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Natural Resources, R&D and Economic Growth

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Abstract

This paper studies the long-run economic impact of natural resources by constructing a Schumpeterian endogenous growth model that incorporates an upstream resource intensive sector. Natural resources are extracted, processed and utilized to produce intermediate capital goods which are essential inputs for producing a final consumption good. R&D activities are targeted at improving the quality of existing intermediate products. In this context, we characterize balanced growth paths and examine the issues of sustainability and long-run growth associated with these competitive equilibrium solution trajectories. The analysis is conducted through the comparison of the two natural resource types: renewable versus non-renewable and two optimal equilibrium conditions: social versus private. We show that negative growth is possible, however, only applied to an economy that is endowed with non-renewable resources. In addition, we derive conditions under which an economy experiences permanent stagnant growth. We also show that having a strong innovative sector is essential for escaping this stagnant growth trap. We then identify conditions under which growth is larger under renewable resources as compared to their non-renewable counterpart and vice versa.

Keywords: non-renewable resources, renewable resources, R&D-based growth, stagnant growth, vertical innovation.

JEL classification: O13, O31, O41.

1 Introduction

Prior to the twentieth century, natural resources, usually comprising primary commodities, played a pivotal role in world trade. Many countries, such as Australia, the United States, and Canada, benefited greatly from significant primary

commodity exports in the early stages of their economic development (North and Thomas, 1973; Auty and Mikesell, 1998). However, since the turn to the twentieth century, natural resources have often been treated as less important than labour and capital in generating economic growth and development. In fact, natural resource abundance may be harmful to the economic development of low and middle income countries due to the so-called resource curse puzzle (Nankani, 1979; Sachs and Warner, 1997, 2001).

Several conjectures are offered towards explaining the natural resource curse puzzle. The Dutch disease theory postulates that natural resource abundance leads to a decline in the production and export of the manufacturing sector which possibly leads to de-industrialisation and lower economic growth. The export of natural resources generates a substantial inflow of foreign capital which in turn causes an appreciation of the domestic currency and a decline in domestic competitiveness. The damaging consequences are even higher if resource proceeds are mainly used for consumption instead of investment (Burnside and Dollar, 2000; Sachs, 2007). The rent seeking synthesis (e.g. Torvik, 2002) suggests that highly resource abundant countries have a higher incidence of firms engaging in rent seeking activities, leaving only a few to engage in productive ventures. Activities in natural resource sector are likely to crowd out physical capital, human capital, and other more technologically advanced activities (e.g. investment in high-tech manufacturing sectors) which reduce the rate of technological progress, the main driver of output growth in the last century.

Despite the potential harmful impact of natural resources on growth, Jones (2002) indicates that having very few natural resources does not alleviate the negative impact of the resource depletion rate on economic performance. Instead, the issue is how to find an effective use of resources given their role in the production function. In that respect, technological progress could make natural resources 'virtually unlimited' (Tisdell, 1990). Higher productivity and greater rates of innovation could reduce the rate of resource exploitation that may be harmful for future growth (Robson, 1980; Perkins et al., 2006). According to Jones (2002), technological progress can help eliminate the risk of natural resource curse because technological progress could create a structural shift from the natural resource sector to the more industrialised sector.

However, there are also some skeptical views on the role of technological progress in abating the impact of natural resources on economic performance. For instance, Tisdell (1990) points out that the speed of technological improvement may not be enough to offset the decreasing availability of natural resources. Technological change may also raise consumption of natural resources due to the so-called Jervons Paradox (Alcott, 2005). For example, technological improvement in the transportation sector, such as rail road, may raise demand for iron and steel. The higher consumption of natural resources may induce unfavourable structural shift to a much greater reliance on natural resources.

Previous studies often consider natural resources, innovation, and growth separately, either between resource abundance and economic growth (e.g. Sachs and Warner, 1995; Lederman and Maloney, 2007) or between innovation and economic performance (e.g. Romer, 1990; Grossman and Helpman, 1991; Aghion

and Howitt, 1992). Hence, it remains an open question as whether natural resources actually play a significant role in enhancing or inhibiting standards of living over time and to what extent and what direction, technological improvement could affect this process. To the best of our knowledge, Grimaud and Rouge (2003), Lafforgue (2008), Peretto (2008, 2012), Peretto and Valente (2011) are among the few recent attempts to fill this gap. However, these models only focus on non-renewable resources while this paper extends its investigation to renewable resources as well. In particular, we consider whether the economy behaves differently under different resource types and different equilibrium conditions.

In this paper, we attempt to answer the questions on the role of technological progress and natural resources in affecting output growth through the lens of modern Schumpeterian growth theory. To that end, we construct a model of endogenous growth with creative destruction and natural resources. As the model yields closed form solutions for balanced growth paths, we then analyze key properties of these equilibrium paths and derive conditions under which the economy obtains permanent positive growth. We also compare the rates of growth across different types of resources.

The model has two factors of production, labour and natural resources, and four sectors, primary (or resource production), research, intermediate good production, and final consumption good production. The primary sector uses labour to process raw natural resources into materials. Here, both types of resources are considered. Unlike non-renewable resources, renewable resources have the capacity to grow in size over time to provide productive input to the intermediate good sector. However, the size of the resource stock cannot be enlarged without bound. Specifically, it is endogenously determined by the rate of extraction and the intrinsic growth of the resources themselves. The R&D sector hires labour to improve the efficiency of production inputs. The intermediate good sector purchases designs created in the R&D sector and employs labour together with processed materials obtained from primary sector to produce intermediate products which are essential for the production of the final consumption good in the final good sector. The analysis is conducted for both cases of renewable and non-renewable resources and from both viewpoints of socially optimal and privately optimal equilibrium conditions. The resulting structure is quite tractable and yields close form solutions for balanced growth paths.

Our key results concern the following issues. What are the balanced growth paths in the presence of R&D and different types of natural resources? What are the main features of these paths? Along these paths, is positive growth attainable and under what conditions? We show that there exist both socially optimal and privately optimal balanced growth paths. Along these paths, while renewable resources do not affect output growth, non-renewable resources decelerate it. However, output growth under renewable resources does not always dominate that under non-renewable resources. In order to escape from possible negative growth triggered by non-renewable resources, the research sector must be sufficiently productive.

The rest of the paper is organized as follows. Section 2 provides basic setting of the model. Section 3 is devoted to characterizing growth equilibrium paths of the economy. In particular, we consider the long-run implications of natural resources on output growth through different optimal conditions (social and private) and for different types of resources (renewable and non-renewable). Section 4 ends the paper with some concluding remarks.

2 The model

2.1 *The final goods sector*

Final consumption good Y is homogeneously produced and sold on a competitive market. There are a large number of identical firms whose production technology is the following:

$$Y_t = Q^{1-\alpha} \int_0^1 A_{it} x_{it}^{\alpha} di, \ \alpha \in (0,1) \tag{1}$$

where Q is a fixed production factor like land or water surface (which is normalized to 1 for simplicity), x_{it} is intermediate good of vintage i that is indexed on a unit interval, and A_{it} is a productivity parameter attached to the latest version of the intermediate good i.

The final good is taken as a numeraire so that $P_Y = 1$. The final good producers are price takers in the input and output markets and their profit maximization problem is:

$$\max \ \pi_{Yt} = Y_t - \textstyle\int_0^1 p_{xit} x_{it} di$$

where p_{xit} denotes the price of intermediate good i at time t. This gives the (inverse) demand function for each intermediate good as follows:

$$p_{xit} = \alpha A_{it} x_{it}^{\alpha-1}, \ \forall i \in [0,1] \tag{2}$$

This equation says that in a competitive market, each intermediate good receives its marginal product in terms of the final consumption good.

2.2 *The intermediate goods sector*

This sector is monopolistically competitive. Goods are available at time t in a continuum of different varieties indexed on a unit interval. Each intermediate producer faces the following production technology:

$$x_{it} = \frac{M_{it}^{\beta} L_{it}^{1-\beta}}{A_{it}}, \ \beta \in [0,1], \ \forall i \in [0,1] \tag{3}$$

Here, L_{it} is labour employment in industry i at time t and M_{it} is the use of processed natural resource materials. The Cobb - Douglas function of M_{it} and L_{it} is deflated by A_{it} to reflect the fact that successive vintages of the intermediate product, which embody increasingly complex technology, require increasing resources to produce. Following Romer (1990) and Grossman and

Helpman (1991), assume that each intermediate good embodies a design created in the research sector and is protected by a patent law. Because no firm can produce an intermediate product without the consent of the patent holder of the design, each intermediate firm is a monopolist of the product it produces.

Profit maximization problem for the representative monopolist i is:

$$\max\ \pi_{xit} = p_{xit}x_{it} - p_{mt}M_{it} - w_tL_{it}$$

subject to the demand equation (2) and production technology equation (3). In this formulation, p_{mt} is the price of one unit of processed material and w_t is the wage rate paid to one unit of labour. The first order conditions with respect to M_{it} and L_{it} deliver:

$$p_{mt} = \frac{\alpha^2\beta A_{it}x_{it}^{\alpha}}{M_{it}}$$

$$w_t = \frac{\alpha^2(1-\beta)A_{it}x_{it}^{\alpha}}{L_{it}}$$

Rearranging and summing over i gives:

$$M_t = \frac{\alpha^2\beta Y_t}{p_{mt}} \tag{4}$$

$$L_{xt} = \frac{\alpha^2(1-\beta)Y_t}{w_t} \tag{5}$$

where $M_t = \int_0^1 M_{it}di$ is the aggregate stock of materials and $L_{xt} = \int_0^1 L_{it}di$ is the total labour employment used for producing intermediate goods. Plugging these results into equation (3) yields:

$$x_{it} = \left(\frac{M_t^{\beta}L_{xt}^{1-\beta}}{Y_t}\right)^{\frac{1}{1-\alpha}}$$

This implies $x_{it} = x_t, \forall i$. This means that when intermediate firms are identical and face the same input costs, they produce the same amount of output. Using this result, the production function in (1) can now be rewritten as:

$$Y_t = A_t^{1-\alpha}(M_t^{\beta}L_{xt}^{1-\beta})^{\alpha} \tag{6}$$

where $A_t = \int_0^1 A_{it}di$ is the economy's aggregate knowledge level.[1]

The representative intermediate firm's flow of profit will be:

$$\pi_{xit} = \alpha A_{it}x_{it}^{\alpha} - \alpha^2\beta A_{it}x_{it}^{\alpha} - \alpha^2(1-\beta)A_{it}x_{it}^{\alpha} = \alpha(1-\alpha)A_{it}x_{it}^{\alpha} \tag{7}$$

This means that operating profit to each monopolist is proportional to his technology level A_{it}. Substituting the value of Y_t given in (6) into the equation for x_t gives:

[1] A_t is also equal to the economy's average technology level as the number of intermediate industries is indexed on a unit interval.

$$x_t = \frac{M_t^{\beta} L_{xt}^{1-\beta}}{A_t}$$

Plugging this value into the profit function for each intermediate firm in (7) yields:

$$\pi_{xit} = \alpha(1-\alpha) A_{it} \frac{M_t^{\alpha\beta} L_{xt}^{\alpha(1-\beta)}}{A_t^{\alpha}} \tag{8}$$

2.3 *The research sector*

As mentioned above, the production of each intermediate good requires the purchase of specific design made from the research sector. This sector is assumed competitive so that any firm or individual can conduct R&D activities provided that benefits exceed the costs. A successful vertical innovation creates a better version of an existing intermediate product and replaces it in the final good production. Because the design is protected by the patent law, the successful innovator can reap the monopoly profits until the next successful innovator occurs in that industry.

With access to the stock of knowledge, research firms use labour to develop new blueprints. Assume that at any point in time, an R&D firm that hires one unit of labour is successful in discovering the next higher quality product with a Poisson arrival rate $\lambda > 0$. Innovations happen such that product of vintage τ makes the product of previous vintage $\tau - 1$ obsolete and then replaces it in production. Each time, when an innovation is successful, the aggregate knowledge level is improved as the following:

$$A_\tau = \mu A_{\tau-1},\ \mu > 1,\ \forall \tau \tag{9}$$

A same amount is spent on vertical R&D in each industry because the prospective payoff is the same in each industry. If L_{rt} is the total amount of labour devoted to doing research then the expected value of A at time $t + \triangle t$ is:

$$E(A_{t+\triangle t}) = \lambda L_{rt} \triangle t \mu A_t + (1 - \lambda L_{rt} \triangle t) A_t = A_t + \lambda(\mu - 1) L_{rt} A_t \triangle t$$

Rearranging and taking the time limit gives:

$$\dot{A}_t = \lim_{\triangle t \to 0} \frac{E(A_{t+\triangle t}) - A_t}{\triangle t} = \lambda(\mu - 1) L_{rt} A_t \tag{10}$$

Under the assumption of free entry, new firms will enter until all profit opportunities are exhausted. Hence, the level of labour employed in research is determined by the arbitrage condition which equates the marginal cost of an extra unit of labour, w_t, to its expected marginal benefit λV_t where V_t is the value of a vertical innovation:

$$\lambda V_t = w_t \tag{11}$$

As the market for design is competitive, the value of vertical innovation at date t will be bid up to the expected present value of future operating profits to be earned by the incumbent intermediate monopolist before being replaced by the next innovator in the industry. The time until replacement is distributed

exponentially with parameter $I_t = \lambda L_{rt}$ which is the rate of successful innovation arrival. As a result, the value of vertical innovation is:

$$V_t = \int_t^{\infty} \pi_{xt\tau} e^{-\int_t^{\tau}(r_s+I_s)ds} d\tau \tag{12}$$

where r_s is the instantaneous interest rate at date s, and $\pi_{xt\tau}$ is the flow of operating profit at date τ to any firm in the sector whose technology is of vintage t. The instantaneous discount rate contains the interest rate and the rate of creative destruction I_s which captures the probability of being displaced by a new innovator.

Following Caballero and Jaffe (1993) and Howitt and Aghion (1998), assume that the leading-edge technology parameter $A_t^{\max} \equiv \max\{A_{it},\ \forall i \in [0,1]\}$ is available to any successful innovator. Growth of this leading-edge technology parameter is due to knowledge spillovers produced by innovations. Similar to Howitt and Aghion (1998), it can be shown that the ratio of the leading-edge technology $A_t^{\max}$ to the average technology A_t will be constant. Indeed, each innovation replaces a randomly chosen A_{it} with the leading edge $A_t^{\max}$. As innovation occurs at rate $I_t = \lambda L_{rt}$ per product and the average change across innovating sectors is $A_t^{\max} - A_t$ so:

$$\dot{A}_t = \lambda L_{rt}(A_t^{\max} - A_t)$$

Dividing both sides by A_t gives:

$$\frac{\dot{A}_t}{A_t} = \lambda L_{rt}\left(\frac{A_t^{\max}}{A_t} - 1\right)$$

This together with (10) implies that $\frac{A_t^{\max}}{A_t} = \mu$. Therefore, using (8), the flow of profit at date τ to the innovator who performed a vertical innovation at date t is:

$$\pi_{xt\tau} = \alpha(1-\alpha)\frac{A_{t\tau}^{\max}}{A_{t\tau}} M_{t\tau}^{\alpha\beta} L_{xt\tau}^{\alpha(1-\beta)} A_{t\tau}^{1-\alpha} = \alpha(1-\alpha)\mu M_{t\tau}^{\alpha\beta} L_{xt\tau}^{\alpha(1-\beta)} A_{t\tau}^{1-\alpha} \tag{13}$$

2.4 *The primary or resource sector*

Assume that the resources are freely accessible in the economy. Following Schaefer (1957), assume at each point in time, the amount of materials extracted is:

$$M_t = BL_{mt}R_t \tag{14}$$

where L_{mt} represents labour input in the resource sector, R_t is the stock of resources, and B is the productivity of resource production. This equation indicates that harvest production exhibits increasing returns to scale to all production factors. Harvest output not only depends on labour employment but also on the existing stock of resources.

The dynamics of the stock of resources are as follows:

$$\dot{R}_t = f(R_t) - M_t \tag{15}$$

Here, $f(R_t)$ is the natural growth of the resources that takes the following logistic growth form:

$$f(R_t) = \eta R_t \left(1 - \frac{R_t}{\overline{R}}\right), \ \eta \geq 0 \tag{16}$$

In this formulations, $\overline{R}$ is the carrying capacity of the environment and η represents the intrinsic growth rate of resources. When $\eta > 0$, the natural resources are renewable and when $\eta = 0$, they are nonrenewable. It can be seen from (15) that when $f(R_t) > M_t$, the natural growth of resources is greater than the amount of resources extracted so the resource stock will rise. On the contrary, when $M_t > f(R_t)$, the resource stock will fall. The stock of resources will stay constant when $M_t = f(R_t)$. This implies a possible sustainable yield for the economy when the same level of resource stock is unchanged.

The resource processing firms maximize their lifetime profit $\int_t^\infty \pi_{m\tau} e^{-\int_t^\tau r_s ds} d\tau$, $\forall t$ subject to the dynamics of resource stock given in (15) where π_{mt} is the flow of instantaneous profit:

$$\pi_{mt} = p_{mt} M_t - w_t L_{mt} \tag{17}$$

2.5 *Consumers' behaviour*

Assume constant population and normalize the size of population to 1 for simplicity ($L = 1$). There are a large number of households each of which contains one infinitely lived agent. Each agent supplies one unit of labour to the market and earns the wage rate w_t. The representative household's lifetime utility takes the following form:

$$U = \int_0^\infty \log(C_t).e^{-\rho t} dt \tag{18}$$

where $\rho > 0$ is the rate of time preference and C_t is the aggregate consumption. Households derive utility from consumption only and there is no preference for leisure.

As households earn income from assets (e.g. bonds), labour, and dividends distributed from firms, their budget constraint is:

$$\dot{a_t} = r_t a_t + w_t + \pi_{Yt} + \hat{\pi}_{xt} + \pi_{mt} - C_t \tag{19}$$

In this equation, $r_t a_t$ is the interest income from renting out the asset a_t at interest rate r_t, w_t is the labour income, and π_{Yt}, $\hat{\pi}_{xt}$, π_{mt} are profits distributed to households from firms producing final goods, intermediate goods, and resource materials respectively. The representative household will maximize utility given in (18) subject to the constraint given in (19).

3 Equilibrium of the economy

Assume full employment for simplicity. Hence, the labour market equilibrium requires that:

$$L_{xt} + L_{rt} + L_{mt} = 1 \tag{20}$$

3.1 *The social planner's problem*

Knowing that $Y_t = C_t$, the program of the social planner is to maximize the utility:

$$U = \int_0^\infty \log(Y_t).e^{-\rho t}dt$$

subject to the dynamic equations of technology and natural resources in (10) and (15) respectively. Using (6) and (14), the utility function can be rewritten as:

$$U = \int_0^\infty \log(A_t^{1-\alpha}B^{\alpha\beta}L_{mt}^{\alpha\beta}R_t^{\alpha\beta}L_{xt}^{\alpha(1-\beta)}).e^{-\rho t}dt$$

Definition 1. *An equilibrium of this centralized economy is an infinite sequence of quantity allocations* $\{C_t,\ Y_t,\ A_t,\ R_t,\ L_{xt},\ L_{mt},\ L_{rt}\}_{t=0}^{\infty}$ *such that consumers' welfare is maximized subject to intertemporal constraints facing the social planner.*

In this section, we focus on equilibrium paths where all variables grow at constant rates or balanced growth paths (BGP). In particular, we analyze BGPs which are defined as follows:

Definition 2. *A BGP is an equilibrium path where all variables grow at a constant rate and the allocations of labour across the intermediate goods, resource, and the R&D sectors are also constant.*

Specifically, along this BGP, $L_{xt},\ L_{mt},\ L_{rt}$ are all constant; $C_t,\ Y_t,\ A_t,\ R_t$ grow at constant rates $g_C,\ g_Y,\ g_A$, and g_R respectively. As a matter of convenience, from now and henceforth, the time index will be dropped for those variables that do not vary over time.

Proposition 1 *Assume*

- $1 - \frac{\alpha\rho}{(1-\alpha)\lambda(\mu-1)} + \frac{\sqrt{\triangle}-(1-\alpha)\lambda(\mu-1)\eta}{2B(1-\alpha)\lambda(\mu-1)} \geq 0$ *for renewable resources*
- $1 - \frac{\alpha\rho}{\lambda(\mu-1)(1-\alpha)} + \frac{B\alpha^2\beta^2\rho}{\lambda(\mu-1)(1-\alpha)[\lambda(\mu-1)(1-\alpha)+B\alpha\beta]} \geq 0$ *for non-renewable resources*

where $\triangle = 4B^2\alpha^2\beta^2\rho^2 + \lambda^2(\mu-1)^2(1-\alpha)^2\eta^2$ *then, for each type of resources, there exists a unique optimal BGP in which output, technology, and resources grow at constant rates and labour is optimally allocated among different sectors of the economy.*

Proof.
We will separately prove the result for each type of resources below.
(i) When resources are renewable:
On the BGP, we have $A_t = A_0e^{tg_A}$ where A_0 is the initial level of technology and g_A is the constant rate of growth of technology. Given that $g_R = \eta\left(1 - \frac{R_t}{\bar{R}}\right) - BL_m$ is constant on the BGP then $R = \bar{R}\left(1 - \frac{BL_m}{\eta}\right)$ is also constant. Note that the condition $0 \leq L_m \leq \frac{\eta}{B}$ must hold for $R \geq 0$.

The utility on the BGP is:

$$\begin{aligned} U &= (1-\alpha)\int_0^\infty [\log(A_0)+tg_A]e^{-\rho t}dt + \alpha\beta\int_0^\infty \log(B)e^{-\rho t}dt \\ &+ \alpha\beta\int_0^\infty \log(L_m)e^{-\rho t}dt + \alpha\beta\int_0^\infty \log(R)e^{-\rho t}dt \\ &+ \alpha(1-\beta)\int_0^\infty \log(L_x)e^{-\rho t}dt \end{aligned}$$

Using $\int_0^\infty te^{-\rho t}dt = \frac{1}{\rho^2}$, $\int_0^\infty e^{-\rho t}dt = \frac{1}{\rho}$ and noting $g_A = \lambda(\mu-1)L_r = \lambda(\mu-1)(1-L_x-L_m)$ the above utility function becomes:

$$\begin{aligned} \rho U &= (1-\alpha)\log(A_0) + \alpha\beta\log(B) + \alpha\beta\log(L_m) \\ &+ \alpha\beta\log(\bar{R}) + \alpha\beta\log(1-\frac{BL_m}{\eta}) + \alpha(1-\beta)\log(L_x) \\ &+ \frac{(1-\alpha)\lambda(\mu-1)(1-L_x-L_m)}{\rho} \end{aligned}$$

We now maximize U with respect to L_x and L_m. The first order conditions with respect to these choice variables give:[2]

$$\frac{B}{\eta - BL_m} = \frac{1}{L_m} - \frac{(1-\alpha)\lambda(\mu-1)}{\alpha\beta\rho} \tag{21}$$

and

$$L_x = \frac{\alpha\rho(1-\beta)}{(1-\alpha)\lambda(\mu-1)} \tag{22}$$

The left hand side (LHS) of equation (21) is increasing in L_m, equal $\frac{B}{\eta}$ when $L_m = 0$ and approaching $+\infty$ when $L_m \to \frac{\eta}{B}$. The right hand side (RHS) is, in contrast, decreasing in L_m, approaching $+\infty$ when $L_m \to 0$ and equal $\frac{B}{\eta} - \frac{(1-\alpha)\lambda(\mu-1)}{\alpha\beta\rho}$ when $L_m = \frac{\eta}{B}$. Hence, this equation has a unique solution $L_m \in (0,\, \frac{\eta}{B})$.

Actually, equation (21) can be solved explicitly to get the expression of L_m. Indeed, rearranging (21) gives a quadratic equation of L_m:

$$B\lambda(\mu-1)(1-\alpha)L_m^2 - [2B\alpha\beta\rho + \lambda(\mu-1)(1-\alpha)\eta]\, L_m + \alpha\beta\rho\eta = 0$$

This equation has two distinct real roots but one of them has to be ruled out due to violating $L_m < \frac{\eta}{B}$.[3] The accepted root is $L_m = \frac{2B\alpha\beta\rho+\lambda(\mu-1)(1-\alpha)\eta-\sqrt{\triangle}}{2B\lambda(\mu-1)(1-\alpha)}$ where $\triangle = 4B^2\alpha^2\beta^2\rho^2 + \lambda^2(\mu-1)^2(1-\alpha)^2\eta^2$.

[2] One can check that the second order conditions with respect to these variables are satisfied for a maximum.

[3] The ruled out root is $L_m = \frac{2B\alpha\beta\rho+\lambda(\mu-1)(1-\alpha)\eta+\sqrt{\triangle}}{2B\lambda(\mu-1)(1-\alpha)}$.

Having obtained L_x and L_m, we can calculate $L_r = 1 - L_x - L_m = 1 - \frac{\alpha\rho}{(1-\alpha)\lambda(\mu-1)} + \frac{\sqrt{\triangle}-(1-\alpha)\lambda(\mu-1)\eta}{2B(1-\alpha)\lambda(\mu-1)}$. Since $L_x \geq 0$, $L_m \geq 0$, the condition $L_r \geq 0$ or $1 - \frac{\alpha\rho}{(1-\alpha)\lambda(\mu-1)} + \frac{\sqrt{\triangle}-(1-\alpha)\lambda(\mu-1)\eta}{2B(1-\alpha)\lambda(\mu-1)} \geq 0$ is sufficient for L_x, L_m, $L_r \leq 1$. Using these results, the growth rates of technology, natural resources, output, and consumption are calculated as follows:

$$g_A = \lambda(\mu - 1)L_r$$

$$g_R = 0$$

$$g_Y = g_C = (1 - \alpha)g_A = (1 - \alpha)\lambda(\mu - 1)L_r$$

(ii) When resources are non-renewable ($\eta = 0$):
On the BGP, we now have $R_t = R_0 e^{-tBL_m}$ where R_0 is the initial stock of natural resources. With a note that $\int_0^\infty te^{-\rho t}dt = \frac{1}{\rho^2}$ and $\int_0^\infty e^{-\rho t}dt = \frac{1}{\rho}$, the utility function on the BGP is:

$$\begin{aligned}\rho U &= (1-\alpha)\log(A_0) + \alpha\beta\log(B) + \alpha\beta\log(L_m) + \alpha\beta\log(R_0) \\ &+ \alpha(1-\beta)\log(L_x) - \frac{\alpha\beta BL_m}{\rho} + \frac{(1-\alpha)\lambda(\mu-1)(1-L_x-L_m)}{\rho}\end{aligned}$$

L_x and L_m will be chosen to maximize this utility functions. The first order conditions give:[4]

$$L_x = \frac{\alpha\rho(1-\beta)}{\lambda(\mu-1)(1-\alpha)} \tag{23}$$

$$L_m = \frac{\alpha\beta\rho}{\lambda(\mu-1)(1-\alpha) + B\alpha\beta} \tag{24}$$

Clearly, $L_x \geq 0$, $L_m \geq 0$. Hence, the value of L_r is:

$$\begin{aligned}L_r = 1 - L_x - L_m &= 1 - \frac{\alpha\rho}{\lambda(\mu-1)(1-\alpha)} \\ &+ \frac{B\alpha^2\beta^2\rho}{\lambda(\mu-1)(1-\alpha)\left[\lambda(\mu-1)(1-\alpha) + B\alpha\beta\right]}\end{aligned} \tag{25}$$

When $L_r \geq 0$ we automatically have L_x, L_m, $L_r \leq 1$. This implies the condition stated previously in the proposition. With these obtained results, growth rates of technology, natural resources, output, and consumption are:

$$g_A = \lambda(\mu - 1)L_r$$

$$g_R = -BL_m$$

[4] Again, it can be verified that the second order conditions are satisfied for a maximum.

$$g_Y = g_C = g_A - \alpha\beta g_R = (1-\alpha)\lambda(\mu-1)L_r - \alpha\beta B L_m$$

■

Results indicate that along the optimal BGP, the rate of growth of technology is always larger than the rate of growth of output (and consumption) and technological progress is the key driver of output growth. Natural resources grow at a negative rate when they are non-renewable because their stock is being depleted through time. However, natural resources grow at rate zero when they are renewable. The reason is that although natural resources can grow over time, they are capped by the maximum carrying capacity imposed by the environment, $\bar{R}$. Hence, in the long run, the stock of natural resources will peak at its optimal level $R = \bar{R}\left(1 - \frac{BL_m}{\eta}\right)$ at which the rate of extraction is equal to the rate of the natural growth. This is the level of stock that can be maintained while allowing a sustainable optimal yield of resources.

Proposition 2 *Other things equal, along the optimal BGP for each type of resources, output growth and welfare are increasing in parameters characterizing productivity of the R&D sector (λ and μ) but decreasing in the rate of time preference (ρ).*

Proof.
(i) When resources are renewable:
When λ (or μ) increases, the RHS of (21) decreases while its LHS does not change. The graph of the RHS shifts down implying that L_m decreases. It is obvious that in this case L_x decreases. Hence, L_r increases since $L_r = 1 - L_x - L_m$. As a result, g_Y increases.

When ρ increases, L_x increases as per (22). The graph of the RHS of (21) shifts up while the LHS does not change implying an increase in L_m. Therefore, L_r decreases and, thus, g_Y decreases as well.

As for the welfare effects, we have:

$$\begin{aligned}
\frac{\partial U}{\partial \lambda} &= \frac{1}{\rho}.\frac{\partial L_m}{\partial \lambda}\left[\frac{\alpha\beta}{L_m} - \frac{B\alpha\beta}{\eta - BL_m} - \frac{(1-\alpha)\lambda(\mu-1)}{\rho}\right] \\
&+ \frac{1}{\rho}.\frac{\partial L_x}{\partial \lambda}\left[\frac{\alpha(1-\beta)}{L_x} - \frac{(1-\alpha)\lambda(\mu-1)}{\rho}\right] \\
&+ \frac{(1-\alpha)(\mu-1)(1-L_x-L_m)}{\rho^2}
\end{aligned}$$

Observe that the first two terms are equal to zero along the optimal BGP as per (21) and (22). Hence, $\frac{\partial U}{\partial \lambda} = \frac{(1-\alpha)(\mu-1)(1-L_x-L_m)}{\rho^2} > 0$. Similarly, we have $\frac{\partial U}{\partial \mu} = \frac{(1-\alpha)\lambda(1-L_x-L_m)}{\rho^2} > 0$ and $\frac{\partial U}{\partial \rho} = -\frac{1}{\rho}U - \frac{(1-\alpha)(\mu-1)(1-L_x-L_m)}{\rho^3} < 0$.
(ii) When resources are non-renewable:
It is obvious from (23) and (24) that when λ (or μ) increases, L_x and L_m both decrease meaning L_r increases and g_Y increases (because $g_Y = (1-\alpha)\lambda(\mu -$

$1)L_r - \alpha\beta BL_m)$. By contrast, when ρ increases, L_x and L_m both increase implying L_r decreases and g_Y decreases.

Regarding the welfare effect, we have:

$$\begin{aligned}\frac{\partial U}{\partial \lambda} &= \frac{1}{\rho}.\frac{\partial L_m}{\partial \lambda}\left[\frac{\alpha\beta}{L_m} - \frac{B\alpha\beta}{\rho} - \frac{(1-\alpha)\lambda(\mu-1)}{\rho}\right] \\ &+ \frac{1}{\rho}.\frac{\partial L_x}{\partial \lambda}\left[\frac{\alpha(1-\beta)}{L_x} - \frac{(1-\alpha)\lambda(\mu-1)}{\rho}\right] \\ &+ \frac{(1-\alpha)(\mu-1)(1-L_x-L_m)}{\rho^2}\end{aligned}$$

With a note that the first two terms are zero according to (23) and (24) then $\frac{\partial U}{\partial \lambda} = \frac{(1-\alpha)(\mu-1)(1-L_x-L_m)}{\rho^2} > 0$. A similar result applies for μ as $\frac{\partial U}{\partial \mu} = \frac{(1-\alpha)\lambda(1-L_x-L_m)}{\rho^2} > 0$. However, $\frac{\partial U}{\partial \rho} = -\frac{1}{\rho}U - \frac{\alpha\beta BL_m}{\rho^3} - \frac{(1-\alpha)(\mu-1)(1-L_x-L_m)}{\rho^3} < 0$.

■

The results are quite intuitive. When λ or μ increases, it becomes more socially efficient to invest in the R&D sector (relatively to other sectors) so the social planner will choose a higher level of L_r which then enhances growth of technological knowledge and output. An increase in ρ means households value current consumption relatively more than future consumption. In order to produce more output to meet higher consumption demand today, the social planner will direct more labour to work in the resource sector (L_m increases) and the intermediate goods sector (L_x increases). As a result, there will be a fall in L_r meaning lower growth of technology and output. Consumption growth will also be lower because consumers increase current consumption relatively to future consumption.

Because and increase in either λ or μ raises output and consumption so welfare rises. However, an increase in ρ reduces welfare as it makes the whole path of utility fall below the one before the shock.

Proposition 3 *Other things equal, along each optimal BGP, an improvement in the productivity of the resource sector (an increase in B):*

- *increases both welfare and output growth if resources are renewable.*
- *increases welfare but has no impact on output growth if resources are non-renewable.*

Proof.
(i) When resources are renewable:
From (21) and (22), it can be seen that an increase in B does not affect L_x. However, it reduces L_m as the graph of the LHS of (21), which is increasing in L_m, shifts up while the RHS of that equation, which is decreasing in L_m, stays the same. Hence, L_r rises and so does g_Y.

With respect to welfare, using (21), we have $\frac{\partial U}{\partial B} = \frac{\alpha\beta L_m}{\rho B}\left[\frac{1}{L_m} - \frac{B}{\eta - BL_m}\right] = \frac{(1-\alpha)\lambda(\mu-1)L_m}{\rho^2 B} > 0$ meaning welfare rises with B.

(ii) When resources are non-renewable:

Plugging the values of L_m and L_r from (24) and (25) into the equation for the growth rate of output along the BGP we obtain:

$$g_Y = \lambda(\mu - 1)(1 - \alpha) - \alpha\rho$$

It can be seen that B does not appear in the result for g_Y. Hence, an increase in B does not have any impact on long-run output growth.

As for the welfare, we have $\frac{\partial U}{\partial B} = \frac{\alpha\beta}{\rho}\left[\frac{1}{B} - \frac{L_m}{\rho}\right]$. From (24), it can be figured out that $L_m < \frac{\rho}{B}$. Therefore, $\frac{\partial U}{\partial B} > 0$ or U is increasing in B.

■

The results can be explained as follows. An increase in B makes it more productive to extract natural resources. Equivalently, less labour is needed for producing resource material to meet the existing market demand. Hence, the social planner will allocate less labour to the resource sector (L_m decreases) and more into the R&D activities (L_r increases).[5] This change will increase welfare as there is more output and consumption created. It will also increase output growth for the case of renewable resources because the growth rate of technology is higher. However, it does not affect output growth under non-renewable resources. The reason is that an increase in B, on the one hand, increases L_r and, hence, technological change, will also exhaust resources at a faster rate on the other (the fall in L_m is less than the increase in B). These two opposing effects cancel out each other at optimum.

Proposition 4 *Assume parameters are such that there exists an optimal BGP for each type of resources then*

- *Output growth is generally higher under renewable resources than under non-renewable resources.*
- *Negative output growth may only happen to non-renewable resources.*

Proof.

Under renewable resources, output growth is:

$$g_Y = (1 - \alpha)\lambda(\mu - 1) - \alpha\rho + \frac{\sqrt{\triangle} - (1-\alpha)\lambda(\mu-1)\eta}{2B}$$

where $\triangle = 4B^2\alpha^2\beta^2\rho^2 + \lambda^2(\mu-1)^2(1-\alpha)^2\eta^2$. Under non-renewable resources, output growth is:

$$g_Y = (1 - \alpha)\lambda(\mu - 1) - \alpha\rho$$

[5] Another way of looking at this is that as the social planner always knows the optimal level of natural resources to be $R = \bar{R}(1 - \frac{BL_m}{\eta})$, he will reduce L_m in accordance with the amount of increase in B.

Clearly, $\sqrt{\triangle} - (1-\alpha)\lambda(\mu-1)\eta \geq 0$ implying a generally higher output growth for renewable resource case. The two rates are equal only when $\sqrt{\triangle} - (1-\alpha)\lambda(\mu-1)\eta = 0$ or $\beta = 0$ meaning there is absolutely no utilization of natural resources in intermediate good production.

Obviously, along the optimal BGP for renewable resources, output growth is non-negative. However, along the optimal BGP for non-renewable resources, output growth may be negative under the following condition:

$$0 \geq 1 - \frac{\alpha\rho}{\lambda(\mu-1)(1-\alpha)} \geq -\frac{B\alpha^2\beta^2\rho}{\lambda(\mu-1)(1-\alpha)[\lambda(\mu-1)(1-\alpha)+B\alpha\beta]}$$

In other words, stagnation (non-positive growth) will happen if $\lambda(\mu-1) \leq \frac{\alpha\rho}{1-\alpha}$. Note that $\lambda(\mu-1)$ is the productivity of the research sector. To escape from this possible stagnation, it requires that $\lambda(\mu-1) > \frac{\alpha\rho}{1-\alpha}$ or the R&D sector must be sufficiently productive.

■

In this economy, output growth comes from two different sources: the evolution of technological knowledge and the natural resource dynamics. Because natural resources cannot grow without bound, the best trajectory that the social planner can choose is to reach the optimal level of resources at which the rate of resource extraction is equal to the rate of natural growth. However, this policy is only achievable in case of renewable resources. With non-renewable resources, the rate of resource extraction always soften the rate of growth of output as output needs to increase to make up for the amount of natural resources that has been depleted. Given that technological progress is the key driver of the economy, it in turns requires the evolution of technological knowledge be strong enough to lift the economy up out of the stagnation trap.

3.2 *The decentralized version equilibrium*

At each point in time, intermediate good producers borrow an amount of $w_t L_{rt}$ from households in the financial market to finance research activities. When an innovation is successful, the intermediate good producers use their profit π_{xt} to make interest payment so $\pi_{xt} = r_t a_t + \hat{\pi}_{xt}$. Observe that on aggregate $\pi_{Yt} = (1-\alpha)Y_t$; $\hat{\pi}_{xt} = \pi_{xt} - r_t a_t = \alpha(1-\alpha)Y_t - r_t a_t$; $\pi_{mt} = \alpha^2\beta Y_t - w_t L_{mt}$; and $C_t = Y_t$. Plugging these into (19) and using (5) then (20) we obtain $\dot{a}_t = w_t - w_t L_{mt} - w_t L_{xt} = w_t L_{rt}$ which is the intermediate good producers' borrowing to finance the R&D activities. The households' utility maximization exercise described in (18) and (19) provides the usual condition on consumption growth:

$$g_C = \frac{\dot{C}_t}{C_t} = r_t - \rho \tag{26}$$

Definition 3. *An equilibrium of this decentralized economy is an infinite sequence of quantity allocations,* $\{C_t, Y_t, A_t, R_t, a_t, M_t, x_t, L_{xt}, L_{mt}, L_{rt}, \pi_{Yt}, \pi_{xt}, \pi_{mt}\}_{t=0}^{\infty}$,

and prices, $\{p_{xt},\, p_{mt},\, w_t,\, r_t\}_{t=0}^{\infty}$*, such that consumers, final goods producers, intermediate firms, and research firms maximize their objective functions taking prices as given and all markets clear.*

Similar to the previous part on centralized economy, we focus our attention on the BGP of this economy along which our interested variables such as $L_{xt},\ L_{mt},\ L_{rt}$ are all constant; $R_t,\ A_t,\ C_t$, and Y_t grow at constant rates g_R, g_A, g_C, and g_Y respectively; and interest rate $r_t = r,\ \forall t$. As a matter of convenience, the time index will be dropped for those variables that do not vary over time.

From (12) and (13), as soon as $[r + \lambda L_r - \alpha\beta g_R - (1-\alpha)g_A] > 0$, the value of a vertical innovation is:

$$V_t = \alpha(1-\alpha)\mu A_t^{1-\alpha}(M_t^{\beta} L_x^{1-\beta})^{\alpha} \int_0^{\infty} e^{-[r+\lambda L_r - \alpha\beta g_R - (1-\alpha)g_A]\tau} d\tau = \frac{\alpha(1-\alpha)\mu Y_t}{[r+\lambda L_r - \alpha\beta g_R - (1-\alpha)g_A]}$$

This together with the arbitrage condition in (11) yield the following:

$$\frac{\lambda\alpha(1-\alpha)\mu Y_t}{[r + \lambda L_r - \alpha\beta g_R - (1-\alpha)g_A]} = w_t \tag{27}$$

Proposition 5 *Assume* $\mu\lambda \geq \frac{\alpha\rho}{1-\alpha}$*, for each type of resources, there exists a unique competitive equilibrium BGP in which the growth rates of output, technology, consumption, and resources are constant and the allocations of labour across different sectors take constant values.*

Proof.
(i) When natural resources are renewable:
From (14)-(16), we have $g_R = \eta(1 - \frac{R_t}{\bar{R}}) - BL_m$. Given that L_m is constant in steady state, g_R will be constant if and only if R_t is constant or $\dot{R}_t = 0$. This implies $R = \frac{\bar{R}}{\eta}(\eta - BL_m)$ and $g_R = 0$. Because the stock of resources is non-negative, $L_m \leq \frac{\eta}{B}$. Given that $L_m \leq 1$ (total labour devoted to the resource sector cannot exceed the total labour force), the constraint on L_m will be $L_m \leq \min(1, \frac{\eta}{B})$.

From resource processing firms' profit in (17), after substituting $R = \frac{\bar{R}}{\eta}(\eta - BL_m)$ and maximizing with respect to the choice variable L_m we get:

$$L_m = \frac{\eta(1-\tilde{w})}{2B}$$

where $\tilde{w} = \frac{w_t}{\bar{R}Bp_{mt}} \in (0,\ 1]$.[6] This yields $R = \frac{\bar{R}}{2}(1 + \tilde{w})$ and $M = BL_mR = \frac{\eta\bar{R}}{4}(1 - \tilde{w}^2)$. For R being constant, $\tilde{w}$ must be constant. In other words, p_{mt} and w_t must both grow at a same rate or $g_w = g_{p_m}$. As a result, M is also constant.

From (4) and (5) we have:

[6] It should be noted that since R is constant, there is no intertemporal issue. Hence, maximizing the lifetime profit is equivalent to maximizing instantaneous profit at each point in time.

$$Y_t = \frac{Mp_{mt}}{\alpha^2\beta} = \frac{\eta\bar{R}}{4\alpha^2\beta}p_{mt}(1-\tilde{w}^2)$$

$$L_x = \frac{\eta(1-\beta)}{4B\beta}.\frac{(1-\tilde{w}^2)}{\tilde{w}}$$

These imply $g_Y = g_w = g_{p_m}$. Because M and L_x are constant along the BGP, according to (6), $g_Y = (1-\alpha)g_A$. Also along the BGP, $g_Y = g_C = r - \rho$ from (26) so $r = \rho + (1-\alpha)g_A$. Therefore, (27) becomes:

$$L_r = \frac{(1-\alpha)\mu\eta}{4B\alpha\beta}.\frac{(1-\tilde{w}^2)}{\tilde{w}} - \frac{\rho}{\lambda}$$

Imposing labour market clearing condition given in (20) with a note that L_x, L_m, L_r are all decreasing in $\tilde{w}$, we have:

$$\frac{\eta(1-\tilde{w})}{2B} + \frac{\eta(1-\beta)}{4B\beta}.\frac{(1-\tilde{w}^2)}{\tilde{w}} + \frac{(1-\alpha)\mu\eta}{4B\alpha\beta}.\frac{(1-\tilde{w}^2)}{\tilde{w}} - \frac{\rho}{\lambda} = 1 \qquad (28)$$

We now investigate conditions to be imposed so that $0 \leq L_m,\ L_x,\ L_r \leq 1$. Let $\bar{a}$ satisfy the following:

$$\frac{(1-\alpha)\mu\eta}{4B\alpha\beta}.\frac{(1-\bar{a}^2)}{\bar{a}} = \frac{\rho}{\lambda}$$

The LHS of this equation is a decreasing function with respect to $\bar{a}$. It approaches $+\infty$ when $\bar{a}$ tends to 0 and equals to 0 when $\bar{a} = 1$. Thus, there exists a unique solution $\bar{a} \in (0,1]$ to this equation. For $\tilde{w} < \bar{a}$ we have $\frac{(1-\alpha)\mu\eta}{4B\alpha\beta}.\frac{(1-\tilde{w}^2)}{\tilde{w}} > \frac{\rho}{\lambda}$. Since the function $\frac{\eta(1-\tilde{w})}{2B} + \frac{\eta(1-\beta)}{4B\beta}.\frac{(1-\tilde{w}^2)}{\tilde{w}}$ is also decreasing in $\tilde{w}$, if the LHS of (28) is smaller than or equal to 1 when $\tilde{w} = \bar{a}$ we can conclude that this equation has a unique solution $\tilde{w} < \bar{a}$. In this case, $0 \leq L_m(\tilde{w}),\ L_x(\tilde{w}),\ L_r(\tilde{w}) \leq 1$.

We have:

$$\frac{(1-\bar{a}^2)}{\bar{a}} = \frac{\rho}{\lambda}.\frac{4B\alpha\beta}{(1-\alpha)\mu\eta}$$

Therefore:

$$\frac{\eta(1-\bar{a})}{2B} + \frac{\eta(1-\beta)}{4B\beta}.\frac{(1-\bar{a}^2)}{\bar{a}} + \frac{(1-\alpha)\mu\eta}{4B\alpha\beta}.\frac{(1-\bar{a}^2)}{\bar{a}} - \frac{\rho}{\lambda} = \frac{\eta(1-\bar{a})}{2B} + \frac{\rho(1-\beta)\alpha}{(1-\alpha)\mu\lambda}$$

Since $\bar{a}$ satisfies the equation:

$$\bar{a}^2 + \frac{4B\alpha\beta\rho}{(1-\alpha)\mu\eta\lambda}\bar{a} - 1 = 0$$

then $\bar{a}$ satisfy $1-\bar{a} = \frac{4B\alpha\beta\rho}{(1-\alpha)\mu\eta\lambda}.\frac{\bar{a}}{1+\bar{a}} \leq \frac{2B\alpha\beta\rho}{(1-\alpha)\mu\eta\lambda}$ as $\frac{\bar{a}}{1+\bar{a}} \leq \frac{1}{2}$ (with equality when $\bar{a} = 1$). Therefore:

$$\frac{\eta(1-\bar{a})}{2B} + \frac{\eta(1-\beta)}{4B\beta}.\frac{(1-\bar{a}^2)}{\bar{a}} + \frac{(1-\alpha)\mu\eta}{4B\alpha\beta}.\frac{(1-\bar{a}^2)}{\bar{a}} - \frac{\rho}{\lambda} \leq \frac{\rho\alpha}{(1-\alpha)\lambda\mu}$$

One can conclude that if $\frac{\rho\alpha}{(1-\alpha)\lambda\mu} \leq 1$ then $0 \leq L_m(\tilde{w}),\ L_x(\tilde{w}),\ L_r(\tilde{w}) \leq 1$. Using these results, growth rates of interested parameters can be calculated, for example, $g_A = \lambda(\mu-1)L_r(\tilde{w})$, $g_Y = g_C = (1-\alpha)\lambda(\mu-1)L_r(\tilde{w})$.
(ii) When natural resources are non-renewable:
From (14)-(16), we have $g_R = -BL_m$. From (4)-(6) and (14), we have $w_t = \alpha^2(1-\beta)A_t^{1-\alpha}L_m^{\alpha\beta}R_t^{\alpha\beta}L_x^{\alpha(1-\beta)-1}$ and $p_{mt} = \alpha^2\beta A_t^{1-\alpha}L_m^{\alpha\beta-1}R_t^{\alpha\beta-1}L_x^{\alpha(1-\beta)}$. Define $\hat{w} = \frac{w_t}{p_{mt}R_t}$. It can be seen that $\hat{w} \geq 0$ and is constant along the BGP.

Now we turn to resource processing firms' profit maximization problem. The current value Hamiltonian function can be established as follows:

$$H = p_{mt}BL_mR_t - w_tL_m - \lambda_tBL_mR_t$$

where λ_t is the co-state variable. The optimality conditions are:

$$p_{mt}BR_t(1-\hat{w}) - \lambda_tBR_t = 0$$

$$\dot{\lambda}_t = r\lambda_t - p_{mt}BL_m + \lambda_tBL_m$$

$$\lim_{t\to\infty} e^{-rt}\lambda_tR_t = 0$$

together with the dynamic equation $\dot{R}_t = -BL_mR_t$. These lead to the following result:

$$g_{p_m} = r + (1 - \tfrac{1}{1-\hat{w}})BL_m$$

From (26), along the BGP we have $g_Y = g_C = r - \rho$. In addition, from (4) we have $g_Y = g_{p_m} + g_R$. Substituting these into the above equation gives:

$$L_m = \tfrac{(1-\hat{w})\rho}{B}$$

Clearly, as $L_m \geq 0$ we need $\hat{w} \leq 1$. Also from (4) and (5), after some simple calculations, we have:

$$L_x = \tfrac{1-\beta}{\beta}.\tfrac{L_m}{\hat{w}} = \tfrac{(1-\beta)\rho}{B\beta}.\tfrac{(1-\hat{w})}{\hat{w}}$$

From (6) and (14) we get $g_Y = (1-\alpha)g_A + \alpha\beta g_R$. Using this result and also noting $g_Y = r - \rho$, from (27) together with (4) we have:

$$L_r = \tfrac{(1-\alpha)\mu L_m}{\alpha\beta\hat{w}} - \tfrac{\rho}{\lambda} = \tfrac{(1-\alpha)\mu\rho}{B\alpha\beta}.\tfrac{(1-\hat{w})}{\hat{w}} - \tfrac{\rho}{\lambda}$$

Now using the equilibrium condition for the labour market in (20) we have:

$$\frac{(1-\hat{w})\rho}{B} + \frac{(1-\beta)\rho}{B\beta}.\frac{(1-\hat{w})}{\hat{w}} + \frac{(1-\alpha)\mu\rho}{B\alpha\beta}.\frac{(1-\hat{w})}{\hat{w}} - \frac{\rho}{\lambda} = 1 \qquad (29)$$

Obviously, when $\hat{w} \in [0,\ 1]$ then $L_m \geq 0$ and $L_x \geq 0$. In order to have $L_r \geq 0$, we need $\hat{w} \leq \frac{(1-\alpha)\mu\lambda}{B\alpha\beta+(1-\alpha)\mu\lambda} \leq 1$. Therefore, as soon as (29) has a solution $\hat{w} \in [0,\ \frac{(1-\alpha)\mu\lambda}{B\alpha\beta+(1-\alpha)\mu\lambda}]$ then it is sufficient to have $0 \leq L_x,\ L_m,\ L_r \leq 1$.

The LHS of (29) is decreasing in $\hat{w}$, approaching $+\infty$ when $\hat{w} \to 0$ and equal to $\frac{B\alpha^2\beta(1-\beta)\rho+\alpha(1-\alpha)\lambda\mu\rho}{(1-\alpha)\lambda\mu[B\alpha\beta+(1-\alpha)\lambda\mu]}$ when $\hat{w} = \frac{(1-\alpha)\mu\lambda}{B\alpha\beta+(1-\alpha)\mu\lambda}$. We have $\frac{B\alpha^2\beta(1-\beta)\rho+\alpha(1-\alpha)\lambda\mu\rho}{(1-\alpha)\lambda\mu[B\alpha\beta+(1-\alpha)\lambda\mu]} \leq \frac{\alpha\rho}{(1-\alpha)\mu\lambda}$ (equality happens when $\beta = 0$). If $\frac{\alpha\rho}{(1-\alpha)\mu\lambda} \leq 1$, the equation renders a unique positive solution $\hat{w} \in [0,\ \frac{(1-\alpha)\mu\lambda}{B\alpha\beta+(1-\alpha)\mu\lambda}]$ which allows us to compute equilibrium values characterizing the BGP such as $L_x(\hat{w})$, $L_m(\hat{w})$, $L_r(\hat{w})$, $g_A = \lambda(\mu-1)L_r(\hat{w})$, and $g_Y = g_C = (1-\alpha)\lambda(\mu-1)L_r(\hat{w}) - \alpha\beta BL_m(\hat{w})$.

■

Proposition 6 *Key properties of the competitive equilibrium BGPs: other things equal, output growth is increasing in the productivity of the R&D sector (λ and μ) but decreasing in the rate of time preference (ρ). While an increase in the productivity of the resource sector (B) is always growth enhancing under renewable resources, it is growth enhancing under non-renewable resources if:*

$$\frac{(1-\alpha)^2(\mu-1)\mu(\lambda+\rho)}{(\alpha+(1-\alpha)\mu)^2} - \rho > 0$$

In particular, when ρ is small, or either λ or μ is large then this sufficient condition is satisfied.[7]

Proof.

(i) For renewable resources:

An increase in either μ or λ will shift the graph of the LHS of (28) upward while the RHS remains the same resulting in a higher equilibrium value of $\tilde{w}$. Because L_x and L_m are apparently decreasing in $\tilde{w}$ and $L_r = 1 - L_x - L_m$ so L_r increases which induces higher technological change and higher output growth because $g_Y = (1-\alpha)\lambda(\mu-1)L_r$. However, an increase in ρ will shift the graph of the LHS downward resulting in a lower equilibrium value of $\tilde{w}$ and meaning higher L_x and L_m. As a result, L_r is lower which implies a lower output growth.

The impact of an increase in B is not immediately clear. However, we claim that L_r will increase and hence g_Y. First, when B increases, the graph of the LHS of (28) shifts downward. This implies that $\tilde{w}$ decreases. Now assume that $\frac{1}{B}\frac{1-\tilde{w}^2}{\tilde{w}}$ decreases. Since $\frac{1+\tilde{w}}{\tilde{w}}$ increases, it would imply that $\frac{1}{B}(1-\tilde{w})$ decreases. As a result, the LHS of (28) will decrease while the RHS remains unchanged which entails a contradiction. Hence, $\frac{1}{B}\frac{1-\tilde{w}^2}{\tilde{w}}$ increases and L_r increases. Therefore, g_Y rises.

(ii) For non-renewable resources:

Output growth is given by $g_Y = (1-\alpha)\lambda(\mu-1)L_r(\hat{w}) - \alpha\beta BL_m(\hat{w})$. An increase in either μ or λ will shift the graph of the LHS of (29) upward while the RHS

[7] It should be noted that we will not conduct any welfare analysis for the decentralized economy but focus on its long-run growth rate. In the decentralized economy, agents do not choose allocation of labour directly as it is done through market mechanism. This complicates the direction of change of welfare to a very large degree.

remains unchanged resulting in a higher equilibrium value of $\hat{w}$. Because L_x and L_m are decreasing in $\hat{w}$ and $L_r = 1 - L_x - L_m$ so L_r increases while both L_x and L_m decrease. This guarantees that g_Y increases.

The impact of an increase in ρ is not that straightforward. To better examine the impact we rewrite (29) as follows:

$$\frac{(1-\hat{w})}{B} + \frac{(1-\beta)}{B\beta}.\frac{(1-\hat{w})}{\hat{w}} + \frac{(1-\alpha)\mu}{B\alpha\beta}.\frac{(1-\hat{w})}{\hat{w}} = \frac{1}{\lambda} + \frac{1}{\rho} \tag{30}$$

The LHS of (30) is decreasing in $\hat{w}$. When ρ increases, its RHS moves downward. Hence $\hat{w}$ increases. We claim that L_m increases or, equivalently, $(1-\hat{w})\rho$ increases. Suppose that is not true, then from (29), $\frac{\rho(1-\hat{w})}{\hat{w}}$ must increase. Since $(1-\hat{w})\rho$ decreases, it would imply that $\hat{w}$ decreases, which is a contradiction. Hence L_m increases. Now either L_x increases or decreases. In the first case, L_r decreases (because both L_m and L_x have increased). In the second case, it must be that $\frac{\rho(1-\hat{w})}{\hat{w}}$ decreases. Since $L_r = \frac{(1-\alpha)\mu\rho}{B\alpha\beta}.\frac{(1-\hat{w})}{\hat{w}} - \frac{\rho}{\lambda}$, one can see that L_r decreases. Therefore, g_Y decreases.

Let us consider the impact of an increase of B. For that, again consider equation (29). First, it is obvious that $\hat{w}$ will decrease. We define the following:

$$B = \frac{\lambda\rho(\Phi + \hat{w})(1-\hat{w})}{(\lambda+\rho)\hat{w}} \tag{31}$$

where

$$\Phi = \frac{1-\beta}{\beta} + \frac{(1-\alpha)\mu}{\alpha\beta} \tag{32}$$

Then output growth is:

$$\begin{aligned} g_Y &= (1-\alpha)\lambda(\mu-1)\left[\frac{(1-\alpha)\mu\rho}{B\alpha\beta}.\frac{1-\hat{w}}{\hat{w}} - \frac{\rho}{\lambda}\right] - \alpha\beta\rho(1-\hat{w}) \\ &= (1-\alpha)\lambda(\mu-1)\left[\frac{(1-\alpha)\mu(\lambda+\rho)}{\lambda\alpha\beta(\Phi+\hat{w})} - \frac{\rho}{\lambda}\right] - \alpha\beta\rho(1-\hat{w}), \text{by using (31)} \end{aligned}$$

Differentiating this relation with respect to B we get:

$$\begin{aligned} \frac{\partial g_Y}{\partial B} &= -\frac{\partial\hat{w}}{\partial B}\left[\frac{(1-\alpha)^2(\mu-1)\mu(\lambda+\rho)}{\alpha\beta(\Phi+\hat{w})^2} - \alpha\beta\rho\right] \\ &\geq -\frac{\partial\hat{w}}{\partial B}\left[\frac{(1-\alpha)^2(\mu-1)\mu(\lambda+\rho)}{\alpha\beta(\Phi+1)^2} - \alpha\beta\rho\right], \text{since } \hat{w} \leq 1,\ \frac{\partial\hat{w}}{\partial B} < 0 \end{aligned}$$

Using (32) we have:

$$1 + \Phi = \frac{\alpha + (1-\alpha)\mu}{\alpha\beta}$$

Substituting this result into the above inequality delivers:

$$\frac{\partial g_Y}{\partial B} \geq -\alpha\beta\frac{\partial\hat{w}}{\partial B}\left[\frac{(1-\alpha)^2(\mu-1)\mu(\lambda+\rho)}{[\alpha+(1-\alpha)\mu]^2} - \rho\right]$$

Hence, if

$$\left[\frac{(1-\alpha)^2(\mu-1)\mu(\lambda+\rho)}{(\alpha+(1-\alpha)\mu)^2}-\rho\right]>0\Rightarrow\frac{\partial g_Y}{\partial B}>0. \tag{33}$$

Obviously, when ρ is small or λ is large then (33) is satisfied. When μ converges to infinity, we get:

$$\frac{(1-\alpha)^2(\mu-1)\mu(\lambda+\rho)}{[\alpha+(1-\alpha)\mu]^2}\to\lambda+\rho$$

The conclusion follows. When μ is close to 1, the expression inside the brackets of the RHS of the equation for $\frac{\partial g_Y}{\partial B}$ becomes negative implying $\frac{\partial g_Y}{\partial B}\leq 0$.

■

The key properties of this competitive equilibrium resemble most of the results obtained under the centralized version: the rate of growth of output is increasing in λ and μ but decreasing in ρ. The impact of these parameters this time can be explained through market mechanisms. An increase in either λ or μ implies a more productive R&D sector which induce R&D firms to employ more workers. According to (10), the rate of growth of technology, g_A, increases. Technological progress will result in higher output growth as $g_Y=(1-\alpha)g_A$. From the demand side, more research in the R&D sector means more borrowing from R&D firms to finance their research which boosts up the interest rate. An increase in interest rate entails a higher rate of consumption growth and, hence, a higher rate of output growth.

An increase in ρ means consumers relatively prefer present consumption to future consumption so they will lend less money and thus r will rise. Because R&D firms need to borrow money to finance their research in the first place, L_r falls and so do g_A and g_Y. From the demand side, as ρ increases more than r, consumers prefer present consumption to future consumption so they are not interested in increasing future consumption. Hence, $g_C=g_Y$ decreases.

Unlike the centralized setting, a change in B affects output growth for both types of resources. An increase in B makes the harvest production of natural resources more productive. Because resource firms optimize their extraction over time, they will reduce the labour used. As a result, there will be more labour for the production of intermediate products and R&D activities. Technological progress will induce higher output growth for the case of renewable resources as resource extraction is fully offset by their natural growth. However, for the case of non-renewable resources, whether output growth is higher or not depends on whether technological change is able to generate enough growth to make up for the amount of natural resources that has been depleted (μ and λ must be sufficiently large or ρ must be sufficiently small).

All these results highlight the role of R&D activities in driving economic growth. If technological innovation is strong, natural resources will be turned into good use and an improvement in the productivity of the resource sector enhances growth. If technological progress is weak (μ is close to 1), such a change will only result in a harmful growth impact of non-renewable resources.

Proposition 7 *Along the competitive equilibrium BGPs, under renewable resources, output growth is always non-negative. Under non-renewable resources, further assume* $\lambda \geq \frac{\alpha\rho}{1-\alpha}$*, output growth may be negative if* $\mu \to 1$*.*

Proof.
(i) For renewable resources:
Because $g_Y = (1-\alpha)\lambda(\mu-1)L_r$ and $L_r \geq 0$, it is obvious that $g_Y \geq 0$.
(ii) For non-renewable resources:
We have:

$$g_Y = (1-\alpha)\lambda(\mu-1)\left[\frac{(1-\alpha)\mu\rho}{B\alpha\beta}.\frac{1-\hat{w}}{\hat{w}} - \frac{\rho}{\lambda}\right] - \alpha\beta\rho(1-\hat{w})$$

We claim that when μ is close to 1 then g_Y will be negative. Indeed, one can see that when μ tends to 1, $\hat{w}$ converges to $0 < \hat{w}^* \leq \frac{(1-\alpha)\lambda}{B\alpha+(1-\alpha)\lambda} < 1$ and $L_r(\hat{w}) \to L_r(\hat{w}^*) \leq 1$ under the assumption $\lambda \geq \frac{\alpha\rho}{1-\alpha}$. Thus, $g_Y \to -\alpha\beta\rho(1-\hat{w}^*) < 0$. ■

This proposition again highlights the connection between output growth, technological change, and natural resource dynamics. Because long-run growth of renewable resources is zero, output growth is solely and positively determined by technological progress which is non-negative by construction. However, it is negatively affected by the speed of non-renewable resource extraction. More importantly, it will be negative if the speed of resource depletion is more than the rate of change of technology. This scenario happens when the magnitude of technological improvement due to innovation is small (μ is close to 1).

Proposition 8 *Along the competitive equilibrium BGPs, if additionally* $\eta < \rho$ *then output growth is higher under renewable resources than under non-renewable resources (*$g_Y(\tilde{w}) > g_Y(\hat{w})$*).*

Proof. From (28) and (29), we get:

$$\eta\left[\frac{1-\tilde{w}}{2} + \Phi.\frac{1-\tilde{w}^2}{4\tilde{w}}\right] = \rho\left[(1-\hat{w}) + \Phi.\frac{(1-\hat{w})}{\widehat{w}}\right] \tag{34}$$

Now assume $g_Y(\tilde{w}) \leq g_Y(\hat{w})$ which means $L_r(\tilde{w}) < L_r(\hat{w})$ or:

$$\frac{\eta(1-\tilde{w}^2)}{4\tilde{w}} < \rho\frac{1-\hat{w}}{\hat{w}} \tag{35}$$

This together with (34) implies the following simultaneous condition:

$$\frac{\eta}{2}(1-\tilde{w}) > \rho(1-\hat{w}) \tag{36}$$

$$\Leftrightarrow \tilde{w} < (1-\frac{2\rho}{\eta}) + \frac{2\rho}{\eta}\hat{w} \tag{37}$$

These two simultaneous conditions (35) and (36) give:

$$\rho\frac{1-\hat{w}}{\hat{w}} > \frac{\eta(1-\tilde{w}^2)}{4\tilde{w}} = \frac{\eta(1-\tilde{w})(1+\tilde{w})}{4\tilde{w}} > \frac{\eta}{4}.\frac{2\rho}{\eta}(1-\hat{w})\frac{1+\tilde{w}}{\tilde{w}}$$

or

$$\hat{w} < \frac{2\tilde{w}}{1+\tilde{w}} \tag{38}$$

From (37) and (38) we have:

$$\tilde{w} < (1 - \tfrac{2\rho}{\eta}) + \tfrac{4\rho}{\eta} \cdot \tfrac{\tilde{w}}{1+\tilde{w}}$$

which is equivalent to:

$$\tilde{w}^2 - \frac{2\rho}{\eta}\tilde{w} - (1 - \frac{2\rho}{\eta}) \quad < \quad 0 \tag{39}$$

Let $H(\tilde{w}) = \tilde{w}^2 - \frac{2\rho}{\eta}\tilde{w} - (1 - \frac{2\rho}{\eta})$. We have $H'(\tilde{w}) = 2\tilde{w} - \frac{2\rho}{\eta}$ and $H''(\tilde{w}) = 2 > 0$. Hence, $H(\tilde{w})$ has a global minimum at $\frac{\rho}{\eta} > 1$ because $H'(\frac{\rho}{\eta}) = 0$. In addition, $H(0) > 0$ and $H(1) = 0$ meaning that $H(z) > 0, \forall z \in (0, 1)$. This contradicts with (39). We conclude that $g_Y(\tilde{w}) > g_y(\hat{w})$.

■

Unlike in the centralized economy where there is monotonic ordering of growth rates, in the decentralized economy, other things equal, renewable resources only result in higher output growth if the intrinsic growth of natural resources is smaller than the rate of time preference. The intuition is as follows. When η is high, more labour is attracted to the (renewable) resource sector leading to a contraction of the research sector so output growth is reduced. When ρ is high, as consumers are more impatient, more (non-renewable) resources will be extracted and output growth will be reduced. Which factor reduces growth more depends on the comparison between them.

Proposition 9 *There exists $\bar{\lambda} > 0$ such that, for any $\lambda > \bar{\lambda}$, one can find $\eta^*(\lambda)$ which has the following property:*
$\eta < \eta^*(\lambda) \Rightarrow g_Y(\tilde{w}) > g_Y(\hat{w})$

$\eta > \eta^*(\lambda) \Rightarrow g_Y(\tilde{w}) < g_Y(\hat{w})$

$\eta = \eta^*(\lambda) \Rightarrow g_Y(\tilde{w}) = g_Y(\hat{w})$

Proof.
Let $\Omega = \frac{(1-\alpha)^2\lambda\mu(\mu-1)}{B\alpha\beta}, \Phi = \frac{1-\beta}{\beta} + \frac{(1-\alpha)\mu}{\alpha\beta}$ and $\triangle = g_Y(\tilde{w}) - g_Y(\hat{w})$. From (29), $\hat{w}$ depends on ρ but is independent of η. Using (34), we obtain:

$$\triangle \quad = \quad \frac{\Omega}{\Phi}\left[\rho(1-\hat{w}) - \frac{\eta}{2}(1-\tilde{w})\right] + \alpha\beta\rho(1-\hat{w})$$

or

$$\triangle = \rho(1-\hat{w})\left[\frac{\Omega}{\Phi} + \alpha\beta\right] - \frac{\Omega\eta}{2\Phi}(1-\tilde{w})$$

Observe that $\tilde{w}$ increases when η increases. We claim that $\eta(1-\tilde{w})$ increases too. If not, when η increases, together $\eta(1-\tilde{w})$ and $\frac{\eta(1-\tilde{w}^2)}{\tilde{w}}$ decrease meaning that the LHS of (28) decreases while its RHS remains unchanged and a contradiction occurs. Another observation is that $\triangle$ decreases when η increases.

Let $l = \lim_{\eta\to+\infty} \eta(1-\tilde{w})$. From (28), we get:

$$l = \frac{2B(\lambda+\rho)}{\lambda(\Phi+1)}$$

Hence, when η goes to infinity,

$$\triangle \to (1-\hat{w})\rho\left[\frac{\Omega}{\Phi}+\alpha\beta\right] - \frac{\Omega B(\lambda+\rho)}{\Phi\lambda(1+\Phi)} \tag{40}$$

Note that (29) is equivalent to

$$(1-\hat{w}) = \frac{B(\lambda+\rho)}{\lambda\rho(1+\frac{\Phi}{\hat{w}})}$$

Hence, (40) becomes:

$$\begin{aligned}
\triangle \to z &= \frac{B(\lambda+\rho)}{\lambda}\left[\frac{\frac{\Omega}{\Phi}+\alpha\beta}{(1+\frac{\Phi}{\hat{w}})} - \frac{\Omega}{\Phi(1+\Phi)}\right] \\
&= \frac{\beta(\lambda+\rho)}{\lambda}\left[\frac{\Omega+\alpha\beta(1+\Phi)-\frac{\Omega}{\hat{w}}}{(1+\frac{\Phi}{\hat{w}})(1+\Phi)}\right] \\
&= \frac{\beta(\lambda+\rho)}{\lambda}\left[\frac{\Omega+\alpha+(1-\alpha)\mu-\frac{\Omega}{\hat{w}}}{(1+\frac{\Phi}{\hat{w}})(1+\Phi)}\right]
\end{aligned}$$

Let $\bar{w} = \frac{\Omega}{\alpha+(1-\alpha)\mu+\Omega}$. It is obvious that $z<0 \Leftrightarrow \hat{w}<\bar{w}$.

Equation (29) can be rewritten as: $G(\hat{w}) = 1+\frac{\rho}{\lambda}$, where

$$G(x) = \frac{(1-x)\rho}{B} + \frac{(1-\beta)\rho}{B\beta}.\frac{(1-x)}{x} + \frac{(1-\alpha)\mu\rho}{B\alpha\beta}.\frac{(1-x)}{x}$$

In computing $G(\bar{w})$, tedious calculations give:

$$G(\bar{w}) = \frac{[\alpha+(1-\alpha)\mu]\rho}{B}\left[\frac{1}{[\Omega+\alpha+(1-\alpha)\mu]}+\frac{\Phi}{\Omega}\right]$$

When $\lambda\to+\infty$, we have $G(\bar{w})\to 0<1$. That means for any λ large enough then $G(\bar{w})<G(\hat{w})$. This is equivalent to $\hat{w}<\bar{w}$ since $G(.)$ is decreasing. Choose $\bar{\lambda}$ such that $G(\bar{w})<G(\hat{w})$ for any $\lambda>\bar{\lambda}$. For any $\lambda>\bar{\lambda}$, we have:
(i) From (28), when $\eta\to 0$, then $\tilde{w}\to 0$. This implies $\triangle \to \rho(1-\hat{w})\left[\frac{\Omega}{\Phi}+\alpha\beta\right] > 0$.
(ii) $\lim_{\eta\to+\infty}\triangle = z < 0$.
Since $\triangle$ is decreasing in η, the conclusion follows. ■

Proposition 10 *There exists $\bar{\rho}$ such that for any $\rho < \bar{\rho}$ then $g_Y(\tilde{w}) < g_Y(\hat{w})$.*

Proof.
Again, let $\Omega = \frac{(1-\alpha)^2\lambda\mu(\mu-1)}{B\alpha\beta}, \Phi = \frac{1-\beta}{\beta} + \frac{(1-\alpha)\mu}{\alpha\beta}$ and $\triangle = g_Y(\tilde{w}) - g_Y(\hat{w})$. As in the proof of Proposition 9:

$$\triangle = \rho(1-\hat{w})\left[\frac{\Omega}{\Phi} + \alpha\beta\right] - \frac{\Omega\eta}{2\Phi}(1-\tilde{w})$$

From (28), we obtain that $\hat{w} \to 0$ when $\rho \to 0$ and $\lim_{\rho\to 0}\frac{\rho}{\hat{w}} = \frac{B}{\Phi}$. Define $\tilde{w}(0) = \lim_{\rho\to 0}\tilde{w}$. (34) now becomes:

$$\eta\left[\frac{1-\tilde{w}(0)}{2} + \Phi.\frac{1-\tilde{w}(0)^2}{4\tilde{w}(0)}\right] = \frac{1}{B}$$

We have $\tilde{w}(0) \in (0,1)$ and

$$\lim_{\rho\to 0}\triangle = -\frac{\Omega\eta}{2\Phi}(1-\tilde{w}(0)) < 0$$

The proof is complete. ■

Similar to what is considered in Proposition 8, the last two propositions draws further attention to the comparison of output growth under different types of resources. While η captures factor reducing output growth under renewable resources, ρ represents what may hamper growth when resources are non-renewable. Results say that the order of output growth rates are not monotonic but change for some critical value of either η or ρ.

4 Conclusions

In this paper, we have considered a simple endogenous growth with creative destruction. We considered balanced growth paths under different optimal conditions (social and private) and for different types of natural resources (renewable and non-renewable). We showed that under either optimal conditions, at the steady state, the dynamics of non-renewable resources hamper output growth while those of renewable resources are not a real concern. We then indicated that equilibrium growth will be positive if parameters capturing the efficiency of the R&D sector are sufficiently high.

We also found that an increase in the productivity of the research sector has an additional positive effect on the long-run rate of output growth. A more patient society chooses a low harvesting rate and reaches a higher long-run output growth. In the centralized economy, while an increase in the productivity of resource extraction is growth and welfare enhancing with renewable resources, it has no impact on growth although still enhances welfare with non-renewable resources. By contrast, in the decentralized economy, such a change can be growth enhancing under non-renewable resources as well.

In comparing long-run rates of output growth, renewable resources result in a higher rate in the centralized economy. However, in the decentralized economy,

non-renewable resources are able to induce a higher growth rate under some conditions involving intrinsic growth of renewable resources, productivity of research activities, and/or the rate of time preference.

According to Gylfason et al. (1999), natural resource endowment is a mixed blessing. Whether growth will be negative, positive, lower, or higher is an endogenously determined outcome reflected people's choice. What matters most is how to use these resources in the most effective way. In that respect, the model presented in this paper, although simple, provides a good starting point for considering the long-run economic implications of natural resource dynamics and the efficiency of their management. A possible extension could be the consideration of horizontal innovation in parallel with vertical innovation activities. It would also be interesting to investigate the transitional dynamics of the model.

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