreases with neighborhood size, n . Intuitively, any factor or combination of factors increasing ℓ makes it harder for efficient behavior to spread in a given network, since it means a larger extra-number of C -players is necessary to make C more attractive than D at the individual level (as reflected in Equation 4). This is confirmed by the literature which has identified the benefit-cost ratio, b/c , as the fundamental driver of cooperation in the long run (in the literature it is often assumed that $p = 0$). Jun and Sethi (2007), for instance, generalize the findings of Eshel et al. (1998) on a circle by allowing agents to interact with an arbitrary number of players symmetrically in either direction. They prove that for each value of interaction distance (the distance on the circle at which neighborhoods extend) there is a critical value of b/c below which cooperation cannot be sustained, and this critical value falls with the interaction radius. This point is further generalized in Ohtsuki et al. (2006) on circles, lattices, random regular networks, random networks with Poisson degree distributions, and scale-free networks (i.e., with power law degree distributions). Using a combination of approximation techniques and numerical simulation, with a stochastic learning rule, they find that “natural selection” favors cooperation if the benefit-cost ratio exceeds the average number of neighbors, which in our case would be written $(b - p)/c > n$, or equivalently $\ell < 1$. Indeed, as we have pointed above, this corresponds to the smallest possible difference of one cooperative player to make C more attractive than D , and intuitively constitutes the most favorable environment for cooperation in our setting.

Equation 4 can directly be used to formulate a necessary and sufficient condition which characterizes shell-wise contagion in terms of the properties of the iterated neighborhoods of a starting set X . To avoid overly complicated dynamics and maintain a simple representation of the problem, we make throughout the paper the simplifying assumption that X is connected.

Proposition 1 *A finite-sized set $X \subset S$ seeds efficient shell-wise contagion in (S, g) if and only if*

$$\max_{k \in \{i\} \cup N_{i,X}^0} \{n_{k,X}^{-1}\} + \ell \leq \max_{j \in N_{i,X}^{-1}} \{n_{j,X}^{-1} + n_{j,X}^0\} \quad (5)$$

for all $i \in S - X$.

Proof. Suppose the efficient action is played in the $(d-1)$ -shell of X , $d \geq 1$. Under what condition does the d -iterated neighborhood of X convert? For any D -player i in N_X^d , it must be the case that the best C -player seen by i interacts with at least ℓ more C -players than the best D -player seen by i . The C -players seen by i are in $N_{i,X}^{-1}$, and the best of them plays with $\max_{j \in N_{i,X}^{-1}} \{n_{j,X}^{-1} + n_{j,X}^0\}$ cooperative players. Similarly, the D -players seen by i are in $N_{i,X}^0$, and the best of them plays with $\max_{k \in \{i\} \cup N_{i,X}^0} \{n_{k,X}^{-1}\}$ cooperative players. From Equation 4 then, i switches to C if and only if $\max_{k \in \{i\} \cup N_{i,X}^0} \{n_{k,X}^{-1}\} + \ell \leq \max_{j \in N_{i,X}^{-1}} \{n_{j,X}^{-1} + n_{j,X}^0\}$.

Is there a C -player i in N_X^{d-1} willing to convert back to D ? No, because for any $d \geq 1$, i sees another C -player who only interacts with C -players. This is true when $d = 1$ by the

assumption that each player is connected to (at least) one player whose entire neighborhood is included in X . And it is true for $d > 1$ by definition of shell-wise contagion. So for any $d \geq 1$, player i in N_X^{d-1} sees another C -player who only interacts with C -players, and therefore switches to D if and only if the best D he can observe in $N_{i,X}^{+1} \subseteq N_X^d$ is such that $\max_{k \in N_{i,X}^{+1}} \{n_{k,X}^{-1}\} + \ell > n$. But from Condition 5, we know that any player k in N_X^d has $n_{k,X}^{-1} + \ell \leq n$, and so i does not switch back to D .

As a consequence, the (imitative) best response to all and only players in the seed group X playing C has all and only players in $X \cup N_X = X_1$ playing C if and only if Condition 5 holds. In the next period, under Condition 5, the players in N_X^2 convert to C while the players in X_1 stick to C , and so all and only players in X_2 play C . Cooperation therefore propagates through successive iterated neighborhoods of X if and only if Condition 5 holds. Note that Proposition 1 is also applicable to finite networks. $\square$

It is interesting to see that Proposition 1 essentially formulates a condition on intra-degree. Players in N_X^d convert to C if and only if each of them has in-degree less than the largest sum of in- and intra-degree among its neighbors in N_X^{d-1} , decreased by ℓ . Informally put, in-degree should be fairly low for players in N_X^d . Assume now all players in N_X^d have converted to C . This time the conversion of players in N_X^{d+1} to C requires that enough players in N_X^d have a large sum of in- and intra-degree. Considered jointly, the conversion of all players in N_X^d and the subsequent conversion of all players in N_X^{d+1} require that enough players in N_X^d have both small in-degree and a large sum of in- and intra-degree. This means sufficiently large intra-degree for these players. Intra-degree ensures that as contagion unfolds through the iterated neighborhoods of X , players interact with a sufficient number of others having the same strategy they have. A C -player interacting mostly with C -players generates a sufficiently high payoff to be attractive, which makes C attractive, particularly to a D -player who interacts mostly with D -players and thus generates a low payoff.

Proposition 2 formulates two conditions on network structure which further emphasize the importance of intra-degree.

Proposition 2 *A finite-sized set $X \subset S$ seeds efficient shell-wise contagion in (S, g) if $n_{i,X}^0 \geq (n + \ell)/2 - 1, \forall i \in S - X$; a set X does not seed efficient contagion if $n_{i,X}^0 < \ell, \forall i \in S - X$.*

Proof. For the first part, assume $n_{i,X}^0 \geq k$ for all $i \in S - X$. Then $1 \leq n_{i,X}^{-1} \leq n - 1 - k$ and Condition 5 holds if the largest possible value of the left-hand side expression is smaller than the smallest possible value of the right-hand side expression, i.e., $n - 1 - k + \ell \leq k + 1$ or $k \geq (n + \ell)/2 - 1$.

For the second part, observe that if $n_{i,X}^0 \leq \ell - \epsilon$, with $\epsilon > 0$, Condition 5 implies that $\max_{k \in \{i\} \cup N_{i,X}^0} \{n_{k,X}^{-1}\} + \ell \leq \max_{j \in N_{i,X}^{-1}} \{n_{j,X}^{-1} + n_{j,X}^0\} \leq n + \ell - \epsilon, \forall i \in N_X^1$. So $n_{k,X}^{-1} \leq n - \epsilon, \forall k \in N_X^1$. Turn to the players in N_X^2 . Condition 5 yields $\max_{k \in \{i\} \cup N_{i,X}^0} \{n_{k,X}^{-1}\} + \ell \leq \max_{j \in N_{i,X}^{-1}} \{n_{j,X}^{-1} + n_{j,X}^0\} \leq n - \epsilon + \ell - \epsilon$. So $n_{k,X}^{-1} \leq n - 2\epsilon$, for all $k \in N_X^2$. Repeating the

argument will imply the existence of a finite d after which all players' in-degree become negative, a contradiction which proves the result. $\square$

The first part of Proposition 2 formulates a sufficient condition for contagion which is only in terms of intra-degree of the iterated neighborhoods of X , namely that players' intra-degree are larger than $(n + \ell)/2 - 1$. The second part of Proposition 2 is a necessary condition again on intra-degree, namely that all intra-degrees cannot be strictly less than ℓ . Proposition 2 is about "extreme" values of intra-degree, i.e., very small values (all less than ℓ) and very large values (all more than $(n + \ell)/2 - 1$). Between ℓ and $(n + \ell)/2 - 1$ intra-degree alone does not permit to conclude, and the more general Condition 5 applies.

A suitable choice of the initial seed group X can greatly facilitate the analysis, especially when it yields iterated neighborhoods of homogenous in-, out- and intra-degree.³ Like in Proposition 2 the role of intra-degree is highlighted.

Proposition 3 *Assume a choice of $X \subset S$ such that:*

1. *in-, out- and intra-degree are homogeneous across all players, equaling n_X^{-1} , n_X^{+1} and n_X^0 respectively; X seeds efficient shell-wise contagion if and only if $n_X^0 \geq \ell$.*
2. *in-degree is homogenous across all players, equaling n_X^{-1} ; X seeds efficient shell-wise contagion if and only if $n_{i,X}^0 \geq \ell, \forall i \in S - X$.*
3. *out-degree is homogenous across all players, equaling n_X^{+1} ; X seeds efficient shell-wise contagion if and only if $n_{i,X}^0 \geq \ell, \forall i \in S - X$.*
4. *intra-degree is homogenous across all players, equaling n_X^0 ; X seeds efficient shell-wise contagion only if $n_X^0 \geq \ell$.*

Proof. Item 1. is Condition 5 rewritten. As for item 2., Condition 5 is rewritten $\ell \leq \max_{j \in N_{i,X}^{-1}} \{n_{j,X}^0\}, \forall i \in S - X$, which obviously holds if $n_{i,X}^0 \geq \ell$ for all $i \in S - X$. Regarding item 3., Condition 5 is rewritten as $n_{k,X}^{-1} \leq n - \ell - n^{+1}$, or equivalently $n_{k,X}^0 \geq \ell, \forall k \in S - X$. Item 4. is a direct consequence of Proposition 2. $\square$

To summarize, when the choice of X leads to homogeneous in-, out- and intra-degree for all players, cooperation spreads across the entire network if and only if there are sufficient interactions within iterated-neighborhoods. How the remaining edges are allocated across the in- and out-degree is unimportant, as long as enough edges exist between nodes which are at the same distance from the starting set X . Trees fall in the category of networks with homogenous iterated neighborhoods for any connected starting set X — the iterated

³Observe however that in-, out- and intra-degree cannot take any values (even when these add up to n). To see this, notice there are two ways of counting the number of edges across two adjacent iterated neighborhoods: as the total number of edges going from N_X^d to N_X^{d+1} , and the other way round. This is written $\sum_{j \in N_X^d} n_{j,X}^{+1} = \sum_{k \in N_X^{d+1}} n_{k,X}^{-1}$. In homogeneous networks the expression boils down to $n_X^{+1} \cdot n_X^d = n_X^{-1} \cdot n_X^{d+1}$ for all $d \geq 0$, implying that $n_X^d = (n_X^{+1}/n_X^{-1})^d$. If $n_X^{-1} > n_X^{+1}$ the previous expression becomes strictly less than 1 after some finite d which contradicts our assumption of (S, g) being of infinite order. Thus we must have $n_X^{-1} \leq n_X^{+1}$ (linear growth if $n_X^{-1} = n_X^{+1}$, geometric growth if $n_X^{-1} < n_X^{+1}$). Only such networks are relevant to our analysis.

neighborhoods of X are characterized by homogenous intra-, in- and out-degree of 0, 1 and $n - 1$. The previous proposition clearly rules out the possibility of efficient contagion in trees.

A final comment is that although the process we study bears important resemblances with an epidemic, during which a disease spreads locally through a network, it is very different. Having multiple sources of exposure (having large in-degree in our case) increases the likelihood of the disease being transmitted in an epidemic. In the strategic context we study here, large in-degree will lower the odds of efficient contagion since they mean larger payoffs to the defectors. The condition for contagion is of a different nature. For a (collectively) desirable behavior to invade a large population, those who are exposed to the new behavior should not be isolated but rather densely interconnected. This way they both see the relative advantage of the new behavior while they still are committed to the old one, and make the new behavior desirable to the next potential generation of adopters. One policy implication is that in order to promote the adoption of a new (dominated) behavior, it is important to guarantee that those in a position to adopt have sufficient interactions among themselves, but are protected from too many interactions with the other camp.

3.1.2 Network propensity to efficient contagion

The results above have highlighted the role played by intra-degree. Intra-degree, however, is a property of the edges among players located within a given iterated neighborhood of the starting set X , and therefore is always contingent upon X . Proposition 1 does not say anything about how to find a starting set X that will permit efficient contagion, it only provides a test that can be applied to a candidate seed group. To check whether a set X seeds efficient contagion in a given network, one has to construct the iterated neighborhoods of X and see whether each node does not have “too many” neighbors within the previous iterated neighborhood of X , where “too many” is defined in Condition 5. It would be highly interesting to find a more general way of characterizing the propensity of a network to efficient contagion.

The literature provides some elements, mostly by means of computer simulations. Roca et al. (2009) perform a systematic factorial experiment to explore various dimensions of the problem: 4 learning rules, 2-dimensional lattices with 3 (Newman, Moore and triangular) neighborhood structures and random regular graphs of the same degree (4, 8 and 6), synchronicity and asynchronicity, over a 2-parameter space encompassing the Prisoner’s Dilemma, the Hawk-Dove game and the Snowdrift games. Relevant to our purpose is that imitation of the best player is very sensitive to network structure and very favorable to the establishment of cooperation. Specifically the Moore and triangular neighborhoods, with significant clustering in both, support cooperation over a much wider range of payoff configurations than the “equivalent” random graphs, under both synchronous and asynchronous learning. Abramson and Kuperman (2001) locate players on the vertices of a family of networks tunable between order (a regular periodic lattice) and randomness (a regular random graph, see the algorithm in Watts and Strogatz 1998). Controlling for the

Figure 2: A tree (left panel) and a connected caveman graph (right panel).

degree of disorder (the proportion of rewired edges in the starting lattice) they examine how cooperation is affected by network architecture, varying also the cost of cooperation. They find that intermediate amounts of disorder (a proportion of 0.1 rewired edges) is most favorable to cooperation in intermediate regions where the cost of cooperation is neither too small (yielding overall cooperation) nor too large (yielding overall defection). They also find that networks of larger density (larger neighborhoods) are less conducive to cooperation in general, due to the need for larger groups of cooperators to cluster in order to resist the invasion of defectors. In a similar line, Jun and Sethi (2009) find that clustering in network is beneficial to cooperation when locating the players on a similar family of networks.

Within our framework, an alternative way to look at intra-degree edges is to observe that they are systematically parts of cycles. Indeed, for any choice of X , if i and j both belong to the d -iterated neighborhood of X and are neighbors of each other, then i and j are part of at least one cycle ($d_{i,X} = d_{j,X} = d$ so i and j are both connected to X , X itself is connected and $ij \in g$).

We have remarked earlier in Subsection 3.1.1 that the propensity of trees to efficient contagion is nil. A tree is a network without cycles and thus for any starting set X , all intra-degrees are equal to zero which, from Proposition 2 is sufficient to rule out efficient contagion. An example of a tree with $n = 4$ is provided on the left panel of Figure 2. It is worth noting that the infinite line is a tree, thus in particular it has a nil propensity to efficient contagion, which is at odds with the results in Eshel et al. (1998). The difference arises because Eshel et al. base imitative learning on the average rather than on the largest payoff earned by each action, which permits then a group of at least three adjacent C -players to seed efficient contagion on the line.⁴ Imitation of the best player turns out to be

⁴If we were to base imitation on average payoffs, then with our notations a C -player at the border of the seed group X (at least three adjacent players) would see himself and his C -playing neighbor within the cluster, which yields an average payoff of $(3b + p - 4c)/2$ for C and of $b + p$ for action D . By contrast, the D -player at the border sees himself and his D -playing neighbor, yielding an average payoff of $(b + 3p)/2$ for D and $b + p - 2c$ for action C . Both players prefer C over D in the next period if and only if $b \geq p + 4c$, and

less favorable to cooperation.

Consider now two connected players i and j . In principle the edge ij can be part of many cycles. If it belongs to none then the edge ij is said to be critical: a critical edge is one which, once removed, disconnects the network. In a tree for instance all edges are critical, but many networks possess critical edges without being trees. Critical edges have a simple relation to a network's propensity to contagion, which the proposition below makes explicit.

Proposition 4 *Efficient contagion cannot take place in a network containing critical edges — it is necessary that every edge be part of at least one cycle.*

Proof. A critical edge is part of no cycle, thus efficient contagion, when reaching it, can only reach it coming from one “side”. Suppose ij is a critical edge and shell-wise contagion, starting with a connected set X , has led i and then j to play C . Contagion cannot go any further, since any D -playing neighbor of j , say k , now plays with exactly one C (namely j), just like j himself (with i), so that $\gamma_{c_k} = \gamma_j = 1 = \gamma_{d_k} = \gamma_k$ which implies that C is not more attractive than D to any of j 's neighbors. $\square$

This result can be generalized a little further by observing that in the proposition above it would take at least $\lceil \ell \rceil + 1$ individuals to play C with j in order to make C more attractive than D to the D -playing neighbors of j . All is needed is an adaptation of the notion of p -cohesion: a group of players in a network is p -cohesive (where p is between 0 and 1) if all players have at least a percentage p of their interactions within the group (cohesion has an essential role in the results of Young 1998 and Morris 2000 on contagion in the Coordination Game). Slightly alter the definition to say that a group of players is m -noncohesive if the group has all except m interactions within itself, with m an integer number. For instance, if the edge ij is critical, then the two subsets of S on either “side” of ij are 1-noncohesive.

Proposition 5 *Efficient contagion cannot take place in a network containing m -noncohesive groups, with $m \leq \lceil \ell \rceil$.*

Proof. Consider an m -noncohesive group $S_1 \subset S$, with $m \leq \lceil \ell \rceil$. This means there are m edges connecting S and S_1 . Suppose shell-wise contagion has led all players involved in these edges to play C , while all the other players in S_1 still play D . Contagion cannot go any further in S_1 , since any C -player in S_1 plays with m other C -players at most, when for C to be attractive to the other D -players in S_1 , this number should be at least $\lceil \ell \rceil + 1$, an impossibility when $m \leq \lceil \ell \rceil$. $\square$

A caveat is in order regarding these two propositions. They hold only provided the starting set X is not chosen so that all critical edges (Proposition 4) and more generally all

so provided this inequality holds, any initial group X of at least three adjacent C -players seeds efficient contagion.

m -noncohesive groups (Proposition 5) belong to X . In that sense, the propensity of such networks to efficient contagion is probably minimal rather than nil.

The two propositions above bear significant implications for the study of social networks. It has been showed in a large number of empirical studies that social networks tend to be organized as small worlds: they are sparse and consist of densely interconnected subsets of the population connected to one another by a few clique-spanning ties (see Uzzi et al. 2007 for a review of the empirical literature on small world networks and their prevalence in social science and management). The densely interconnected subsets arise from the transitivity of social relationships while at the same time a few edges provide shortcuts across the network. But Propositions 4 and 5 are exactly stating that such an organization of edges hinders efficient contagion. In small world networks, cooperation could persist within cooperative clusters, but would not spread from one cluster to the next. For cooperation to spread, the clusters would have to be fairly densely interconnected: a set of individuals which is only sparsely connected to the rest of the network will resist efficient contagion. The connected caveman network in the right panel in Figure 2 consists of sparsely interconnected clusters. Starting with a disconnected caveman network consisting of cliques of $n + 1$ players arrayed on the infinite line, break one edge in each cave (the middle one in the figure) and connect the two spokes to the spokes emanating from the adjacent caves (one on each side). The caves are 2-noncohesive so from Proposition 5 contagion is impossible for $\ell > 1$. But even for $\ell \in (0, 1]$ contagion is impossible because the two clique-spanning edges in a given clique emanate from two different, disconnected players.

Finally, we (verbally) analyze the propensity to contagion of a network in relation to the length of the cycles in this network. Focus for a moment on a minimum cycle of length p in (S, g) , i.e., a cycle such that none of its constitutive p edges is part of a shorter cycle. This cycle exists irrespectively of the choice of X , but the choice of X affects the contribution of this cycle to the intra-degree of the p agents involved. This means that, depending on the choice of X , it is possible that the cycle lies entirely in a given iterated neighborhood of X , thus making a contribution of 2 to the intra-degree of each of the p agents involved. But with a different choice of X it is also possible that the cycle does not contribute to the intra-degree of any of the p agents involved, as is for instance the case of the cycle of length 4 ab, bc, cd, da with a in N_X^k , b and c both in N_X^{k+1} and d in N_X^{k+2} for some positive k . Any cycle of even length can make a nil contribution to the intra-degree of the agents involved, whereas a minimum cycle of uneven length always makes a contribution of 1 to the intra-degree of at least two of the agents involved. A triangle (a cycle of length 3) always contributes to intra-degree, regardless of how it is reached by the epidemic (either it is entirely in some iterated neighborhood of the starting set X , or it has one edge connecting two agents in some iterated neighborhood of X , contributing by 1 to the intra-degree of these two agents).

This discussion suggests is that, in order to make a significant contribution to intra-degree and increase the network's propensity to contagion, long minimum cycles have to be organized carefully (by a proper choice of X). Or, put another way, that long minimum

cycles are more sensitive to the choice of the starting set X than short cycles. In addition, given all nodes have identical degree n , creating intra-degree by carefully organizing long cycles is likely to be more “consuming” in terms of degree than resorting to shorter cycles. So without providing a strict proof, it is (intuitively) very likely that short cycles are a more robust (with respect to the choice of a starting set X) way of creating intra-degree than longer cycles. And so networks with short, rather than long, cycles should have higher propensities to efficient contagion.

3.1.3 Illustrations and discussion

To illustrate the previous arguments, we now look at a few examples. Figure 3 displays a 2-dimensional periodic lattice on an infinite cylinder in which each player interacts with 4 neighbors (a standard Newman neighborhood). Taking as the starting set X three adjacent C -playing rings, we get homogeneous in-, out- and intra-degree $(n_X^{-1}, n_X^0, n_X^{+1}) = (1, 2, 1)$ and from Proposition 3 we see that X seeds shell-wise contagion if and only if $\ell \leq 2$.

Figure 3: Contagion on the torus with Newman neighborhood.

The initially segregated cooperators do well in subsequent periods as they keep playing mostly among themselves, and for that same reason defectors do poorly. All edges in the network are part of 2 cycles of length 4, and each of these cycles, given the choice of the starting set X , makes a contribution of 1 to the intra-degree of the four agents involved. Suppose instead that X is assumed to be the union of a player and his neighborhood (not displayed). Then all the minimum cycles of length 4 are wasted: no player has any intra-degree in any iterated neighborhood of X , and thus X does not seed contagion. This shows the importance of a proper choice of X . It is worth noting that there are no shorter cycles in the network, and specifically no triangle, which also shows that clustering (the accumulation of cycles of length 3) is not necessary for contagion. Consider now enlarging neighborhoods by connecting all the players in a given ring. If the starting set X again consists of the three adjacent C -playing rings then $(n^{-1}, n^0, n^{+1}) = (1, 5, 1)$, and X seeds shell-wise contagion for $\ell \leq 5$. Agents’ neighborhoods are now highly clustered: among the

$7 \cdot 6/2 = 21$ possible edges which can form, $5 \cdot 4/2 = 10$ exist, i.e., neighborhood clustering amounts to $10/21 = 0.48$ for any $i \in S$. Clustering is good for contagion in the sense that efficient contagion is now possible over a much wider range of values of ℓ . Also note that neighborhood growth is slow in the network considered: each new iterated neighborhood of X adds the same number of agents as the previous one.

Figure 4: Contagion on two networks with geometric neighborhood growth.

The two panels in Figure 4 represent two networks with geometric neighborhood growth in which contagion is possible. It is interesting to examine them since Morris' (2000) analysis of contagion highlights that contagion is maximal when neighborhood growth is slow, i.e., not geometric (the learning rule is different in his paper: players resort to myopic best-reply rather than imitation). Leaving aside the seed group $X = \{r\} \cup N_r$ (one player and his 4 neighbors), the network in the left panel has minimum cycles of length 4 and 5. Efficient contagion is possible: taking $X = \{r\} \cup N_r$ we get homogenous in-, out- and intra-degree of $(n_X^{-1}, n_X^0, n_X^{+1}) = (1, 1, 2)$ if and only if $\ell \leq 1$. Nonetheless there is geometric neighborhood growth as $n_X^d = n_X^{d-1} (n_X^{+1}/n_X^{-1}) = n \times 2^d$ for all $d \geq 1$. The network on the right panel in Figure 4 displays exactly the same rate of neighborhood growth although edges are organized differently: all players belong to minimum cycles of length 3, i.e., there is a significant neighborhood clustering of $1/3$. The same starting set X also seeds efficient contagion if and only if $\ell \leq 1$. This again illustrates that clustering is not necessary, but cycles definitely are. Networks organized like the one in the right panel are generalized: they consist of cliques of any size s organized like trees with overlap of the cliques restricted to one node. In such a network there is efficient shell-wise contagion if and only if $\ell \leq s - 2$. Indeed, whenever a clique is reached by efficient contagion, only one node is concerned, and given the tree structure of cliques this happens only via one of the cliques which overlap at this node. Therefore for any starting set $X \subset S$, $\max_{k \in \{i\} \cup N_{i,X}^0} \{n_{k,X}^{-1}\} + \ell = 1 + \ell$ and $\max_{j \in N_{i,X}^{-1}} \{n_{j,X}^{-1} + n_{j,X}^0\} = s - 1$, so that contagion takes place if and only if $\ell \leq s - 2$.

From these examples we are able to conclude that, unlike what the literature sometimes suggests, clustering (the accumulation of cycles of length 3) is not necessary for efficient

contagion but cycles are very necessary. As ℓ increases, players must part of more cycles to achieve efficient contagion. These cycles do not necessarily have to be short, as we have showed in examples. But one reason why short cycles can still be expected to increase a network's propensity to contagion is their robustness to the choice of the seed set: their contribution to players' intra-degree can be expected to be larger, in the sense of materializing more frequently across different choices of the seed set X .

3.2 Other forms of contagion

3.2.1 Inefficient shell-wise contagion

In this subsection, we take an alternative perspective, asking whether a finite group of D -players immersed in a sea of cooperation can start an epidemic which leads to defection being played everywhere in the population.⁵ So we explore the conditions for inefficient contagion, starting from an initial seed group X of inefficient players. We make the simplifying assumption that each player in a given iterated neighborhood of X always has at least one link to the next iterated neighborhood of X . In that context, it is straightforward to obtain a necessary and sufficient condition similar to Condition 5.

Proposition 6 *A finite-sized set $X \subset S$ seeds efficient shell-wise contagion in (S, g) if and only if*

$$\min_{k \in N_{i,X}^{-1}} \{n_{k,X}^{-1} + n_{j,X}^0\} < \ell \quad (6)$$

for all $i \in S - X$.

Proof. The proof is similar to that of Proposition 1. Suppose the inefficient action is played in the d -shell of X , $\omega_i = D$ for all $i \in X_d$, where d is a non-negative integer number. Under what condition does the next iterated neighborhood of X convert? Consider any C -player $i \in N_X^{d+1}$. Player i has a link to the next iterated neighborhood of X , N_X^{d+2} , and thus sees a C -player interacting only with C -players, thereby collecting the largest possible payoff from playing C . The highest scoring D -players which i can see are in $N_{i,X}^{-1}$, and the best of them play(s) with $\max_{k \in N_{i,X}^{-1}} \{n_{k,X}^{+1}\}$. Thus i will switch to D if and only if $\max_{k \in N_{i,X}^{-1}} \{n_{k,X}^{+1}\} + \ell > n$, which can be rewritten as Condition 6 to look similar to Condition 5. $\square$

Inefficient and efficient contagion follow different paths. The trick for D to invade the population is that the iterated neighborhoods of the seed group should be characterized by low in- and intra-degree (equivalently large out-degree), which is in stark contrast with the demands of efficient contagion (large intra-degree).

First consider a choice of X yielding homogenous in-, out- and intra-degree. Condition 6 is then written as $n_X^0 + n_X^{-1} < \ell$, which implies that if X seeds inefficient contagion then it cannot seed efficient contagion, and the other way round, since the condition for efficient

⁵The points raised by one of the referees have lead us to examine inefficient contagion as well.

contagion is $n_X^0 \geq \ell$ in Proposition 3. Put another way, in a homogeneous network, if a group X seeds one type of contagion, it cannot seed the other type. When the payoff configuration is most favorable to cooperation, $\ell \leq 1$, inefficient contagion is precluded as it requires $n_X^0 + n_X^{-1} < 1$, an impossibility for any seed set X . Also note how easy inefficient contagion is in trees, since the condition becomes $n_X^0 + 1 = 1 < \ell$ for any X , i.e., inefficient contagion takes place as soon as ℓ is larger than 1.

Condition 6 is related to, but does not exactly match the notion of cohesion (used among other by Young 1998 and Morris 2000). The condition says that cohesion should be small at least for some people. If cohesion is high for X or its subsequent iterated neighborhoods then Condition 6 cannot hold. This in turn is associated with fast neighborhood growth. So inefficient contagion, unlike efficient contagion, is adversely affected by slow neighborhood growth. In Figure 3 for instance, Condition 6 becomes $n_X^0 + n_X^{-1} = 3 < \ell$, which means the payoff configuration must be very favorable to defection for it to invade a population of cooperators.

3.2.2 Discerning interaction from information

Recent literature has explored the consequences of distinguishing interaction and information neighborhoods (see Alós-Ferrer and Weidenholzer 2008; Mengel 2009; Stark and Behrens 2010). Players interact with their neighbors on a network, but exchange payoff information beyond their neighborhoods. Alós-Ferrer and Weidenholzer (2008) consider agents who imitate the best maximal payoff in the Coordination Game. They show that provided the network is connected and interaction neighborhoods are small enough relative to the population size (interaction is not “too global”), having information neighborhoods extended (slightly) beyond interaction neighborhoods guarantees that the efficient behavior spreads to the entire population, even when it is risk-dominated by the other behavior. Mengel (2009) extends Eshel et al.’s (1998) paper on the Prisoner’s Dilemma played on a circle, finding that whenever payoff information from beyond the interaction neighborhoods is considered by players, defection spreads to the entire population. Introducing a conformism bias (a follow-the-majority component) restores the prevalence of cooperation. Eshel et al. (1999) also consider extended information neighborhoods on the circle, with an additional assumption of conservative behavior: though information is acquired beyond the interaction neighborhood, adopting the new behavior is only possible if it is played in the interaction neighborhood. This distinction is particularly relevant in the Prisoner’s Dilemma, as payoffs to the defectors are larger at the border than inside clusters of defectors. Without the conservative assumption, one could imagine defectors switching to cooperation within a cluster of defectors, while defectors at the border would not, the resulting diffusion process being very difficult to characterize. Finally, Stark and Behrens (2010) locate again players on a circle, restrict interactions to nearest neighbors and vary the information radius, finding that a larger information neighborhood is favorable defection. They refer to this effect as “the curse of global information”, a finding in line with those in Mengel (2009).

The networks we consider in this paper are much more general, and so we resort to the conservative assumption in order to avoid overly complicated, discontinuous types of contagion. Let the information distance be $\delta \geq 2$. The proposition below straightforwardly obtains.

Proposition 7 *There is efficient shell-wise contagion in (S, g) if and only if there exists a finite-sized set $X \subset S$ such that*

$$n_{i,X}^{-1} + \ell \leq n \quad (7)$$

for all $i \in S - X$.

Proof. Suppose the efficient action is played in the d -shell of X , where d is a non-negative integer number. When $\delta \geq 2$, the C -players in N_X^d and the D -players in N_X^{d+1} are in exactly the same situation. Each of them sees at least one player in N_X^{d-1} who only play with C s. In addition, each of them sees the players who belong to the intersection of his augmented neighborhood with N_X^{d+1} . If all the D -players in N_X^{d+1} play with (strictly) less than $n - \ell$ C s, the D -players convert to C . If there is (at least) one D -player in N_X^{d+1} who plays with more than $n - \ell$ C s, him and possibly others who see him will not switch. The same is true for the C -players in N_X^d , under exactly the same condition: if there is (at least) one D -player in N_X^{d+1} who plays with more than $n - \ell$ C s, those C s who see him will switch to D . As a consequence, the imitative best response to the configuration in which all and only players in X_d play C is the configuration in which all and only players in X_{d+1} play C for any $d \geq 0$ if and only if $n_{i,X}^{-1} + \ell \leq n$ for all $i \in S - X$. $\square$

This very simple condition contrasts with the one obtained when the interaction and information neighborhoods coincide. Here, the players who are in a position to learn all see a cooperator collecting the largest possible payoff. Therefore the only issue is about the performance of D -players, and this translates into a constraint on in-degree only. Like in the previous subsection, Condition 7 is related to the notion of cohesion. That the in-degree of all players must be small means that the cohesion of the complementary sets of (all the) iterated neighborhoods of X should be high. It is also interesting to note that Condition 7 is compatible with fast neighborhood growth (in the case when X is such that intra-degrees are small and thus out-degrees are large).

The simplest illustration obtains again for trees. Any seed set X (containing at least one agent and his neighbors) has iterated neighborhoods characterized by $(n_X^{-1}, n_X^0, n_X^{+1}) = (1, 0, n - 1)$. Condition 7 is simply written $\ell \leq n - 1$. So the n -regular network with fastest neighborhood growth (since a tree has no cycles) is also the network in which efficient contagion obtains in the most adverse payoff configurations, i.e., when cooperation needs to be played by as many as $n - 1$ additional players to outperform defection.

Considering simultaneously Condition 5 (no distinction between neighborhoods) and Condition 7, it is immediately seen that if the former holds, the latter holds as well. Thus in our setting, a network's propensity to efficient contagion increases when information neighborhoods are extended beyond interaction neighborhoods.

4 Conclusion

In this paper we have examined the conditions under which cooperation diffuses in a network, starting from an initial seed group and eventually reaching the entire population. The focus has been on imitative learning, with players imitating the best observable action in their neighborhoods. For arbitrary regular networks, we find that efficient contagion in a Prisoner’s Dilemma depends on the extent to which interactions happen mostly within (rather than across) the iterated neighborhoods of the initial seed group — the intra-degree of agents. These interactions involve players who are confronted with the opportunity to adopt the new behavior at the same time, i.e., who are at the same distance from the initial seed group. Interactions with players who were confronted with the decision in the previous round or players who will be confronted with it in the next round (and thus still use the “old” behavior) is less critical. Depending on the network architecture, intra-degree can act as a sufficient or a necessary condition for contagion by efficient behavior. In all cases, intra-degree play a central part.

We show that the existence of intra-degree is related to the presence of cycles in the network. This permits us to consider a network’s propensity to contagion more generally, i.e., without reference to a particular seed set. We find that the existence of critical links prevents contagion, and that networks consisting of dense clusters of individuals connected to one another by few clique-spanning ties do not allow efficient contagion. We contend that longer cycles are more sensitive to the choice of the initial seed group than shorter ones, and thus short cycles, though by no means necessary, are likely to contribute more to agents’ intra-degree across different choices of X than longer cycles. In that sense, although clustering (neighbor commonality) is not necessary, it can support contagion.

We have also examined the situation in which information and interaction neighborhoods differ, and found that the propensity to efficient contagion of networks increases when information neighborhoods are extended beyond interaction neighborhoods. In this setting in-degree should be low for efficient contagion to happen, a configuration in which typically neighborhood growth can be very fast. Therefore, extended information neighborhoods are not only favorable to cooperation, but actually permit the most rapid diffusion of efficient behavior on regular networks, as contagion takes place in networks in which neighborhoods grow fast.

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