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Benjamin Hamidi
Bertrand Maillet
Jean-Luc Prigent

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IPAG Business School
184, Boulevard Saint-Germain
75006 Paris
France

“A Dynamic AutoRegressive Expectile for Time-Invariant Portfolio Protection Strategies”*

Benjamin Hamidi[†] Bertrand Maillet[‡] Jean-Luc Prigent[§]

- April 2009 -

Preliminary Draft

Abstract

Among the most popular techniques for portfolio insurance strategies that are used nowadays, the so-called “Constant Proportion Portfolio Insurance” (CPPI) allocation simply consists in reallocating the risky part of a portfolio according to the market conditions. This general method crucially depends upon a parameter - called the multiple - guaranteeing a predetermined floor whatever the plausible market evolutions. However, the unconditional multiple is defined once and for all in the traditional CPPI setting; we propose in this article an alternative to the standard CPPI method, based on the determination of a conditional multiple. In this time-varying framework, the multiple is conditionally determined in order the risk exposure to remain constant, but depending on market conditions. We thus propose to define the multiple as a function of Expected Shortfall.

After briefly recalling the portfolio insurance principles, the CPPI framework and the main properties of the conditional or unconditional multiples, we present a Dynamic AutoRegressive Expectile (DARE) class of models for the conditional multiple in a time-varying strategy whose aim is to adapt the current exposition to market conditions following a traditional risk management philosophy. We illustrate this approach in a Time-Invariant Portfolio Protection (TIPP) strategy, as introduced by Estep and Kritzman (1988), which aims to increase the protected floor according to the insured portfolio performance. Finally, we use an option valuation approach for measuring the gap risk in both conditional and unconditional approaches.

Keywords: CPPI, Expectile, VaR, CAViaR, Quantile Regression, Dynamic Quantile Model, Expected Shortfall, Extreme Value.

JEL Classification: G11, C13, C14, C22, C32.

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[†] A.A.Advisors-QCG (ABN AMRO), Variances and University of Paris-1 (CES/CNRS). E-mail: benjamin.hamidi@gmail.com

[‡] A.A.Advisors-QCG (ABN AMRO), Variances and University of Paris-1 (CES/CNRS and EIF). Correspondence to: Dr. Bertrand B. Maillet, CES/CNRS, MSE, 106 bv de l'hôpital F-75647 Paris cedex 13. Tel: +33 144078189. E-mail: bmaillet@univ-paris1.fr

[§] University of Cergy (THEMA). Email: jean-luc.prigent@u-cergy.fr

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After briefly recalling the portfolio insurance principles, the CPPI framework and the main properties of the conditional or unconditional multiples, we present a Dynamic AutoRegressive Expectile (DARE) class of models for the conditional multiple in a time-varying strategy whose aim is to adapt the current exposition to market conditions following a traditional risk management philosophy. We illustrate this approach in a Time-Invariant Portfolio Protection (TIPP) strategy, as introduced by Estep and Kritzman (1988), which aims to increase the protected floor according to the insured portfolio performance. Finally, we use an option valuation approach for measuring the gap risk in both conditional and unconditional approaches.

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1 Introduction

Leland and Rubinstein (1976) first show that an optional asymmetric performance structure can be reached using some portfolio insurance strategies. Thanks to dynamic allocation strategies, insured portfolios are protected against large falls by a contractually guaranteed predetermined floor and they take partially advantage of market performances. A portfolio insurance trading strategy is defined to guarantee a minimum level of wealth at a specified time horizon, and to participate in the potential gains of a reference portfolio (Grossman and Villa, 1989; Basak, 2002). Thus the investor reduces her downside risk and participates to market rallies. The most prominent examples of dynamic versions are the Constant Proportion Portfolio Insurance (CPPI) strategies and Option-Based Portfolio Insurance (OBPI) strategies with synthetic puts[1]. On a micro-economic perspective, such strategies using insurance properties are thus rationally preferred by individuals that are specially concerned by extreme losses and completely risk averse for values below the guarantee (or floor). We propose in this paper a new applied financial strategy that helps the investor in realizing her objectives in most of the market conditions.

The Constant Proportion Portfolio Insurance (CPPI) is introduced by Perold (1986) on fixed income assets. Black and Jones (1987) extend this method by using equity based underlying assets. In that case, the CPPI is invested in various proportions in a risky asset and in a non-risky one to keep the risk exposure constant. CPPI strategies are very popular: they are commonly used in hedge funds, retail products or life-insurance products. The main difficulty of the CPPI strategy is to determine the parameter defining the portfolio risk exposure, known as the multiple. Banks directly bear the risk of the guaranteed portfolios they sell: at maturity, if the guaranteed floor is not reached, the banks have to compensate the loss with their own capital. The sharpest determination of the multiple is the main actual challenge of the CPPI strategy. Unconditional multiple determination methods have been developed in the literature such as the extreme value approach to the CPPI method (see Bertrand and Prigent, 2002 and 2005; Prigent and Tahar, 2005). But all these unconditional setting methods of the multiple reduce the risk exposure to the risky asset exposure. Thus these traditional unconditional methods do not take into account that the risk of the risky underlying asset changes according, for instance, to market conditions. We develop here a setting method of conditional multiple to the

[1] Option-Based Portfolio Insurance (OBPI) with synthetic puts is introduced in Leland and Rubinstein (1976). Synthetic is understood here in the sense of a trading strategy in traded assets that create the put. In a complete financial market model, a perfect hedge exists (*i.e.* a self–financing and duplicating strategy). In contrast, the introduction of market incompleteness impedes the concept of perfect hedging.

CPPI strategy to keep a constant risk exposure. We illustrate it in a Time Invariant Portfolio Protection (TIPP) strategy form, as introduced by Estep and Kritzman (1988), which is based on an increase of the protected floor according to the performance of the insured portfolio.

The aim of this paper is to provide a general model and compare different ways for estimating the multiple, conditionally to market evolutions, when keeping a constant risk exposure defined by the Expected Shortfall.
For each τ-expectile there is a corresponding α-quantile (see Efron, 1991; Jones, 1994; Abdous and Remillard, 1995; Yao and Tong, 1996). Thus, we can use Conditional AutoRegressive Expectile (CARE) models based on quantile regression estimations (see Koenker and Basset, 1978; Mukherjee, 1999) for estimating dynamic conditional quantile models as extensions of CAViaR models (see Engle and Manganelli, 2004). Taylor (2009-a and 2009-b) explicits the link between Expectile and Expected Shortfall. Moreover quantile estimates can be linearly combined through a DAQ model (see Gouriéroux and Jasiak, 2008), the Expected Shortfall can therefore be expressed as a function of quantile combination whose associated probabilities can be precisely estimated using a Dynamic AutoRegressive Expectile (DARE) approach.
In this article we introduce, develop and apply this original DARE approach of the Expected Shortfall to propose a new way to model the conditional multiple.

After having briefly recalled portfolio insurance principles, the CPPI method and presented properties of conditional or unconditional multiple (section 2), we justify, describe and estimate the conditional multiple DARE model when keeping a constant exposition to the risk (section 3). We illustrate then the Dynamic AutoRegressive Expectiles Time-Varying Proportion Portfolio Insurance in a TIPP framework in section 4. The introduction of market incompleteness and model risk may impede the concept of dynamic portfolio insurance. Measuring the risk that the value of a CPPI strategy is less than the floor (or guaranteed amount) is therefore of practical importance. For example, the introduction of trading restrictions (liquidity problem...) is one justification to take into account the gap risk in the sense that a CPPI strategy cannot be adjusted adequately. Thus, an additional option is often written. The option is exercised if the value of the CPPI strategy is below the floor. Using this hedging approach, we finally evaluate the gap risk between conditional and unconditional approaches in a proportion portfolio insurance framework according to different option valuation model (section 4).

2 On the Proportion Portfolio Insurance Principles

Portfolio insurance is defined to allow investors to recover, at maturity, a given proportion of their initial capital. One of the standard Proportion Portfolio In-

surance (PPI) methods is the Constant Proportion Portfolio Insurance (CPPI). This strategy is based on a specific dynamic allocation on a risky asset and on a riskless one to guarantee a predetermined floor.
The properties of CPPI strategies are extensively studied in the literature, (Bookstaber and Langsam, 2000; Black and Perold, 1992). A comparison of OBPI and CPPI is given in Bertrand and Prigent (2005). The literature also deals with the effects of jump processes, stochastic volatility models and extreme value approaches on the CPPI method (Bertrand and Prigent, 2002 and 2003).

The optimality of an investment strategy depends on the risk profile of the investor. In order to determine the optimal rule, one has to decide what strategy to adopt according to the expected utility criterion. Thus, portfolio insurers can be modeled by utility maximizers where the optimization problem is given under the additional constraint that the value of the strategy is above a specified wealth level. In a complete market, the PPI can be characterized as expected utility maximizing when the utility function is piecewise HARA and the guaranteed level is growing with the risk less interest rate (Kingston, 1989). This argument can be no more valid if additional frictions are introduced such as for example trading restrictions[2]. Mostly, the solution of the maximization problem is given by the unconstrained problem including a put option. Obviously, this is in the spirit of the OBPI method. But, the introduction of various sources of market incompleteness in terms of stochastic volatility and trading restrictions makes the determination of an optimal investment rule under minimum wealth constraints quite difficult (if not impossible)[3]. For example, if the payoff of a put (or call) option is not attainable, the standard OBPI approach is no more viable since a dynamic option replication must be introduced. It explains why the PPI method has become so popular among practitioners. In incomplete markets, hedging strategies depend on some dynamic risk measure (Schweizer, 2001). In this framework, the use of a precise definition of the expected maximum loss has to be used to model the conditional multiple.

The management of cushioned portfolio follows a dynamic strategic portfolio allocation. The floor, denoted F_t, is the minimum value of the portfolio, which is acceptable for an investor at maturity. The value of the covered portfolio, denoted V_t, is invested in a risky asset denoted by S and a non-risky asset denoted by B. The proportion invested in the risky asset varies relatively

[2]For works without completeness assumption, we refer to the works of Cox and Huang (1989), Brennan and Schwartz (1989), Grossman and Villa (1989), Grossman and Zhou (1993, 1996), Basak (1995), Cvitanić and Karatzas (1995 and 1999), Browne (1999), Tepla (2000 and 2001) and El Karoui *et al.* (2005).

[3]The market imperfection can be caused by trading restrictions. The price process of the risky asset, is driven by a continuous-time process, while trading is restricted to discrete time. Therefore, the effectiveness of the OBPI approach is given by the effectiveness of a discrete-time option hedge. The error of time-discretizing a continuous-time hedging strategy for a put (or call) is extensively studied in the literature (*Cf.* Boyle and Emanuel, 1980; Russel and Schachter, 1994; Bertsimas *et al.*, 1998; Mahayni, 2003; Talay and Zheng, 2003; Hayashi and Mykland, 2005).

to the amount invested in the non-risky asset, in order to insure at any time the guaranteed floor. Hence, even if the market is downward sloping at the investment horizon T, the portfolio will keep at maturity a value equal to the floor, (*i.e.* a predetermined percentage of the capital deposit at the beginning of the management period). At maturity, the theoretical guaranteed value cannot be obviously higher than the value initially invested and capitalized at the non-risky rate r^B, denoted by $V_0 \mathrm{Exp}(r^B T)$.

The cushion, denoted by c_t, is defined as the spread (which can vary across time) between the portfolio value and the value of the guaranteed floor. Therefore, it satisfies, $\forall t \in [0, \ldots, T]$:

$$c_t = V_t - F_t. \tag{1}$$

Thus, the cushion is the maximal theoretical amount, which we can loose over a period without reducing the guaranteed capital. The ratio between the risky asset and the cushion corresponds, at each time, to the target multiple, denoted by m_t. The multiple reflects the maximal exposure of the portfolio. The cushioned management strategy aims at keep a constant proportion of risk exposure. The position in the risky asset, denoted e_t, has to be proportional to the cushion. Thus, we have at any time, $\forall t \in [0, \ldots, T]$:

$$e_t = m_t \times c_t. \tag{2}$$

It means that the amount invested in the risky asset is determined by multiplying the cushion by the multiple.

At any time, the fluctuating multiple moves away from its target value. This is the reason why a third parameter is introduced, the tolerance, denoted by τ, to determine when the portfolio has to be rebalanced. If after the fluctuation of the risky asset, the remaining multiple, denoted by m_t^*, moves away from its target value of a superior percentage of the tolerance, there will exist adjustments (thus transaction fees).
Therefore, we have, $\forall t \in [0, \ldots, T]$:

$$m_t^* \in \left[m_t \times (1-\tau)\,,\ m_t \times (1+\tau)\right]. \tag{3}$$

Time-Invariant Protected Portfolio (TIPP, see Estep and Kritzman, 1988) strategy is a modified version of CPPI. The main difference between CPPI and TIPP is a stochastic time-varying definition of the floor. Actually the TIPP floor is defined as the maximum between the traditional CPPI Floor and a percentage of the maximum past cushioned portfolio value. Thus the modified TIPP floor keeps gains and absolutely protects the original CPPI Floor.

Whatever, the problem of the cushion management is the determination of the target multiple. For instance, if the risky asset price drops, the value of the cushion must remain (by definition) superior or equal to zero. Therefore, the portfolio based on the cushion method will have (theoretically) a value superior or equal to the floor. However, in case of a drop of the risky underlying asset price, the higher the multiple, the higher the cushion and the exposure tends rapidly to zero. Nevertheless, before the manager can rebalance his portfolio, the cushion allows absorbing a shock inferior or equal to the inverse of the superior limit of the multiple.

To be protected in a PPI setting, the multiple of the insured portfolio has to stay smaller than the inverse of the underlying asset maximum drawdown, until the portfolio manager can rebalance his position. Additionally, to obtain a convex cash flow with respect to the risky asset return, the investor has to require a multiple higher than one[4]. Moreover, to keep a portfolio value superior to its floor, the cushion has to be positive. Thus, we have, $\forall t \in [0, \ldots, T-1]$:

$$c_{t+1}/c_t \geq 0 \Rightarrow X_t \leq \; 1 + (1 - m_t) r^B_{t+1}/m_t, \tag{4}$$

and:

$$c_{t+1}/c_t \geq 0 \Rightarrow X_t \leq (m_t)^{-1}, \tag{5}$$

where c_t is the cushion value at time t, X_t is the opposite of the underlying risky asset rate of return r^s_t (*i.e.* $X_{t+1} \leq -\Delta S_{t+1}/S_t$), m_t is the multiple and r^B_t is the risk free rate.

To be protected, a PPI based portfolio has to fulfill, at time t, the following condition:

$$m_t \quad \leq \quad \left[\bar{X}_t\right]^{-1}, \tag{6}$$

with $\overline{X}_t = \max \{X_t\}_{\; 0<t\leq T}$, is the maximum of the random variable at time t, for $t = [0, ..., T]$.

3 A DARE Model for a Conditional Multiple

The main parameter of a PPI dynamic strategy is the multiple. The first purpose of this paper is to determine a conditional multiple model and to study it using several estimation methods.

For a capital guarantee constraint at a significance level α, the multiple must be lower than the inverse of the α conditional-quantile of the asset return distribution, that is, $\forall t \in [0, \ldots, T]$:

$$Prob_t\left(c_{t+1}/c_t \geq 0\right) \geq 1 - \alpha \;\Leftrightarrow\; m_t\left(1 + \tau\right) \leq -\left\{Q_{\alpha,t+1}\right\}^{-1}, \tag{7}$$

with:

$$\begin{cases} Q_{\alpha,t+1} = \underset{r_{t+1}\in\mathbb{R}}{Inf}\ \{r_{t+1}\,|\,G_{t+1}\left(r_{t+1}\right) \geq \alpha\}, \\ m_t\left(1 - \tau\right) \geq 1. \end{cases}$$

[4]By definition, the multiple is strictly positive. With a multiple equal to 1, the protection is absolute: the risky asset exposition is then equal to the value of the cushion. A multiple inferior to one is therefore irrational. Nevertheless, even if the multiple was smaller than one, $(1 - m_t) \times r^B_t$ is neglectible relative to m_t and this property would be further verified.

where T is the terminal date, $Q_{\alpha,t+1}$ is the α-*th* quantile of the conditional distribution denoted $G_t(.)$ of the risky periodic asset returns and r^S_{t+1} is a future periodic return.

Thus, the target multiple can be interpreted as the inverse of the maximum loss that can bear the cushioned portfolio before the re-balancing of its risky component, at a given confidence level. For a given parametric model, explicit solutions for the upper bound on the multiple can be provided from Relation (7). An example when the risky asset price follows a general marked point process is developed in Appendix 1. However, without specific assumptions on the market dynamics, upper bounds on the multiple can also be determined from market conditions.

Hamidi *et al.* (2009-a and 2009-b) propose a first conditional multiple model based on Value-at-Risk (VaR). This risk measure is a quantile function (*i.e.* an inverse of the cumulative distribution function), it measures the potential loss of a given portfolio over a prescribed holding period at a given confidence level. A quiet impressive literature about Value-at-Risk (VaR) calculation has been developed, VaR is thus preconised by financial regulation and is therefore considered as a standard measure to quantify market risk.

Moreover, the VaR is now directly used in profit and loss paradigm. Thus, in portfolio selection model under shortfall constraints introduced in the work by Roy (1952) on safety-first theory and developed by Lucas and Klaassen (1998), the shortfall constraint is defined such that the probability of the portfolio value falling below a specified disaster level is limited to a specified disaster probability. Campbell *et al.* (2001) have used the VaR to define the shortfall constraint in order to develop a market equilibrium model for portfolio selection.
VaR can be used for modeling the conditional multiple, which allows the hedged portfolio to keep a constant exposure to risk. It leads to:

$$m_t = \left| VaR_{\alpha,t}\left(r^S_{t-1};\beta\right)\right|^{-1}, \tag{8}$$

where $VaR_{\alpha,t}\left(r^S_{t-1};\beta\right)$ is the Value-at-Risk of the conditional distribution of returns of the underlying asset r^S_t, corresponding to the periodic return of the risky part of the portfolio covered and β is the vector of unknown parameters of the conditional VaR model.
Different approaches have been proposed for estimating conditional tail quantiles of financial returns. The common approach to model dynamic quantiles is to specify a conditional quantile at a risk level α as a function of conditioning variables known at a given time t (using previous notations):

$$G_t^{-1}(\alpha) := Q_{\alpha,t}\left(r^S_{t-1},\beta\right), \tag{9}$$

with β a vector of some real parameters. The quantile function $Q_{\alpha,t}\left(r^S_{t-1},\beta\right)$ is to be an increasing function of the risk level α. The quantile estimators have to fulfill the monotonicity property with α. Thus, a well specified quantile model is expected to provide estimators that behave like true quantiles and to increase for any values of parameters and conditioning variables according to the risk

level α. Quantile functions follow other properties as for instance the following examples.
1) If Q is a quantile function defined on [0, 1] then:

$$Q(\alpha)^* = -Q(1-\alpha) \quad \text{and} \quad Q(\alpha)^* = Q(\alpha^a), \tag{10}$$

with $a > 0$, are also quantile functions, denoted Q_α.
2) If Q_k, $k = [1, ..., n]$ are quantile functions with identical real domains, then:

$$Q_\alpha^* [Q_k(\alpha), a_k] = \sum_{k=1}^{n} a_k Q_k(\alpha) \tag{11}$$

where a_k are positive, is also a quantile function. These properties can be used to derive new quantile functions from an existing one. But VaR is also criticized for only reporting a quantile and thus disregarding outcomes beyond the quantile. In addition VaR is not a subadditive risk measure and can therefore dissuade from diversification. This property concerns the idea that the total risk on a portfolio should not be greater than the sum of the risks of the constituent parts of the portfolio.

Other risk measures as Expected Shortfall (denoted ES) overcome these weaknesses and are becoming increasingly widely used. The Expected Shortfall (also called Conditional Value at Risk or Expected Tail Loss) is the average loss that a portfolio can suffer in a predefine horizon for a specified level of probability, such as:

$$ES_{\alpha,t} = -\frac{1}{\alpha}\int_0^\alpha Q_{p,t}\ dp \tag{12}$$

Expected Shortfall was introduced for its ability to probe how serious are losses of the specified sample of worst case. VaR, on the contrary, only highlights the threshold of such losses neglecting completely what is beyond the threshold. The superiority of Expected Shortfall is revealing extreme losses and the severuty of the tail losses. As shown by Artzner *et al.* (1999), Acerbi (2002), the α-Expected Shortfall correctly defined as the average of the $\alpha\%$ worst losses of a portfolio is a coherent risk measure. The Expected Shortfall in this coherent version can be used as a basic object for obtaining new risk measures (see Acerbi, 2002 and 2007). It is in fact natural to think of the one-parameter family as a basis for expansions which define a larger class of risk measures. Following these works, the risky asset exposition is driven by a conditional multiple determined by the inverse of a shortfall constraint precisely quantified by the Expected Shortfall. The hedging depends in fact on this risk measure. The conditional multiple allows the hedged portfolio to keep a constant exposure to risk defined by the Expected Shortfall.
The inverse of the Expected Shortfall (ES) is a better way to model the multiple. Actually, to be protected in a PPI setting, the multiple of the insured portfolio

has to stay inferior of the inverse of the underlying asset maximum drawdown up to the portfolio manager can rebalance its position (see previous section for more details). Thus multiple can be modelled as:

$$m_t = \left| ES_{\alpha,t}\left(r_{t-1}^S;\beta\right)\right|^{-1}, \qquad (13)$$

where $ES_{\alpha,t}\left(r_{t-1}^S;\beta\right)$ is the Expected Shortfall of the conditional distribution of returns of the underlying asset r_{t-1}^S, corresponding to the periodic return of the risky part of the portfolio covered and β is the vector of unknown parameters of the conditional Expected Shortfall model.
Before introducing the DARE approach for the Expected Shortfall, we briefly review the main methods to estimate the Expected Shortfall.
The most widely used non-parametric Expected Shortfall estimation method is the historical method: the Historical Expected Shortfall is easily determined as the mean of the returns (in the in-sample period) that exceed the Historical VaR estimate (quantile of the empirical distribution). It requires no distributional assumptions but the main difficulty is to choose a relevant estimation period.

Parametric approaches involve a parameterization of the stock prices behavior. Conditional quantiles are estimated using a conditional volatility forecast with an assumption for the shape of the distribution, such as a Student-t or a Gaussian. For these distribution Expected Shortfall can be directly calculated. A GARCH model can be used for example to forecast the volatility (see Poon and Granger, 2003), though GARCH misspecification issues are well known.

Using semi-parametric method as the the Extreme Value Theory, the derived exceedance distribution can be found and it delivers an analytical *formula* for the Expected Shortfall (McNeil *et al.*, 2005).
Koenker (2005) shows that the unconditional shortfall for the α-quantile in the lower tail of the distribution is given by:

$$ES_{\alpha,t}(r_t^S) = E(r_t^S) - \alpha^{-1}E\left\{\left[r_t^S - Q_{\alpha,t}\right]\left[\alpha - I_{\left\{r_t^S < Q_{\alpha,t}\right\}}\right]\right\}, \qquad (14)$$

where $I_{\{.\}}$ is an Heavyside function and $E(.)$ is the expected value operator.

Expectiles can also be used for estimating the Expected Shortfall, since Expectiles are defined by the minimization of:

$$\mu^* = \underset{\mu\in\mathbb{R}}{ArgMin}\left\{E\left[\left|\tau - I_{\left\{r_t^S<\mu\right\}}\right|\left(r_t^S-\mu\right)^2\right]\right\}, \qquad (15)$$

where the population τ-expectile of r_t^S is the parameter μ and $|.|$ is the absolute value.
Parameters of a conditional model for expectile can be estimated using least square Asymmetric Least Square (ALS) regression, which is the least square analogue for quantile regression:

$$\beta^* = \underset{\beta\in\mathbb{R}^n}{ArgMin}\sum_{t=1}^{T}\left\{\left|\tau - I_{\left\{r_t^S<\widehat{\mu}_{\tau,t}\left(r_{t-1}^S;\beta\right)\right\}}\right| \times \left[r_t^S - \widehat{\mu}_{\tau,t}\left(r_{t-1}^S;\beta\right)\right]^2\right\}. \qquad (16)$$

For each τ-expectile there is a corresponding α-quantile (see Efron, 1991; Jones, 1994; Abdous and Remillard, 1995; Yao and Tong, 1996). Thus, we can use expectiles as estimators of quantiles (and VaR). Taylor (2008-a) explicits the link between the Expectile and Expected Shortfall, that leads to:

$$ES_{\alpha,t-1}(r_t^S;\beta) = \left[1 + \tau\left(1-2\tau\right)^{-1}\alpha^{-1}\right]\widehat{\mu}_{\tau,t}\left(r_{t-1}^S;\beta\right). \tag{17}$$

Thus, over time, for a given value of α, the conditional Expected Shortfall is proportional to the conditional γ-quantile model, which is estimated by the τ-expectile.
Moreover, quantile estimates can be linearly combined through the DAQ model proposed by Gouriéroux and Jasiak (2008). This class of dynamic quantile models is defined by:

$$Q_{\alpha,t}\left(\beta, r_{t-1}^S, y_{t-1}\right) = \sum_{k=1}^{K} a_k\left(r_{t-1}^S, y_{t-1}, \beta_k\right) \times Q_{k,\alpha,t}\left(\beta_k\right) + a_0\left(r_{t-1}^S, y_{t-1}, \beta_0\right), \tag{18}$$

where $Q_{k,\alpha,t}\left(\beta_k\right)$ are path-independent quantile functions and $a_k\left(r_{t-1}^S, y_{t-1}, \beta_k\right)$ are non-negative functions of the past returns and other exogenous variables.
A linear dynamic quantile model is linear in the parameters, then:

$$Q_{\alpha,t}\left(\beta, r_{t-1}^S\right) = \sum_{k=1}^{p} \beta_k\, Q_{k,\alpha,t}\left(\beta, r_{t-1}^S\right). \tag{19}$$

Thus DAQ model can use different quantile functions to model a given quantile. To increase the accuracy of the conditional mutliple model, we can also combine different quantile functions, in a multi-quantile method. Actually, every quantile functions can be extended to define a simple class of parametric dynamic quantile models.

The Expected Shortfall can therefore be expressed as a combination of quantile whose associated probabilities are defined through the estimation of equation (16)(for each quantile):

$$\underset{[1\times1]}{ES_{\alpha,t}} = \underset{[1\times n]}{\mathbf{W}_{\alpha,t}}\ \underset{[n\times1]}{\mathbf{Q}_{\gamma,t}}, \tag{20}$$

with, for $i = [1, 2, ..., n]$:

$$\begin{cases} \mathbf{W}_{\alpha,t} = \left[\begin{array}{cccc} w_1 \times \beta_{\alpha,1} & w_2 \times \beta_{\alpha,2} & ... & w_n \times \beta_{\alpha,n} \end{array}\right] \\ \mathbf{Q}_{\gamma,t} = \left[\begin{array}{cccc} Q^1_{\gamma_1,t} & Q^2_{\gamma_2,t} & ... & Q^n_{\gamma_n,t} \end{array}\right]' \end{cases},$$

and:

$$\left\{\begin{array}{l}\overline{ES}_{\alpha,t}=\beta_i Q^i_{\gamma_i,t}\left(\mu_{\tau_i,t},\beta_i\right)\\ \beta_i=\left[1+\tau_i\left(1-2\tau_i\right)^{-1}\alpha^{-1}\right]\\ \sum_{i=1}^{n}w_i=1\end{array}\right.,$$

where $Q^i_{\gamma_i,t}()$ are several quantile functions associated to the probability γ_i. Aggregating linearly several quantile functions and estimating the good correspondence between the probability α associated to the Expected Shortfall and the probability γ_i associated to each quantile functions (thanks to the τ_i-expectile defined by equation 14) leads to the estimation of the α-Expected Shortfall.
Actually, given n coherent measure of risk, any convex linear combination is another coherent measure of risk, thus the DARE approach provide also a coherent measure of risk contained in the genrated convex hull. This new space of coherent measures allows us to define a spectral measure of risk, which in this case is coherent.

Keeping in mind that using this DARE model, the Expected Shortfall can be expressed as a combination of quantile functions, every formalization in appendix concerning the quantile approach is directly used to justify the use of the Expected Shortfall to model the conditional multiple (only the probability level is changed but can be precisely estimated). We review now the main quantile models which can be used in the Dynamic AutoRegressive Expectile (DARE) approach for the conditional multiple.

Our first aim is to compare several conditional estimation methods of the target multiple within the cushioned portfolio framework. We use a quantile hedging approach based on a DARE Expected Shortfall model. Within this framework, the multiple can be modeled by the inverse of the Expected Shortfall conditional on the distribution of the asset return estimated by a combination of several quantiles.
First, we present below the main VaR models and estimation methods used in this paper for estimating this model of multiple depending on the Expected Shortfall expressed as a combination of quantile functions.

Quantile regression estimation methods do not need any distributional assumptions. Conditional AutoRegressive VaR (CAViaR) introduced by Engle and Manganelli (2004) is one of them. They have defined directly the dynamics of risk by means of an auto regression involving the lagged-Value-at-Risk (VaR) and the lagged value of endogenous variable called CAViaR. They present four CAViaR specifications[5]: model with a symmetric absolute value, an asymmet-

[5]Considering CAViaR model without the autoregressive component, the conditional quantile function is well defined if parameters can be considered as quantile functions too. In fact, CAViaR models weight different baseline quantile functions at each date and can be therefore considered as quantile functions. Adding a non-negative autoregressive component of VaR,

ric slope, indirect GARCH(1,1) and an adaptive form, denoted respectively: $Q^{SAV}_{\gamma,t}\left(r^S_t;\beta\right)$, $Q^{AS}_{\gamma,t}\left(r^S_t;\beta\right)$, $Q^{IG}_{,\gamma,t}\left(r^S_t;\beta\right)$, $Q^{A}_{\gamma,t}\left(r^S_t;\beta\right)$ where:

$$\begin{cases} Q^{SAV}_{\gamma,t}\left(r^S_{t-1};\beta\right) = & \beta_1 + \beta_2 \times Q^{SAV}_{\gamma,t-1}\left(r^S_{t-1};\beta\right) + \beta_3 \times \left|r^S_{t-1}\right| \\ Q^{AS}_{\gamma,t}\left(r^S_{t-1};\beta\right) = & \beta_1 + \beta_2 \times Q^{AS}_{\gamma,t-1}\left(r^S_{t-1};\beta\right) \\ & +\beta_3 \times \max\left(0, r^S_{t-1}\right) + \beta_4 \times \left[-\min\left(0, r^S_{t-1}\right)\right] \\ Q^{IG}_{\gamma,t}\left(r^S_{t-1};\beta\right) = & \beta_1 + \beta_2 \times \left[Q^{IG}_{\gamma,t-1}\left(r^S_{t-1};\beta\right)\right]^2 + \beta_3 \times \left(r^S_{t-1}\right)^2 \\ Q^{A}_{\gamma,t}\left(r^S_{t-1};\beta\right) = & Q^{A}_{\gamma,t-1}\left(r^S_{t-1};\beta\right) - \gamma \\ & +\beta_1 \left[1+\text{Exp}\left\{.5 \times \left[r^S_{t-1} - Q^{A}_{\gamma,t-1}\left(r^S_{t-1};\beta\right)\right]\right\}\right]^{-1} \end{cases}, \tag{21}$$

and where β_i are parameters to estimate and r^S_t is the risky asset return at time t.

Kuester *et al.* (2006) have proposed another CAViaR model the Indirect AR-GARCH CAViaR (an AR(1)-GARCH(1,1) specification), which aims to model autocorrelation in the conditional mean of the returns series.

CAViaR model parameters are estimated using the quantile regression minimization (denotes QR Sum) presented by Koenker and Bassett (1978):

$$\beta^* = \underset{\beta\in\mathbb{R}^n}{ArgMin}\left\{\sum_{t=1}^{T}\left\{\left[r^S_t - Q_{\gamma,t}\left(r^S_{t-1};\beta\right)\right]\left[\gamma - I_{\left\{r^S_t < Q_{\gamma,t}\left(r^S_{t-1};\beta\right)\right\}}\right]\right\}\right\}, \tag{22}$$

with $Q_{\gamma,t} = \mathbf{x}_t\beta$ where $\mathbf{x}_t$ is a vector of regressors, β is a vector of parameters and $I_{\{\cdot\}}$ is an Heavyside function.

The procedure proposed by Engle and Manganelli (2004) to estimate their CAViaR models is to generate vectors of parameters from a uniform random number generator between zero and one, or between minus one and zero (depending on the appropriate sign of the parameters). For each of the vectors then evaluated the QR Sum. The ten vectors that produced the lowest values for the function are then used as initial values in a Quasi-Newton algorithm. The QR sum is then calculated for each of the ten resulting vectors, and the vector producing the lowest value of the QR Sum is to be chosen as the final parameter vector.

Moreover, the structure of CAViaR models (see below) can be directly used for conditional autoregressive expectile models (CARE), replacing centile with expectiles in equation (20). The structure of the CAViaR models can be used for conditional autoregressive expectile (CARE) models. The models parameters can be estimated using ALS with a similar non-linear optimization routine to that used by Engle and Manganelli (2004) for CAViaR models. Using equation (19), CARE models can be converted into conditional autoregressive Expected

the CAViaR conditional quantile function becomes a linear combination of quantile functions weighted by non-negative coefficients. Thus CAViaR model satisfies the properties of a quantile function, even if the indirect GARCH(1,1) or adaptive specifications do not satisfy the monotonicity property.

Shortfall models.

In the following section we aggregate CARE models based (for example on CAViaR models) in respect with equation (10) to estimate conditional multiple for a TIPP based PPI Strategy.
Conditional multiple estimation methods and the performance of their PPI are compared.

4 The DARE Approach for PPI: Some Empirical Evidences

In this section, we describe implementation methods for the conditional multiple model, presented in section 3, then we compare the performances of cushioned portfolio using traditional unconditional multiples, Expected Shortfall based conditional multiple and the DARE conditional multiple (developped in section 3).

4.1 Empirical Evidences

The sample period used in our study consisted of 58 years of daily data of the Dow Jones Index, from 2 January 1950 to 20 september 2008. This period delivered 15,327 returns. We have used a rolling window of 1,000 returns to estimate dynamically method parameters. Figure 1 presents the data that are used for the empirical analysis.

- Please, insert Figure 1 somewhere here -

The multiple depends at each date on the inverse of the 99% Expected Shortfall (denoted $ES_{99\%}$). Using this model the portfolio $ES_{99\%}$ is controlled and extreme returns are taken into consideration[6].
To compute this conditional multiple, we use main methods of quantile estimation exposed in the literature or used by operational. Eight methods of ES calculation are combined: one non-parametric method using the "naive" historical simulation approach, three methods based on distributional assumptions, and the four Expected Shortfall CARE based on CAViaR specifications.

The $ES_{99\%}$ based on historical (or "naive") simulation is denoted $H\_ES_{99\%}$. $ES_{99\%}$ based on distributional assumptions are the normal $ES_{99\%}$ (denoted $Normal\_ES_{99\%}$), the *Risk-metrics* $ES_{99\%}$ (denoted $RM\_ES_{99\%}$), and a GARCH(1,1)

[6] The probability of 1% associated to the Expected Shortfall was chosen in order for focusing on extremes and having at the same time enough points for backing-out good estimations – see below. It is also a standard in Risk Management for defining extreme loss. We tested several higher probability levels in the following.

$ES_{99\%}$ (denoted GARCH_$ES_{99\%}$). The normal ES is built under assumption of normality of returns. It is computed with the empirical mean and standard deviation of the returns of the in sample period. The *Risk-metrics* ES is a standard for practitioners: an exponential moving-average is used to forecast the volatility, and to compute the Expected Shortfall assuming a Gaussian distribution. Another very popular method based on volatility forecasts is presented as the GARCH ES. To compute it, we implement a GARCH(1,1) model. Our choice of the (1,1) specification was based on the analysis of the initial in-sample period of 1,000 returns and on the popularity of this order for GARCH models. We have derived the model parameters using maximum likelihood based on a Student-t distribution and the empirical distribution of standardized residuals. Finally, we present the multiple analysis, using the four CARE CAViaR models (the Symmetric Absolute Value, the Asymmetric Slope, the IGARCH(1,1) and the Adaptive denoted respectively, SAV_$ES_{99\%}$, AS_$ES_{99\%}$, IGARCH_$ES_{99\%}$, and Adaptive_$ES_{99\%}$) using the procedure used by Engle and Manganelli (2004) and ALS, the implementation method was already presented (see section 3).

- Please, insert Figures 2 somewhere here -

- Please, insert Table 1 somewhere here -

4.2 Backtesting Quantile Models

Expected Shortfall are based on quantile estimation methods. To evaluate conditional $ES_{99\%}$ estimation, we first focus on the quantiles evaluation thanks to the hit ratio. We also observe the discrepancy between observations and the conditional $ES_{99\%}$ for only the periods for which the observation exceeds the conditional quantile estimates. Thus excepted Normal $ES_{99\%}$ and RM $ES_{99\%}$, the others $ES_{99\%}$ estimation methods suit well.
The VaR Evaluation tests using in this paper cover the Unconditional and Conditional Tests, the Dynamic Quantile Test, the Distributional Forecast Test and the Exception Magnitudes[7].

4.2.1 Unconditional and Conditional Tests

The most used Backtest VaR models, based on the exception indicator, are the Unconditional and Conditional Tests. This VaR tests cover Unconditional Coverage Test (Kupiec, 1995), Independent Test and Conditional Coverage Test (Christoffersen, 1998). The conditional coverage test combines the Unconditional Coverage Test and the Independent Test.
For the test, the exception indicator variable is define as:

$$I_{Mt+1} = \begin{cases} 1 \text{ if } R_{pt+1} < VaR_{Mt}(\alpha) \\ 0 \text{ otherwise} \end{cases} \tag{23}$$

[7] In this section, we only gives an summary of the test. For more details on these tests, see for instance Christoffersen (1998); Christoffersen and Pelletier (2003) or Ferreira and Lopez (2005).

where M the subscript stands for the model used to estimate VaR, with t the current period, R_{pt+1} the actual portfolio return on the period $t+1$ and $VaR_{Mt}(\alpha)$ the one-period VaR estimates from model M for the portfolio at the α quantile on period t.

Unconditional Coverage Test

The Unconditional test of the coverage of VaR estimates is an simply counts exceptions over the entire period. Kupiec (1995) shows that if the VaR model is well specified then the exceptions occur can be modeled as independent draws from a binomial distribution with a probability of occurrence equal to α percent. The likelihood ratio statistic is:

$$LR_{uc} = 2\{\log[\widehat{\alpha}^x(1-\widehat{\alpha}^{T-x})] - \log[\alpha^x(1-\alpha^{T-x})]\} \tag{24}$$

where x is the number of exception, T the number of observations and $\widehat{\alpha} = x/T$ the unconditional coverage.
For the hypothesis that $\widehat{\alpha} = \alpha$, the likelihood ratio LR_{uc} has an asymptotic $\chi^2(1)$ distribution.

Independent Test

This test detect the serial independence in the VaR model forecast. The Likelihood function for the test is:

$$LR_{ind} = 2[\log L_A - \log L_0] \tag{25}$$

which has an asymptotic $\chi^2(1)$ distribution (Christoffersen, 1998). Where L_A is the likelihood function under the hypothesis of first-order Markov dependence:

$$L_A = (1-\pi_{01})^{T_{00}}\pi_{01}^{T_{01}}(1-\pi_{11})^{T_{10}}\pi_{11}^{T_{11}} \tag{26}$$

with T_{ij} denotes the number of observations in the state j after having been in i in the previous period, $\pi_{01} = T_{01}/(T_{00}+T_{01})$ and $\pi_{11} = T_{11}/(T_{10}+T_{11})$. At the contrary, L_0 is the likelihood function under the hypothesis of independence ($\pi_{01} = \pi_{11}$):

$$L_0 = (1-\pi)^{T_{00}+T_{10}}\pi^{T_{01}+T_{11}} \tag{27}$$

with $\pi = (T_{01}+T_{11})/T$.

Conditional Coverage Test

The Conditional test used here is a test of correct conditional coverage proposed by Christoffersen (1998). In this case, if the VaR estimates have correct conditional coverage, the exception variable must exhibit both correct unconditional coverage and serial independence. The Conditional coverage test (LR_{cc}) is therefore a joint test of these properties, and the relevant statistic is, with the previous notations:

$$LR_{cc} = LR_{uc} + LR_{ind} \tag{28}$$

which is asymptotically distributed as a $\chi^2(2)$.

Exception Magnitudes
The main idea of this test is that the magnitudes of exceptions should be of primary interest to the various users of VaR models (see Hendricks, 1996; Berkowitz, 2001; Ferreira and Lopez, 2005). For this test, as in the LR_{dist} test, Berkowitz (2001) transforms the empirical series into standard normal z_{Mt+1} series. The z_{Mt+1} values are then compared to the normal random variables with the desired coverage level of the VaR estimates. If the VaR model generating the empirical quantiles is correct, the γ_{Mt+1} series, define by 29, should be identically distributed and (μ_M, σ_M)[8] should equal (0,1).

$$\gamma_{Mt+1} = \begin{cases} z_{Mt+1} & \text{if } z_{Mt+1} < \Phi^{-1}(\alpha) \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

where Φ is the standard normal cumulative distribution function. Finally, the corresponding test statistic is:

$$LR_{mag} = 2\ [L_{mag}(\mu_M, \sigma_M) - L(0,1)] \tag{30}$$

which is asymptotically distributed $\chi^2(2)$; where:

$$\begin{aligned} L_{mag}(\mu_M, \sigma_M) = {} & \textstyle\sum_{\gamma_{Mt+1}=0} \log\{1 - \Phi[(\Phi^{-1}(\alpha) - \mu_M)\sigma_M^{-1}]\} \\ & + \textstyle\sum_{\gamma_{Mt+1}\neq 0} \{-(1/2)\log(2\pi\sigma_M^2) - (\gamma_{Mt+1} - \mu_M)^2 (2\sigma_M^2)^{-1} \\ & - \log[\Phi((\Phi^{-1}(\alpha) - \mu_M)\sigma_M^{-1}]\}. \end{aligned}$$

4.2.2 Dynamic Quantile Test

The Dynamic Quantile (DQ) test is proposed by Engle and Manganelli (2004) to detect a good model for VaR estimation. The ideas is that a good model should produce a sequence of unbiased and uncorrelated exception indicator variable. Under the null hypothesis, all coefficients as well as the intercept are zero in the regression (31) of the exception indicator variable on its past values and on current VaR estimate such as[9]:

$$I_{Mt+1} = \delta_0 + \sum_{k=1}^{5} \delta_k I_{Mt-k+1} + \delta_6 VaR_{Mt} + \epsilon_{t+1} \tag{31}$$

where I_{Mt+1} is the exception indicator variable, δ_i are the coefficients of the regression, VaR_{Mt} is the quantile estimates at time t, ϵ_{t+1} is the residual associated to the regression.

4.2.3 Distributional Forecast Test

This test is based on the fact that the VaR models are generally characterized by their distribution forecasts of portfolio returns; thus, the evaluations should

[8] μ_M and σ_M are respectively the unconditional mean and standard deviation of the z_{Mt+1} series.

[9] In this paper, following Ferreira and Lopez (2005), we use 5 lags.

be based directly on these forecasts. The object of interest in these evaluation methods is the observed quantile q_{Mt+1}, which is the quantile under the distribution forecast f_{Mt+1} in which the observed portfolio return R_{pt+1} actually falls. If the underlying VaR model is accurate, then its q_{Mt+1} series should be independent and uniformly distributed over the unit interval. The observed quantile q_{Mt+1} is defined as:

$$q_{Mt+1}(R_{pt+1}) = \int_{-\infty}^{R_{pt+1}} f_{Mt+1}(R_p) dR_p \tag{32}$$

To test the two properties of the series of observed quantile (independence and uniform distribution), Diebold *et al.* (1998) propose the use of CUMSUM statistic, Christoffersen and Pelletier (2003) Duration-based tests and Berkowitz (2001) propose a Likelihood Ratio Test. In this paper, we use a Likelihood Ratio Test.
For this of the two properties simultaneously, Berkowitz (2001) propose the transformation $z_{Mt+1} = \Phi^{-1}(q_{Mt+1})$; which corresponds to the inverse of the standard normal cumulative distribution function of q_{Mt+1}. Under the null hypothesis, the LR statistic is:

$$LR_{dist} = 2[L(\mu_M, \rho_M, \sigma_M^2) - L(0, 0, 1)] \tag{33}$$

which is an asymptotically distributed $\chi^2(3)$, with:

$$\begin{aligned} L(\mu_M, \rho_M, \sigma_M^2) = & \textstyle\sum_{t=1}^{T} \{-(1/2)\log[(2\pi\sigma_M^2)(1-\rho_M^2)^{-1}] - L_z\} \\ & -(1/2)(T-1)\log(2\pi\sigma_M^2) \\ & -\textstyle\sum_{t=1}^{T-1}(2\sigma_M^2)^{-1}(z_{Mt+1} - \mu_M - \rho_M z_{Mt})^2 \end{aligned}$$

with $L_z = \{[z_{Mt+1} - \mu_M(1-\rho_M)^{-1}]^2\}[2\sigma_M^2/(1-\rho_M^2)]^{-1}$ and where (μ_M, ρ_M) are respectively the conditional mean and AR(1) coefficient corresponding to the z_{Mt+1} series; *i.e.* $z_{Mt+1} - \mu_M = \rho_M(z_{Mt} - \mu_M) + \epsilon_{t+1}$, with ϵ_{t+1} the normal random variable with zero mean and variance σ_M^2.

4.3 Conditional Multiples: Model Evidence

These $\mathrm{ES}_{99\%}$ models can be used for estimating conditional multiples.
We use the same notations, which were used for $\mathrm{ES}_{99\%}$ models. Thus for example, the conditional multiple estimated using the GARCH(1,1) $\mathrm{ES}_{99\%}$ (denoted $\mathrm{GARCH\_ES}_{99\%}$) is denoted m_GARCH. Conditional multiple estimations over the 144 post sample periods are represented on Figures 3. The estimations of conditional multiples spread between 1, and 13, which are compatible with multiple values used on this market by practitioners (between 3 and 8) and by the literature (under 13). The multiple is the parameter which determines the exposition of the cushioned portfolio. To guarantee a predetermined floor, the multiple has to be inferior to the inverse of the potential loss that the risky asset could reach before the portfolio manager rebalances its position. If we assume that the portfolio manager can rebalance totally his position during one day, all

estimations of conditional multiple with this first model allow guaranteeing the predetermined floor defined by the investor.

After having evaluated these $ES_{99\%}$ models, we can combine them for estimating conditional multiples according to the first DARE based model presented above.

- Please, insert Figures 3 somewhere here -

- Please, insert Table 2 somewhere here -

If we analyze the empirical distributions of conditional multiple (see Figure 4), we observe that for every conditional multiple estimation methods we obtain several modes. One mode is around a low value of the multiple (between 1 and 2 depending of estimation methods) associated with a more defensive behavior of the cushioned portfolio during turbulent period, and another mode around 6 associated to "classical" and realistic value of the multiple. On one hand, distribution shape of conditional multiple using Asymmetric Slope, IGARCH(1,1), Symmetric Value and DARE of the multiple are very similar, and on the other hand "naive" historical, adaptive and Gaussian conditional multiples densities have comparable characteristics. For the sake of clarity, we have thus only represented on Figure 4 the densities of the DARE model and for the "naive" historical estimation of the multiple.

- Please, insert Figure 4 somewhere here -

4.4 Conditional *versus* Unconditional PI Strategy

We consider in this section a variety of performance measures to compare the different strategies based on economic principles.

TIPP performances based on traditional unconditional multiples, conditional Expected Shortfall multiple and the DARE approach are analysed for 1 to 15 years from 1950. The conditional DARE based cushioned portfolio is always one of the first portfolio according to Sharpe, Sortino, Omega and Kappa performance *criteria*[10]. The ranking between other conditional multiple estimation methods changes according to time and investment period.

- Please, insert Table 6 somewhere here -

[10] Except for the one year TIPP where the ranking is not so clear due to a limited cushion (explained by the short investment horizon) and by a high leverage bias (portfolios with high multiple are not penalized by a long moneterization period).

- Please, insert Table 7 somewhere here -

Values of cushioned portfolio are path dependent. The performances of these guaranteed portfolios, determined by using different estimation methods of the conditional multiple, are often not easy to compare. Actually, the performance of the insured portfolio depends more on its start date, and on the investment horizon than of the estimation of the conditional multiple, moreover some estimation methods are more adapted to particular period. To investigate whether these different estimations methods lead to significantly different performances (for a long term analysis), we propose to introduce an original "multi-start" analysis. The "multi-start analysis" consists for a fixed investment horizon (here one year) in computing every value of insured portfolios beginning at every moment of the post sample period, according to its conditional multiple estimations. An illustration of the method is presented on Figure 5. Table 3 reports the main characteristics of the returns of cushioned portfolio using the "multi-start" analysis.

- Please, insert Figure 5 somewhere here -

- Please, insert Table 3 somewhere here -

On the contrary of what we observe in a classical analysis with a predefined start and end of analysis, the characteristics of covered portfolios returns are not significantly different. But if we choose a specific start and end date, then covered portfolio returns using this conditional multiple can be more different. In following sections, the analysis is limited to the Asymmetric Slope multiple model, for the sake of simplicity and because of the similarity of the behavior of this multiple with the one of the best multiple estimation methods. The Asymmetric Slope multiple based Time-Varying Proportion Portfolio Insurance was dynamically back-tested from 1937: the floor was violated only one time (during the 1987 crash), loosing 0.61% of its guaranteed value.

- Please, insert Table 4 somewhere here -

5 About the Gap Risk Estimation

We have used a probabilistic approach to model the conditional multiple. By construction, the guarantee is associated to a tiny level of probability but it is not an absolute guarantee. The risk of violating the floor protection is called gap risk.

At any rate, even in an unconditional multiple PPI framework, the guarantee depends on the estimation of the maximum potential loss that the risky asset can reach before the portfolio manager is able to rebalance his position. Actually, in continuous time, the PPI strategies provide a value above a floor level unless the price dynamic of the risky asset has jumps. In practice, it is caused by liquidity constraints and price jumps. Both can be modeled in a setup where the price dynamic of the risky asset is described by a continuous–time stochastic process but trading is restricted to discrete time. If the potential loss is underestimated, the predefined guarantee of the portfolio is not anymore insured. The only way to be sure to reach a perfect guarantee is to choice an unconditional multiple equal to one (the potential loss is then supposed to be 100%). For all other cases, we should estimate the gap risk between a perfect insurance and the insurance proposed assuming an estimation of the potential loss. The Gap Risk at time t (denoted GR_t) can thus be defined such as:

$$GR_t = V_{t,DPPI_{m_t}} - V_{t,CPPI_{m=1}} \tag{34}$$

We propose here a way to estimate this gap risk, and we recommend to add it as an additional performance criterion of insured portfolios.

To estimate the gap risk in a PPI framework, we use the multiple at any time to get the assumed estimation of the maximum potential loss. To reach a perfect guarantee assuming no maximum potential loss scenario, we will hedge the gap risk buying a put whose maturity will be the rebalance frequency. The strike (denoted K) will be defined each day thanks to the assumed (or modeled) maximal potential loss at T. The Gap Risk at time t (denoted GR_t) is thus also given by:

$$GR_t = p[K = S_t.(1 - 1/m_t)] \tag{35}$$

The put price denoted $p[.]$ is computed using several pricing methods for European options and realistic transaction costs with the data used in previous section (for the DARE model).

- Please, insert Table 5 somewhere here -

In addition, an OBPI pay–off can be reduced for example to a put and the underlying asset. To reach a perfect guarantee it is also possible (in theory) to build an OBPI using the DPPI as underlying risky asset.

$$V_{OBPI_t} = p[K = S_t.(1 - 1/m_t)] + V_{t,DPPI_{m_t}} \tag{36}$$

It leads to:

$$GR_t = V_{OBPI_t} - V_{DPPI_{m_t}} \tag{37}$$

We assume now that it is not possible not distinguish the small variations of the multiple, *i.e.* it can be considered as a constant value when its variations stay between some bounds. The limit of the variation will be defined by the ratio τ.

We can set a sequence of stopping times $\{\Theta_i\}_{i=1,\ldots,I}$ which correspond to the times the perception of the multiple is modified. We define the stopping times by induction:

$$\Theta_0(\omega) = 0, \tag{38}$$

with:

$$\bar{m}_0 = m_0(\omega).$$

Then, for every $i \in N^*$,

$$\Theta_i(\omega) = \underset{\omega\in\Omega}{ArgMin}\{t |\ t \geq \Theta_{i-1}(\omega),\ |m_t(\omega) - \bar{m}_{i-1}| \geq \bar{m}_{i-1}(1-\tau)\}, \tag{39}$$

and:

$$\bar{m}_i = m_{\Theta_i(\omega)}(\omega).$$

Moreover, we define the index stochastic process I by:

$$I_t(\omega) = \{i \in N |\ \Theta_i(\omega) \leq t, \Theta_{i+1}(\omega) > t\}. \tag{40}$$

Then, the value of the cushion is given by:

$$\begin{aligned} C_t = \quad & C_0 Exp\left\{\sum_{i=0}^{I_t}\left[\int_{\Theta_i}^{\Theta_{i+1}}\left\{r + \bar{m}_i\left[\mu(s,S_s) - r\right] - \tfrac{1}{2}\bar{m}_i^2\sigma^2(s,S_s)\right\} ds \right.\right. \\ & \left.\left. + \int_{\Theta_i}^{\Theta_{i+1}} \bar{m}_i\sigma(s,S_s)dW_s\right]\right\}. \end{aligned} \tag{41}$$

The time can be then divided into intervals between two stopping times ($[\Theta_i;\Theta_{i+1}[$) for which the value of the multiple is constant, $\bar{m}_i$.
When the risky asset price follows a geometric Brownian motion (*i.e.* $\mu(.,.)$ and $\sigma(.,.)$ are constant), by setting up the level of risk, we obtain the following relation between multiples and stopping times (as in case 2):

$$P\left[\ C_{\Theta_{i+1}} > 0 \middle|\ C_{\Theta_i} > 0\right] \geq (1-\alpha) \tag{42}$$

$$\Leftrightarrow$$

$$m_{t_i} \leq \left\{1 - Exp\left[N^{-1}(\alpha)\ \sigma\ (\Theta_{i+1} - \Theta_i)^{-1/2} + \left(\mu - r - 1/2\sigma^2\right)(\Theta_{i+1} - \Theta_i)\right]\right\}^{-1},$$

where $N^{-1}(\alpha)$ denotes the α-quantile of the standard Gaussian distribution.

We can now consider a natural case of process for m. We assume that it follows the Cox-Ingersoll-Ross one, with the property of mean reverting,

$$dm_t = \beta(\theta - m_t)dt + \gamma\sqrt{m_t}dW_t, \tag{43}$$

with $\beta > 0$, $\theta > 0$, $\gamma > 0$. Moreover, we assume that:

$$2\beta\theta \geq \gamma^2. \tag{44}$$

The solution of the equation (52) is obtained through the Laplace transform (see Geman and Yor, 1993) and is given by:

$$m_t = Exp\left(-\beta t\right) B\left\{\gamma^2(4\beta)^{-1}\left[Exp\left(\beta t\right)-1\right]\right\}, \tag{45}$$

where $B[.]$ is a squared Bessel process (BESQ) of dimension $\delta = (4\beta\theta)\gamma^{-2}$. Thanks to the condition (56), the value of m will be always positive. With properties of such process governing hitting times, we can obtain properties on the sequence of stopping times (θ_i).

- Please, insert Figure 6 somewhere here -

6 Conclusions

We model in the paper the multiple as a function of the Expected Shortfall determined as a combination of quantile function. This has the advantages of dealing with a more robust and flexible multiple quantile estimations at the same time, in a coherent risk measure framework. The model proposed in this paper for the conditional multiple, allows us guaranteeing the insured floor according to market evolutions, on a large post sample period. This method provides a rigorous framework to determine the multiple, which is the main parameter of cushioned portfolio preserving a constant exposition to risk. To compute the conditional multiple, according to this model, appropriate method to estimate the quantile of the risky asset return are combined to determine the Expected Shortfall. If the quantile used are well modeled (hit ratio not significantly different from α% and no cluster of exceedances), the resulting Expected Shortfall is correct and the guarantee is insured. A Dynamic AutoRegressive Expectile approach for the TIPP conditional multiple is introduced and appears to be impressive compared to traditional unconditional and other conditional methods.

Moreover, this paper has introduced the use of an original "multi-start / horizon" performance comparison framework, which seems more adapted to path and start dependent financial product such as the constant proportion portfolio insurance portfolio. This comparison framework will be thus second further extended with the introduction of the Multi-start Multi-horizon Time-Varying Proportion Portfolio Insurance Mixed Approach (MTPPI) based on aggregation of various multi-start TVPPI associated to several horizon. At last, we have also proposed an empirical way for estimating the gap risk between theoretical perfect portfolio insurance and the insurance proposed with transaction costs. Additional performance *criteria* will also be introduced to examine the adequacy of such insured portfolios together with the multi-horizon multi-start approach.

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8 Appendix

Appendix 1: Portfolio Insurance based on Quantile *Criterion* (in a Marked Point Process Framework)

As we argue in the text, despite the fact that the multiple is conditional and thus time-varying, the portfolio is still guaranteed under some conditions. Indeed, a guaranteed portfolio is defined so that the portfolio value will always be above a predefined floor at a given high probability level. Assume that the risky price follows a marked point process, which is characterized by the sequence of marks $(S_l)_{l\in\mathbb{N}_+^*}$ and the increasing sequence of times $(T_l)_{l\in\mathbb{N}_+^*}$ at which the risky asset varies.

In the CPPI framework, the first following "global" quantile hedging condition can be considered (see Bertrand and Prigent, 2002):

$$\text{Prob}\left[\forall t \leq T, C_t \geq 0\right] \geq 1-\delta \tag{46}$$

where C_t is the cushion defined as the spread between the portfolio value and the guaranteed floor, Prob[.] stands for the unconditional probability and $(1-\delta)$ for a probability confidence level. Splitting the complete period, denoted $[0, ..., T]$, into various L successive subperiods $[T_l, T_{l+1}[$, the previous equation is equivalent to define the multiple m as such (see Bertrand and Prigent, 2002):

$$m \leq \left[f_T^{-1}\left(1-\delta\right)\right]^{-1} \tag{47}$$

where $f_T^{-1}(.)$ is the quantile function, evaluated at a risky asset return for which the inverse function - denoted $f_T(.)$, is equal to $(1-\delta)$ - a specified unconditional quantile, as such:

$$f_T\left(r\right) = \sum_{l=1}^{\infty}\left\{\text{Prob}\left[M_l \leq r \mid T_l \leq T < T_{l+1}\right] \times \text{Prob}\left[T_l \leq T < T_{l+1}\right]\right\} \tag{48}$$

with Prob$[. \mid T_l \leq T < T_{l+1}]$ denoting the conditional probability given the event $T_l \leq T < T_{l+1}$ and:

$$M_l = \underset{k=[1,\ldots,L]}{Max}\left\{-r_1, ..., -r_k\right\} \tag{49}$$

where $r_t = \left(S_t - S_{t-1}\right)/S_{t-1}$ is the risky asset return at time t.

Following the same principle in a time-varying framework now, another "local" quantile condition can also be introduced, based this time on a conditional quantile corresponding to a conditional probability confidence level denoted $(1-\alpha)$, such as for any time $t \in [T_l, T_{l+1}[$ with $t \leq T$:

$$\text{Prob}\left[C_{T_l} > 0 \mid \Omega_{T_{l-1}}\right] \geq 1-\alpha \tag{50}$$

where $\Omega_{T_{l-1}}$ is the σ-algebra generated by the set of all intersections of $\left\{C_{T_{l-1}} > 0\right\}$ with any subset $\Omega_{T_{l-1}}$ of the σ-algebra generated by the observation of the marked point process until time T_{l-1}.

From previous condition (37), an upper bound on the multiple can be deduced according to specific assumptions (see Ben Ameur and Prigent, 2007) for the special case of GARCH-type models with deterministic transaction times. Consider the systems of auto regressive equations:

$$\begin{cases} R_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \times R_{t-i} + \sigma_t \times \epsilon_t \\ \Lambda(\sigma_t) = \beta + C_0(\epsilon_{t-1}) + C_1(\epsilon_{t-1}) \times \Lambda(\sigma_{t-1}) \end{cases}$$

where σ_t denotes the volatility, the sequence $(\epsilon_t)_t$ is i.i.d with common probability distribution function, Λ, $C_0(.)$ and $C_1(.)$ are deterministic functions. In particular, the function $\Lambda : \mathbb{R}^+ \to \mathbb{R}$ is assumed to be strictly increasing.

We define Z_{t-1} such as:

$$Z_{t-1} = \alpha_0 + \sum_{i=1}^{p} \alpha_i \times R_{t-i} + \sigma_t \times F^{-1}\left[1 - (1-\epsilon)^{1/T}\right]$$

where $F(.)$ denotes the common cumulative distribution function of the random variables ϵ_t.

The quantile condition at time $t-1$ can be satified as soon as the cushion value at time $t-1$ is none negative. In this case, if $Z_{t-1} > 0$, then the multiple can take any positive value, and, if $Z_{t-1} < 0$, the the conditional multiple must satisfy:

$$m_{t-1} \leq \left[1 - \exp(Z_{t-1})\right]^{-1}$$

When the cushion is positive at time $t-1$, the choice of multiple is very flexible. Thus, within the quantile condition at time $t-1$, we can add some other conditions on the multiple to better benefit from market conditions.

When the cushion is negative (which happens with a small probability), the quantile condition generally cannot be satified, except for small values of $(1-\epsilon)$. But in this case, this is not a true insurance condition. Therefore, a possible strategy is to adopt the previous condition when the cushion is positive and to invest the whole portfolio value on the riskless asset, as soon as the cushion is negative.

Appendix 2: Continuous-time Version of Conditional Multiple

We consider a filtered probabilistic space $[\Omega, F, (F_t)_t, P]$. We assume that the risky asset price follows a diffusion process:

$$dS_t = S_t\left[\mu(t, S_t)\, dt + \sigma(t, S_t)\, dW_t\right], \tag{51}$$

where W is a standard Brownian motion and the functions $\mu(.,.)$ and $\sigma(.,.)$ satisfy usual Lipschitz and Growth conditions to ensure the existence and unicity of the solution of the previous stochastic differential equation.

The CPPI portfolio value satisfies:

$$dV_t^{CPPI} = (V_t^{CPPI} - e_t)\frac{dB_t}{B_t} + e_t\frac{dS_t}{S_t}, \tag{52}$$

with:

$$\left\{\begin{array}{l} V_t^{CPPI} = C_t + F_t \\ e_t = m_t C_t \\ dF_t/F_t = dB_t/B_t = rdt \end{array}\right. .$$

Case 1. The multiple m_t evolves in continuous-time.
It is assumed to follow a predictable stochastic process such that next Condition of existence is satisfied:

$$\int_0^T m_s^2 ds < +\infty. \tag{53}$$

Then, the cushion value C satisfies:

$$dC_t = d(V_t^{CPPI} - F_t), \tag{54}$$

Under the previous Condition of existence, we deduce:

$$V_t^{CPPI} = C_t + F_0 Exp\,(rt)\,, \tag{55}$$

with:

$$C_t = C_0 Exp\left\{\int_0^t \left\{r + m_s\left[\mu(s,S_s) - r\right] - \frac{1}{2}m_s^2\sigma^2(s,S_s)\right\} ds + \int_0^t m_s\sigma(s,S_s)dW_s\right\}.$$

Case 2. The multiple and the exposition can only be changed on a discrete-time basis (more realistic case).

Consider the dates $t_0 < t_1 < ... < t_n = T$. Since market prices cannot not be evaluated between two fixed dates, both the cushion and the exposition can be considered as fixed during each time period $[t_i, t_{i+1}[$ (since they are *simple* processes). Then, the multiple m_{t_i} defined on each period $[t_i, t_{i+1}[$, which is equal to the ratio e_{t_i}/C_{t_i}, is also fixed.
Therefore, we get, for any time $t \in [t_i, t_{i+1}[$:

$$\begin{aligned} V_t &= (V_{t_i} - e_{t_i})Exp\left[r(t - t_i)\right] \\ &\quad + e_{t_i} Exp\left\{\int_{t_i}^t \left\{\mu(s,S_s) - \frac{1}{2}\sigma^2(s,S_s)\right\} ds + \int_0^t \sigma(s,S_s)dW_s\right\}, \end{aligned} \tag{56}$$

with:

$$e_{t_i} = m_{t_i}\left\{\, V_{t_i} - F_0\, Exp\left[r(t_i - t_0)\right]\,\right\}.$$

Thus, we can deduce the portfolio value V_{t_i} by induction.
When the risky asset price follows a geometric Brownian motion (*i.e.* $\mu(.,.)$ and $\sigma(.,.)$ are constant), we get:

$$P\left[\left. C_{t_{i+1}} > 0\right| C_{t_i} > 0\right] \geq (1-\alpha) \tag{57}$$

$$\Leftrightarrow$$

$$m_{t_i} \leq \left\{ 1 - Exp\left[N^{-1}(\alpha)\ \sigma\ (t_{i+1}-t_i)^{-1/2} + \left(\mu - r - 1/2\sigma^2\right)\ (t_{i+1}-t_i)\right] \right\}^{-1}$$

where $N^{-1}(\alpha)$ denotes the α-quantile of the standard Gaussian distribution.

Case 3. The multiple is now a stochastic process. We assume that $m : \Omega \times [0,T] \mapsto I\!R$ is P-a.s. bounded and measurable. In this case, the multiple is locally integrable. Then, the value of the cushion is still:

$$\begin{aligned} C_t = \quad & C_0 Exp\left\{\int_0^t \left\{r + m_s\left[\mu(s,S_s) - r\right] - \tfrac{1}{2}m_s^2\sigma^2(s,S_s)\right\} ds \right. \\ & \left. + \int_0^t m_s\sigma(s,S_s)dW_s\right\} \end{aligned} \tag{58}$$

and is perfectly defined.

This case encompasses the case where m is solution of a stochastic differential equation on a set $[0,T]$. Indeed, this solution will be a P-a.s. continuous process then bounded. This is also in relation with the empirical results of this paper about the multiple, *i.e.* it belongs in our case to [1, 13].

Case 4. We can also apply the model of Cont and Tankov (2007) with continuous time and price processes with jumps. For the risky asset, we consider a jump diffusion process given by

$$dS_t = S_{t-}dZ_t \tag{59}$$

where Z_t is a Lévy process. This last one will be writen here in the following form:

$$dZ_t = \mu dt + \sigma dW_t + d\left(\sum_{i=1}^{N_t}(J_i - 1)\right) \tag{60}$$

with N_t a Poisson process with rate λ and with J the jump size. This corresponds to the increment of the compound jump process $\sum_{i=1}^{N(t)} J_i$. The J_i are assumed to be i.i.d. random variables. Moreover, the random processes (W_t), (N_t) $(0 \leq t \leq T)$ and (J_i) $(i = 0, 1\ldots)$ are independent.

The space-time jump process JdN_t has mean $E[J]\lambda dt$, and dN_t is a Poisson discrete distribution:

$$Prob[dN_t = k] = exp(-\lambda dt)(\lambda dt)^k/k!$$

Following Kou (2002), we consider the special case for which $Y = \ln(J)$ has an asymmetric double exponential distribution, i.e.:

$$f_Y(y) = p \cdot \nu_1 e^{-\nu_1 y} 1_{\{y \geq 0\}} + (1-p) \cdot \nu_2 e^{\nu_2 y} 1_{\{y \geq 0\}} \tag{61}$$

with $\nu_1 > 1$, $\nu_2 > 0$, $p, q \geq 0$ and $p + q = 1$. This implies the basic assumption stated by Cont and Tankov (2007): $\Delta Z_t > -1$ almost surely. It insures the price of the risky asset to be always positive.

We notice also that the solution of the stochastic equation is:

$$S_t = S_0 exp[(\mu - \sigma^2/2)t + \sigma W_t] \prod_{i=1}^{N(t)} J_i \tag{62}$$

Moreover, $E[Y] = p\frac{\nu_1}{\nu_1+1} + q\frac{\nu_2}{\nu_2+1}$.

Similarly to case 2, the multiple m_i is defined on every time interval $[t_i; t_{i+1}]$. Then, the portfolio value satisfies for every $t \in [t_i; t_{i+1}]$:

$$dV_t^{CPPI} = m_i(V_{t-}^{CPPI} - B_t)\frac{dS_t}{S_{t-}} + \{V_{t-}^{CPPI} - m_i(V_{t-}^{CPPI} - B_t)\}rdt \tag{63}$$

which implies:

$$dC_t = C_{t-}\{m_i dZ_t + (1 - m_i)rdt\} \tag{64}$$

We define the discounted cushion $C_t^* = \frac{C_t}{B_t}$ and then, with Ito lemma, we obtain:

$$dC_t^* = C_{t-}^* m_i(dZ_t - rdt). \tag{65}$$

We define $L_t = Z_t - rt$. Then, we obtain

$$C_t^* = C_{t_i}^* \varepsilon[m_i(L_t - L_{t_i})] \tag{66}$$

where ε is the Doléans-Dade stochastic exponential defined by:

$$\frac{d\varepsilon(m_i L)_t}{\varepsilon(m_i L)_{t-}} = m_i dL_t. \tag{67}$$

This gives:

$$\varepsilon[m_i(L_t - L_{t_i})] = exp[m_i(L_t - L_{t_i}) - 1/2\sigma^2(t - t_i)] \prod_{t_i \leq s \leq t} (1 + \Delta L_s) exp(-\Delta L_s). \tag{68}$$

Formulae (66) is true as soon as $C_t^* > 0$ (or equivalently $C_t > 0$).

Since $\varepsilon(m_i L)_t = \varepsilon(m_i L)_{t-}(1 + m\Delta L_t)$, the condition $C_t \leq 0$ is equivalent to $m_i \Delta L_t \leq -1$.

We note ν the Lévy measure of the compound process $\sum_{i=1}^{N_t}(J_i-1)$. Then, the probability to go upper the floor is:

$$\begin{aligned} P\left[\, C_{t_{i+1}}>0 \middle| \, C_{t_i}>0\right] &= exp\left(-(t_{i+1}-t_i)\int_{-\infty}^{-1/m_i}\nu(dx)\right) \\ &= exp\left(-(t_{i+1}-t_i)\int_{-\infty}^{\ln(1-1/m_i)}\lambda f_Y(y)(dy)\right) \end{aligned} \tag{69}$$

Appendix 3: Performance Measure

We have considered in this article a variety of performance measures to compare the different strategies based on economic principles.
We have used traditional performance measures, risk measures based on lower partial moments, and other measures describe hereafter.

The Drawdown

Drawdown-based measures are particularly popular in practice. They often are used by commodity trading advisors because these measures illustrate what the advisors are supposed to do best – continually accumulating gains while consistently limiting losses (see Lhabitant, 2004).

The drawdown of a security is the loss incurred over a certain investment period. In describing drawdown-based risk measures, r_{it-T} denotes the return realized over the period from t to T ($t<T\leq T$). For all these returns, MD_{i1} denotes the lowest return and MD_{i2} the second lowest return, and so on. In general, the smallest return, MD_{i1}, is negative and denotes the maximum possible loss that could have been realized in the considered period of time. The *Calmar ratio* (see Young, 1991), Sterling ratio (see Kestner, 1996), and Burke ratio (see Burke, 1994) use the maximum drawdown, an average above the N largest drawdowns (which does not react too sensitively to outliers), and a type of variance above the N largest drawdowns (which takes into account that a number of very large losses might represent a greater risk than several small declines) as risk measures[11]:

$$C_i = \frac{r_i^d-r_f}{-MD_{i1}} \tag{70}$$

$$St_i = \frac{r_i^d-r_f}{\frac{1}{N}\sum\limits_{j=1}^{N}-MD_{ij}} \tag{71}$$

$$B_i = \frac{r_i^d-r_f}{\sqrt[2]{\sum\limits_{j=1}^{N}MD_{ij}^2}} \tag{72}$$

[11] Defining the Calmar and the Sterling ratio, the maximum drawdown is preceded by a minus sign so that the denominator is positive and higher values for the denominator represent a higher risk.

The Transaction Cost

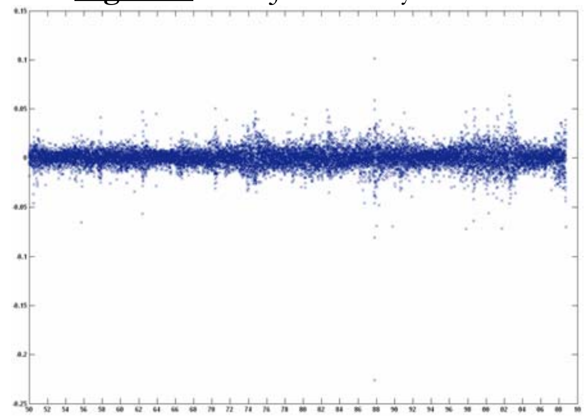
Finally, since we compare dynamic strategies, we must control for the possible effect of transaction costs. Indeed, given that a static strategy generates very low turnover, the gain of dynamic strategies may be partly of set by transaction costs. In practice, it is diffcult to estimate the actual transaction costs, since a wide range of costs may be incurred depending on the asset and the type of customer relation. For this reason, we follow the approach of Han (2006) and compute the breakeven transaction cost, denoted τ^{be}. This parameter measures the level of transaction costs required to make the investor indifferent when choosing between the dynamic and a reference static strategies. If transaction costs are equal to a fixed fraction τ of the value traded in all stocks in the portfolio, tv, the average weekly transaction cost of this strategy is $\tau \times tv$, where:

$$tv = \frac{1}{T} {}_{t=1}^{T} {}_{i=1}^{n} \left| \alpha_{i,t} - \frac{\alpha_{i,t-1} \left(1 + r_{i,t}\right)}{1 + r_{p,t}} \right| \tag{73}$$

Finally, the breakeven transaction cost between the dynamic strategy d and the static strategy s is defined as $\tau^{be} = \left(\overline{r}_p^d - \overline{r}_p^s\right) / \left(tv^d - tv^s\right)$. If the actual transaction cost is lower than τ^{be}; the investor will prefer the dynamic strategy over the static strategy.[12]

[12] Actual transaction costs are difficult to estimate. Marquering and Verbeek (2001) consider 1% as high. Balduzzi and Lynch (1999) consider that 0.5% is a reasonable transaction cost for direct trades in stocks, whereas these costs may be as low as 0.01% for an institutional investor trading in futures. See also the extensive discussion in Han (2006).

Figure 1: Dow Jones Daily Returns



Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008; computations by the authors.

Figures 2: $ES_{99\%}$ Estimates

Fig.2.1: Historical $ES_{99\%}$

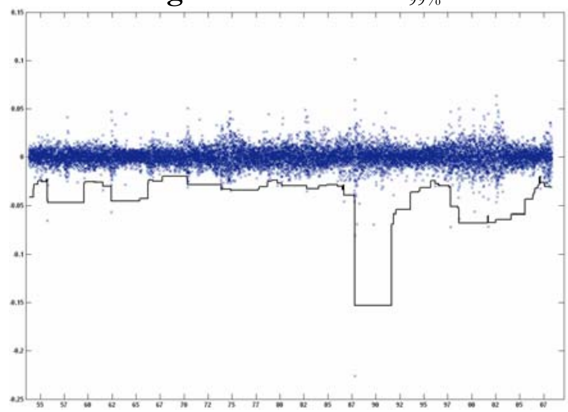
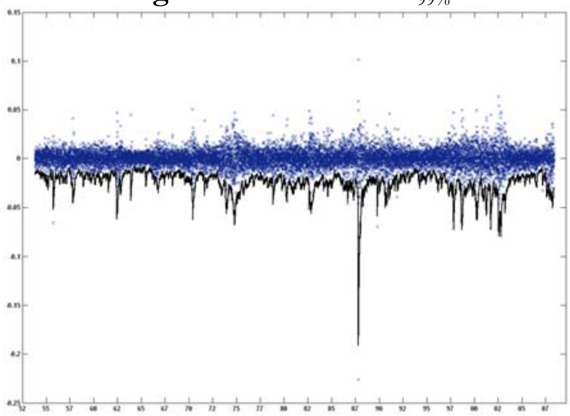


Fig.2.2: *Risk metrics* $ES_{99\%}$



Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, ES are dynamically estimated for 144 post-sample periods; computations by the authors. Black lines represent the evolution of the Expected Shortfall at a 99% confidence level while blue points represent the Dow Jones daily returns.

Fig.2.3: Normal $ES_{99\%}$

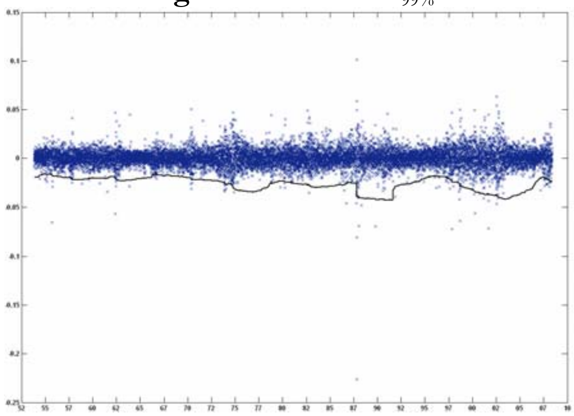
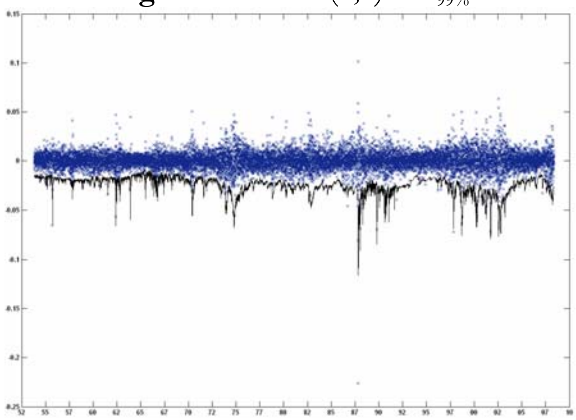


Fig.2.4: GARCH(1,1) $ES_{99\%}$



Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, Expected Shortfalls are dynamically estimated for 144 post-sample periods; computations by the authors. Black lines represent the evolution of the Expected Shortfall at a 99% confidence level while blue points represent the Dow Jones daily returns.

Fig.2.5: Symmetric Abs. Value $ES_{99\%}$

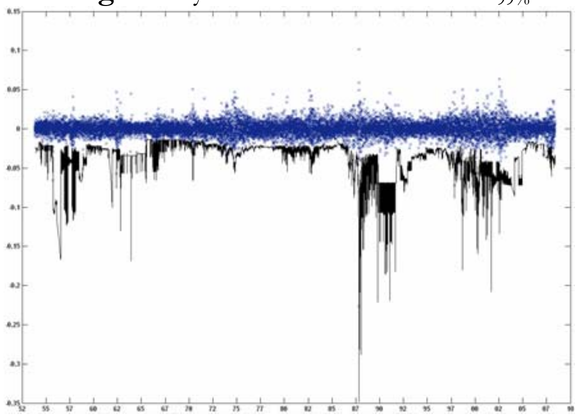
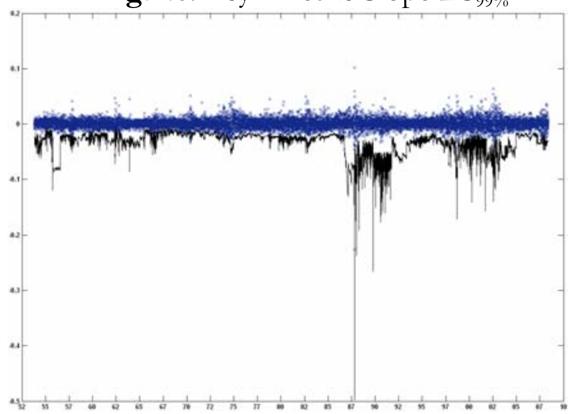


Fig.2.6: Asymmetric Slope $ES_{99\%}$



Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, Expected Shortfalls are dynamically estimated for 144 post-sample periods; computations by the authors. Black lines represent the evolution of the Expected Shortfall at a 99% confidence level while blue points represent the Dow Jones daily returns.

Fig.2.7: IGARCH(1,1) $ES_{99\%}$

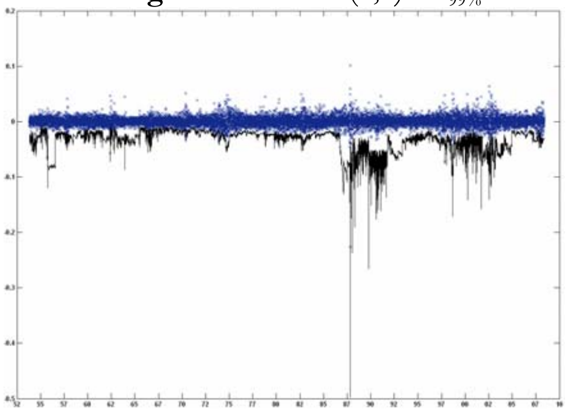
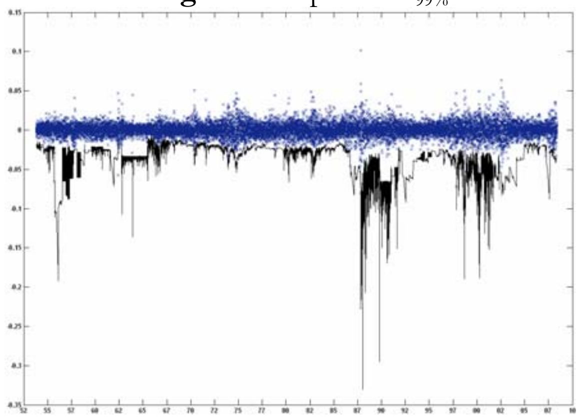
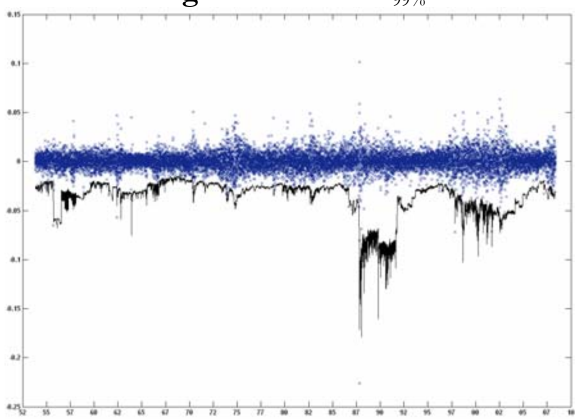


Fig.2.8: Adaptive $ES_{99\%}$



Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, Expected Shortfalls are dynamically estimated for 144 post-sample periods; computations by the authors. Black lines represent the evolution of the Expected Shortfall at a 99% confidence level while blue points represent the Dow Jones daily returns.

Fig.2.9: DARE $ES_{99\%}$



Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, Expected Shortfalls are dynamically estimated for 144 post-sample periods; computations by the authors. Black lines represent the evolution of the Expected Shortfall at a 99% confidence level while blue points represent the Dow Jones daily returns.

Figures 3: Conditional Multiple based on $ES_{99\%}$ Estimates

Fig.3.1: Multiple based on the Historical $ES_{99\%}$

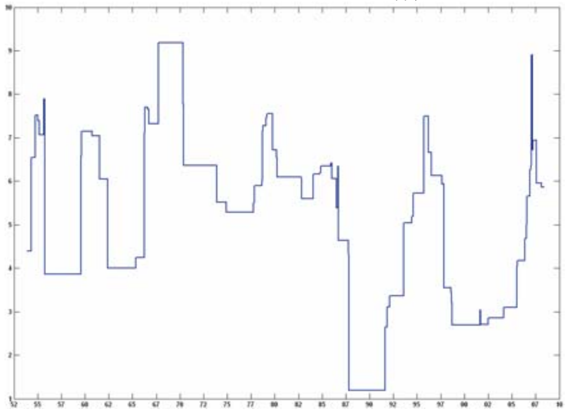
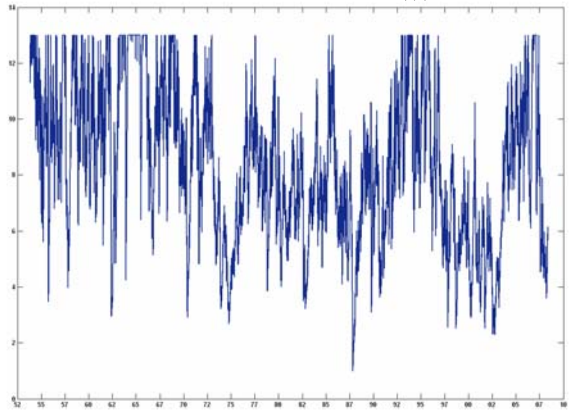


Fig.3.2: Multiple based on the *Riskmetrics* $ES_{99\%}$



Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, conditional multiples are dynamically estimated for 144 post-sample periods; computations by the authors. Multiples have been limited to the extreme value of 13.

Fig.3.3: Multiple based on the Normal $ES_{99\%}$

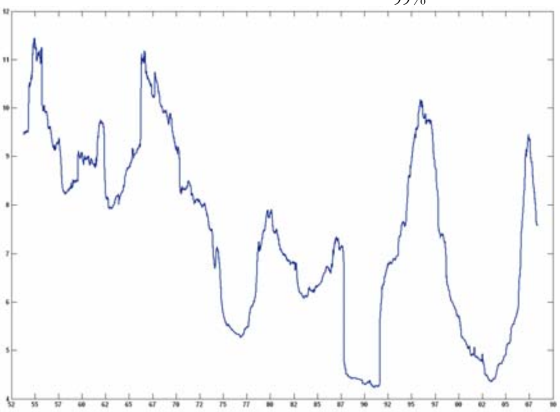
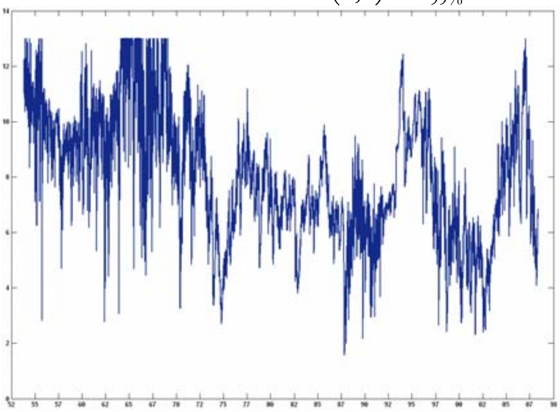


Fig.3.4: Multiple based on the GARCH(1,1) $ES_{99\%}$



Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, conditional multiples are dynamically estimated for 144 post-sample periods; computations by the authors. Multiples have been limited to the extreme value of 13.

Fig. 3.5: Multiple based on the Symmetric Absolute Value $ES_{99\%}$

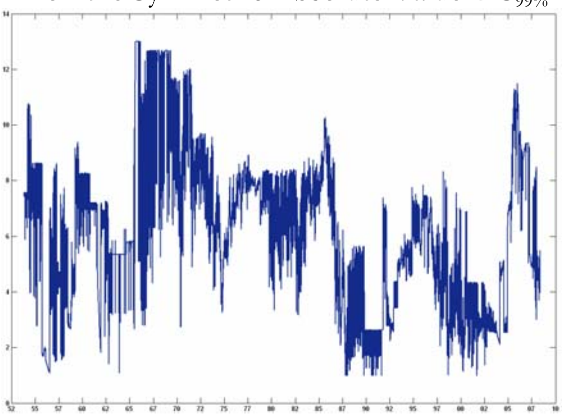
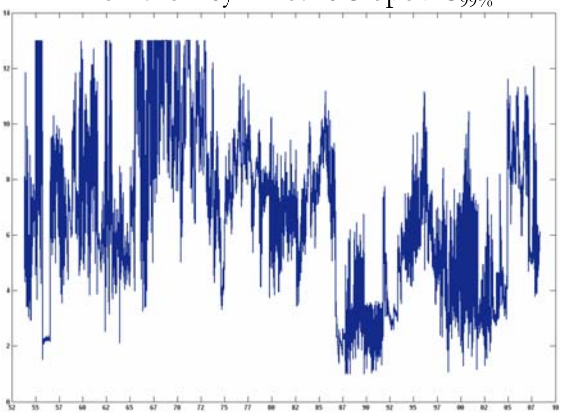


Fig. 3.6: Multiple based on the Asymmetric Slope $ES_{99\%}$



Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, conditional multiples are dynamically estimated for 144 post-sample periods; computations by the authors. Multiples have been limited to the extreme value of 13.

Fig. 3.7: Multiple based on the IGARCH(1,1) $ES_{99\%}$

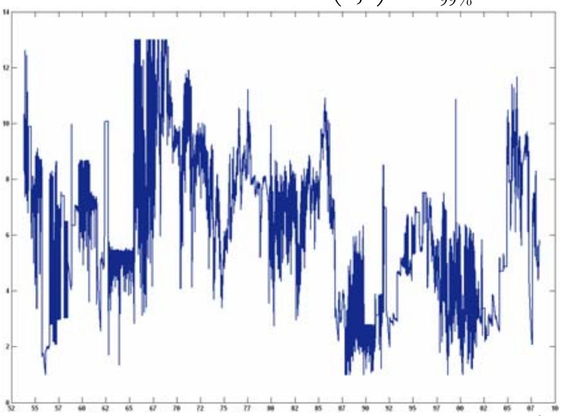
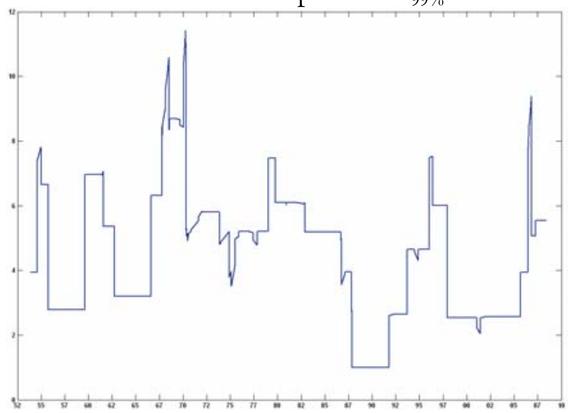
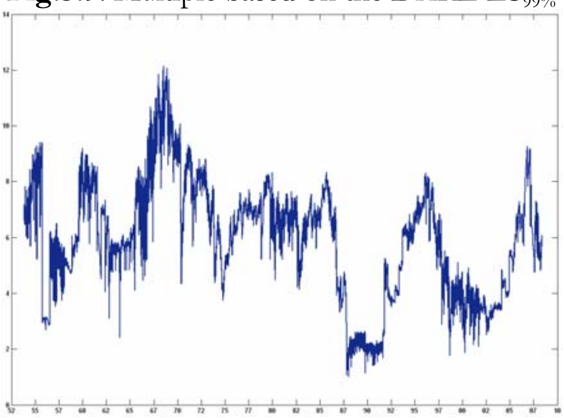


Fig. 3.8: Multiple based on the Adaptive $ES_{99\%}$



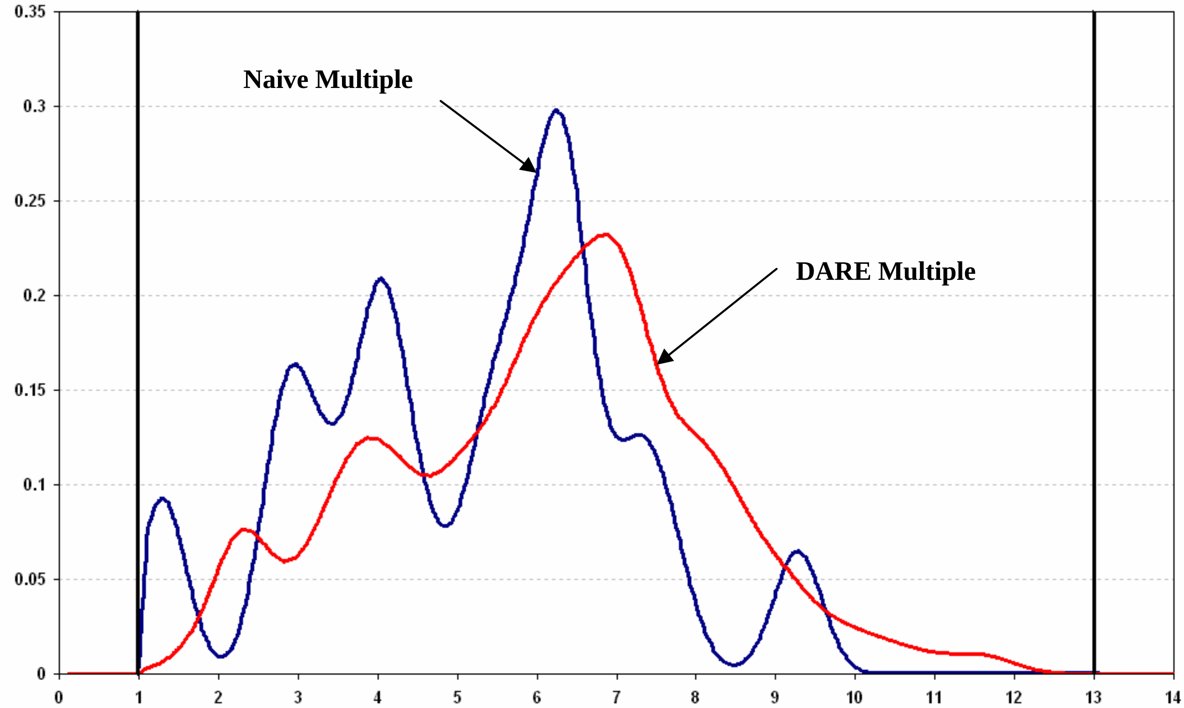
Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, conditional multiples are dynamically estimated for 144 post-sample periods; computations by the authors. Multiples have been limited to the extreme value of 13.

Fig.3.9: Multiple based on the DARE $ES_{99\%}$



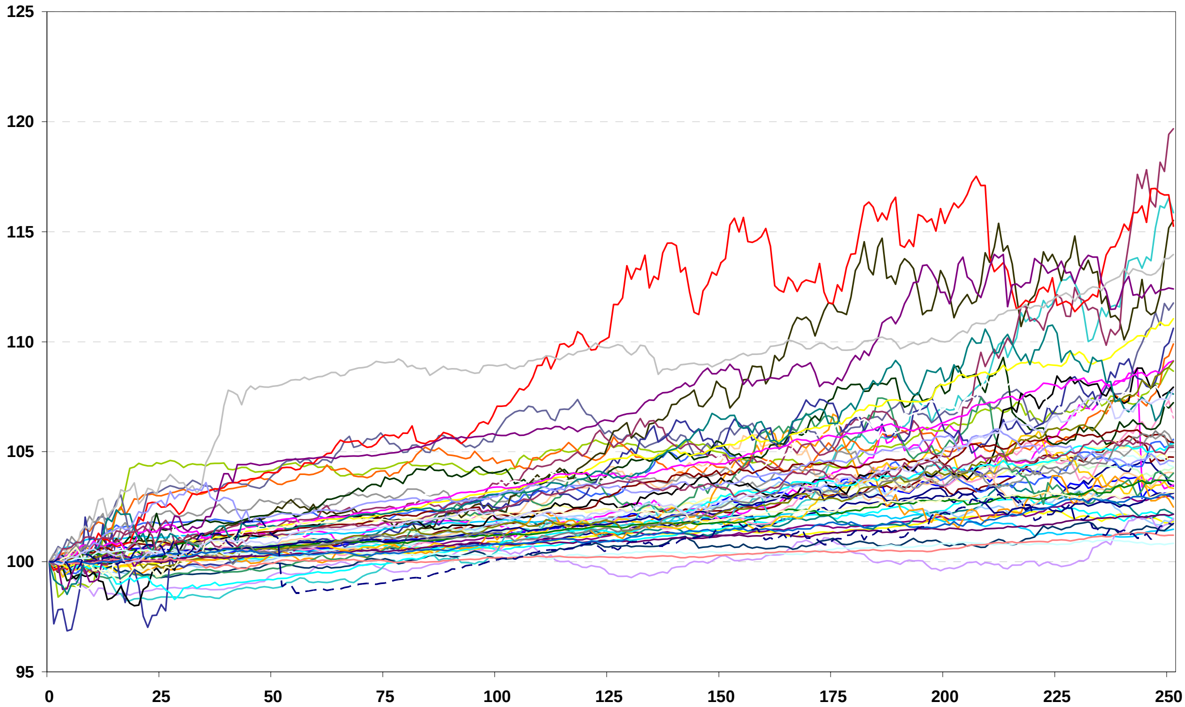
Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, conditional multiples are dynamically estimated for 144 post-sample periods; computations by the authors. Multiples have been limited to the extreme value of 13.

Figure 4: Conditional Multiples Empirical Frequencies based on $ES_{99\%}$ Estimates



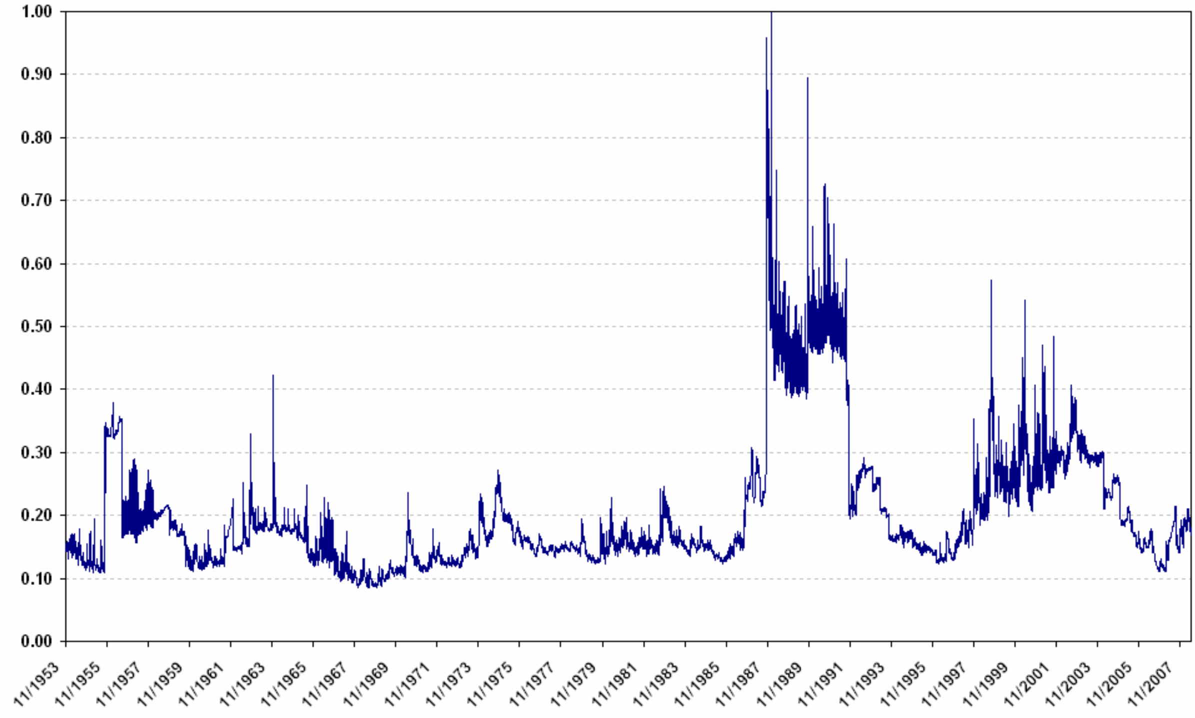
Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, conditional multiples are dynamically estimated for 144 post-sample periods; computations by the authors. The notations "Naive Multiple" and "DARE Multiple" stand respectively for a multiple based on Historical and DARE-modeled Expected Shortfalls. Vertical black lines represent the *maximum* and *minimum* fixed multiples.

Figure 5: Illustration of the Various One-year Covered Portfolios launched at different Starting Dates



Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, conditional multiples are dynamically estimated for 144 post-sample periods; computations by the authors. Colored lines represent every one year covered portfolios evolution in a multi-start framework: every day a new covered portfolio is launched for on year.

Figure 6: Estimation of the Gap Risk (in basis points)
using Corrado and Su (1996) Model for the multiple based on a DARE Model of the Expected Shortfall



Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, conditional multiples are dynamically estimated for 144 post-sample periods; computations by the authors. The insured portfolio Gap Risk is estimated from 01/02/1987 to 05/25/2005 using the DARE Model of the Multiple.

Table 1: Hit Percentage of 1% Conditional Quantile

	H $VaR_{99\%}$	RM $VaR_{99\%}$	Normal $VaR_{99\%}$	GARCH $VaR_{99\%}$	SAV $CAViaR_{99\%}$	AS $CAViaR_{99\%}$	IGARCH $CAViaR_{99\%}$	Adaptive $CAViaR_{99\%}$
Hit ratio	1.12	1.67*	1.64*	1.35	0.98	1.48	1.19	1.43
Unconditional Coverage	-	-	-	-	-	-	-	-
Independence Test	-	-	-	-	-	-	-	-
Conditional Coverage	-	-	-	-	-	-	-	-

Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, *VaR* are dynamically estimated for 144 post-sample periods; computations by the authors. Hit ratios are expressed in percentage. The stars indicate hit ratios significantly different from 1%; computations by the authors. H VaR, RM VaR, Normal VaR, GARCH VaR, SAV VaR, AS CAViaR, IGARCH CAViaR, Adaptive CAViaR stand respectively for Historical VaR, *Risk metrics* VaR, the Gaussian VaR, the GARCH (1,1) VaR, the Symmetric Absolute Value CAViaR, the Asymmetric Slope CAViaR, the IGARCH CAViaR and the Adaptive CAViaR.

Table 2: Under-estimation of the Maximum Drawdown by the various Conditional Multiples

	H	RM	Normal	GARCH (1,1)	SAV	AS	IGARCH (1,1)	Adaptive
Under estimations	Never	Never	Once (-7.79%)	Never	Never	Never	Never	Never

Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, conditional multiples are dynamically estimated for 144 post-sample periods; computations by the authors. The notations H, RM, Normal, GARCH, SAV, AS, GARCH, and Adaptive stand respectively for multiples based on Historical Expected Shortfall, *Riskmetrics* Expected Shortfall, the Gaussian Expected Shortfall, the GARCH (1,1) Expected Shortfall, the Symmetric Absolute Value Expected Shortfall, the Asymmetric Slope Expected Shortfall, the IGARCH(1,1) Expected Shortfall and the Adaptive Expected Shortfall.

Table 3: Performance Analysis using the "Multi-start" Framework

Covered Portfolio using:	H	RM	Normal	GARCH (1,1)	SAV	AS	IGARCH (1,1)	Adaptive
Mean Return	2.47%	2.45%	2.45%	2.46%	2.47%	2.43%	2.48%	2.51%
Standard-Deviation	.08%	.08%	.08%	.08%	.09%	.09%	.09%	.07%
Skewness	-.51	-.31	-.48	-.32	-.38	-.18	-.32	-.54
Kurtosis	-.39	-.53	-.43	-.47	-.53	-.5	-.47	-.36
Sharpe Ratio	5.83	5.44	5.51	5.63	5.51	5.01	5.48	6.85

Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, conditional multiples are dynamically estimated for 144 post-sample periods; computations by the authors. The notations H, RM, Normal, GARCH, SAV, AS, GARCH, and Adaptive stand respectively for multiples based on Historical Expected Shortfall, *Riskmetrics* Expected Shortfall, the Gaussian Expected Shortfall, the GARCH (1,1) Expected Shortfall, the Symmetric Absolute Value Expected Shortfall, the Asymmetric Slope Expected Shortfall, the IGARCH(1,1) Expected Shortfall and the Adaptive Expected Shortfall.

Table 4: Extended Backtest of the Asymmetric Slope PPI Conditional Multiple from 1937

	Date of Floor Violating	Amplitude of Floor Violation	Annualized Excess Return	Volatility
Asym. Slope Multiple PPI	Once (20/10/1987)	.61	1.17	5.58
CPPI Multiple 1	Never	-	.09	.34
CPPI Multiple 2	Never	-	.19	.68
CPPI Multiple 3	Never	-	.31	1.05
CPPI Multiple 4	Once (20/10/1987)	.05	.45	1.48
CPPI Multiple 5	Once (20/10/1987)	.47	.60	1.97
CPPI Multiple 6	Once (20/10/1987)	.78	.77	2.53
CPPI Multiple 7	Once (20/10/1987)	.97	.96	3.18
CPPI Multiple 8	Once (20/10/1987)	1.05	1.16	3.90

Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 10/02/1928 to 12/21/2005, conditional multiple is every week dynamically estimated; computations by the authors. Excess returns are calculated against the risk free rate, volatilities are annualized. Excess returns, volatilities and floor violation amplitude are expressed in percentage. A successive one year investment capital guarantee horizon was used.

Table 5: Annual Cost of Gap Risk Estimation using Classical Options Pricing Models for the DARE Model of the Multiple (in basis points)

Options Model used to estimate the Gap Risk:		Black and Scholes	Merton Jump Diffusion	Corrado and Su	Jarrow and Rudd
Fees (in bp):					
	70	50.72	50.72	50.35	52.92
	80	50.77	50.78	50.40	52.97
	90	50.82	50.83	50.45	53.02
	100	50.87	50.88	50.50	53.07
	120	50.97	50.98	50.60	53.18
	140	51.07	51.08	50.70	53.28
	200	51.38	51.38	51.00	53.60

Source: *Bloomberg*, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008, conditional multiples are dynamically estimated for 144 post-sample periods; computations by the authors. Annual costs are expressed in basis points (bp). No difference can be made between Black and Scholes, Cox and Ross, Rubinstein's binomial tree, Boyle's trinomial tree option models when pricing the insured portfolios Gap Risk from 01/02/1950 to 09/30/2008.

Table 6: Normality tests on DARE and Other TIPP Cushioned Portfolio Strategy Returns On the Dow Jones Index from 1950 to 2008 on a One-year Basis

	Jarque-Bera		Kolmogorov-Smirnov		Lilliefors		Anderson-Darling		Shapiro Wilk		Shapiro Francia		D'Agostino's K-squared		Cramer von Mises	
Risky Asset	.63	.00%	.48	.00%	.06	.00%	206.35	.00%	-	-	-	-	-	-	-	-
DARE Multiple	1.53	.00%	.49	.00%	.16	.00%	210.24	.00%	-	-	-	-	-	-	-	-
Multiple 1	.17	.00%	.50	.00%	.13	.00%	212.40	.00%	-	-	-	-	-	-	-	-
Multiple 2	.28	.00%	.50	.00%	.14	.00%	211.89	.00%	-	-	-	-	-	-	-	-
Multiple 3	.42	.00%	.50	.00%	.14	.00%	211.42	.00%	-	-	-	-	-	-	-	-
Multiple 4	.59	.00%	.50	.00%	.15	.00%	210.98	.00%	-	-	-	-	-	-	-	-
Multiple 5	.80	.00%	.49	.00%	.15	.00%	210.55	.00%	-	-	-	-	-	-	-	-
Multiple 6	1.02	.00%	.49	.00%	.15	.00%	210.12	.00%	-	-	-	-	-	-	-	-
Multiple 7	1.24	.00%	.49	.00%	.16	.00%	209.72	.00%	-	-	-	-	-	-	-	-
Multiple 8	1.47	.00%	.49	.00%	.16	.00%	209.25	.00%	-	-	-	-	-	-	-	-
Multiple 9	1.73	.00%	.49	.00%	.17	.00%	208.88	.00%	-	-	-	-	-	-	-	-
Multiple 10	2.14	.00%	.49	.00%	.18	.00%	208.45	.00%	-	-	-	-	-	-	-	-
Multiple 11	2.32	.00%	.49	.00%	.18	.00%	208.04	.00%	-	-	-	-	-	-	-	-
Multiple 12	2.99	.00%	.49	.00%	.19	.00%	207.67	.00%	-	-	-	-	-	-	-	-
Multiple 13	2.50	.00%	.49	.00%	.19	.00%	207.34	.00%	-	-	-	-	-	-	-	-
H Multiple	2.28	.00%	.49	.00%	.18	.00%	210.28	.00%	-	-	-	-	-	-	-	-
RM Multiple	7.26	.00%	.49	.00%	.15	.00%	209.70	.00%	-	-	-	-	-	-	-	-
Normal Multiple	14.31	.00%	.49	.00%	.17	.00%	209.54	.00%	-	-	-	-	-	-	-	-
GARCH(1,1) Multiple	35.58	.00%	.49	.00%	.15	.00%	209.84	.00%	-	-	-	-	-	-	-	-
SAV Multiple	.95	.00%	.49	.00%	.16	.00%	210.02	.00%	-	-	-	-	-	-	-	-
AS Multiple	1.29	.00%	.49	.00%	.16	.00%	209.84	.00%	-	-	-	-	-	-	-	-
IGARCH(1,1) Multiple	.82	.00%	.49	.00%	.16	.00%	210.03	.00%	-	-	-	-	-	-	-	-
A Multiple	3.25	.00%	.49	.00%	.18	.00%	210.39	.00%	-	-	-	-	-	-	-	-

Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950. See Bontemps and Meddahi (2005) and related articles for the normality tests. The P-statistics are presented in grey.

Table 7: DARE and Unconditional TIPP Cushioned Portfolio Strategy Characteristics on the Dow Jones Index from 1950 to 2008 on a Five-year Basis

	Return (mean annual)	Return (% of risky asset one)	Beta	Volatility	VaR99%	ES99%	Max Drawdown	Skewness	*Kurtosis*	Violating Floor	Sharpe	Sortino	Omega	Kappa	Relative perf/ ES99%	Cond. Sharpe Ratio	Treynor Ratio
Risky Asset	7.06%	100.00%	1.00	14.40%	-2.33%	-3.26%	-	-1.08	35.63	-	.10	.05	1.10	.03	-	.46	.02
DARE Multiple	7.42%	105.05%	.79	11.33%	-2.02%	-3.20%	-	-.68	22.72	0	.16	.06	1.16	.03	.11	.58	.02
Multiple 1	5.91%	83.73%	.47	2.31%	-.39%	-.58%	-	.23	15.37	0	.15	.25	1.67	.15	-1.99	.61	.01
Multiple 2	6.29%	89.02%	.52	4.59%	-.80%	-1.22%	-	-.02	17.21	0	.16	.13	1.32	.07	-.64	.60	.01
Multiple 3	6.69%	94.74%	.59	6.94%	-1.17%	-1.92%	-	-.15	21.97	0	.16	.09	1.23	.05	-.19	.59	.02
Multiple 4	7.00%	99.13%	.67	9.07%	-1.55%	-2.56%	-	-.36	23.69	0	.16	.07	1.19	.04	-.02	.56	.02
Multiple 5	7.16%	101.38%	.75	10.63%	-1.89%	-2.98%	-	-.67	23.60	1	.15	.06	1.16	.03	.03	.54	.02
Multiple 6	7.32%	103.61%	.83	11.90%	-2.21%	-3.35%	-	-.74	21.41	1	.15	.06	1.15	.03	.08	.52	.02
Multiple 7	7.40%	104.83%	.88	12.66%	-2.33%	-3.52%	-	-.69	20.13	1	.15	.06	1.14	.03	.10	.52	.02
Multiple 8	7.35%	104.09%	.91	13.15%	-2.40%	-3.59%	-	-.72	20.31	1	.14	.05	1.13	.03	.08	.50	.02
Multiple 10	7.24%	102.58%	.97	14.05%	-2.62%	-3.76%	-	-.68	17.79	1	.12	.05	1.12	.03	.05	.45	.02
Multiple 11	7.19%	101.79%	1.00	14.46%	-2.71%	-3.83%	-	-.64	16.53	1	.11	.05	1.12	.03	.03	.43	.02
Multiple 13	7.14%	101.14%	1.07	15.29%	-2.90%	-3.97%	-	-.57	14.53	2	.10	.05	1.11	.03	.02	.40	.01

Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950. Returns and Volatility are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The Skewness and kurtosis P-statistics are computed using Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Eling and Schuhmacher (2007) and related literature for other performance measures.

Table 8: DARE and Unconditional Cushioned Portfolio Strategy Characteristics on a Five-year Basis based on Bootstrapped Simulated Series (on the Dow Jones Index from 1950 to 2008)

	Return (mean annual)	**Return (% of risky asset one)**	**Volatility**	**VaR99%**	**Skewness**	***Kurtosis***	**Jarque-Bera**	**Kolmogorov-Smirnov**	**Lilliefors**	**Anderson-Darling**	**Sharpe**	**Sortino**	**Omega**	**Kappa**
Risky Asset	7.04%	10.00%	14.49%	-2.33%	-1.23	39.3	.63	.48	.06	162.32	.10	.05	1.1	.03
							.00%	.00%	.00%	.00%				
DARE Multiple	7.00%	99.36%	1.22%	-2.37%	-.75	28.75	.23	.48	.13	93.95	.13	.05	1.13	.03
							.00%	.00%	.00%	.00%				
Multiple 1	5.95%	84.49%	2.31%	-.38%	-1.17	46.87	.09	.5	.12	92.15	.13	.24	1.64	.12
							.00%	.00%	.00%	.00%				
Multiple 2	6.21%	88.19%	4.70%	-.82%	-1.2	47.2	.12	.49	.12	9.83	.12	.12	1.3	.06
							.00%	.00%	.00%	.00%				
Multiple 3	6.40%	9.88%	7.23%	-1.30%	-1.24	52.46	.21	.49	.13	79.1	.10	.08	1.2	.04
							.00%	.00%	.00%	.00%				
Multiple 4	6.49%	92.25%	9.73%	-1.79%	-1.35	59.33	.25	.49	.13	69.56	.09	.06	1.16	.03
							.00%	.00%	.00%	.00%				
Multiple 5	6.49%	92.22%	11.83%	-2.22%	-1.86	93.25	.25	.49	.12	66.98	.07	.05	1.14	.03
							.00%	.00%	.00%	.00%				
Multiple 6	6.52%	92.59%	13.39%	-2.54%	-1.88	95.27	.2	.48	.12	68.84	.07	.05	1.12	.02
							.00%	.00%	.00%	.00%				
Multiple 7	6.57%	93.32%	14.49%	-2.76%	-1.7	82.47	.18	.48	.12	74.14	.06	.05	1.12	.02
							.00%	.00%	.00%	.00%				
Multiple 8	6.59%	93.56%	15.26%	-2.89%	-1.66	79.04	.18	.48	.12	81.7	.06	.04	1.11	.02
							.00%	.00%	.00%	.00%				
Multiple 10	6.69%	95.03%	16.29%	-3.10%	-1.51	7.43	.13	.48	.12	83.66	.06	.04	1.11	.02
							.00%	.00%	.00%	.00%				
Multiple 11	6.75%	95.89%	16.67%	-3.15%	-1.45	66.82	.11	.48	.13	84.68	.07	.04	1.11	.02
							.00%	.00%	.00%	.00%				
Multiple 13	6.84%	97.22%	17.27%	-3.25%	-1.45	65.06	.08	.48	.13	166.32	.07	.04	1.11	.02
							.00%	.00%	.00%	.00%				

Source: *Bloomberg*, daily data, Daily Returns of the *Dow Jones* Index from 01/02/1950 to 09/30/2008; computation by the authors. The strategies characteristics are calculated using 50 simulations of 14.230 daily returns based on stationary bootstrap (*Cf.* Politis and Romano. 1994): artificial series are composed with Dow Jones random blocks of daily returns determined using a geometric probability law defined by a parameter of .9. Statistics presented here are the average of the statistics computed for each strategy over every simulation. Returns and volatility are annualized. TIPP Strategies use a five year investment horizon. Returns and Volatility are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The Skewness and kurtosis P-statistics are computed using Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Eling and Schuhmacher (2007) and related literature for other performance measures, and Bontemps and Meddahi (2005) and related articles for the normality tests. The P-statistics are presented in grey.

Appendix 2: Supplementary Results available on Request (at the referee's attention)

Table 9: One-year Based DARE and Unconditional TIPP Cushioned Portfolio Strategy Characteristics on the Dow Jones Index from 1950 to 2008

	Return (mean annual)	Return (% of risky asset one)	Beta	Volatility	VaR99%	ES99%	Max Drawdown	Skewness	*Kurtosis*	Violating Floor	Sharpe	Sortino	Omega	Kappa	Relative perf/ ES99%	Cond. Sharpe Ratio	Treynor Ratio
Risky Asset	7.06%	100.00%	1.00	14.40%	-2.33%	-3.26%	-	-1.08	35.63	-	.10	.05	1.10	.03	-	.46	.02
DARE Multiple	5.84%	82.69%	.05	3.30%	-.58%	-.97%	-	-1.35	53.80	0	.08	.16	1.50	.08	-1.26	.29	.06
Multiple 1	5.62%	79.54%	.00	.58%	-.08%	-.12%	-	1.33	19.76	0	.10	1.40	7.04	.71	-12.42	.48	5.59
Multiple 2	5.67%	80.31%	.01	1.11%	-.18%	-.27%	-	.90	24.65	0	.10	.57	2.90	.31	-5.21	.41	.11
Multiple 3	5.73%	81.08%	.01	1.65%	-.28%	-.43%	-	.69	29.43	0	.10	.35	2.09	.19	-3.13	.39	.16
Multiple 4	5.78%	81.84%	.02	2.22%	-.39%	-.60%	-	.52	34.58	0	.10	.26	1.76	.14	-2.14	.36	.11
Multiple 5	5.83%	82.59%	.04	2.81%	-.50%	-.79%	-	.35	39.69	1	.10	.20	1.59	.10	-1.56	.34	.07
Multiple 6	5.88%	83.30%	.06	3.43%	-.63%	-1.00%	-	.17	44.42	1	.09	.16	1.48	.08	-1.18	.32	.05
Multiple 7	5.93%	83.92%	.08	4.08%	-.74%	-1.22%	-	-.06	48.72	1	.09	.13	1.40	.07	-0.93	.30	.05
Multiple 8	5.96%	84.44%	.11	4.77%	-.87%	-1.47%	-	-.31	52.79	1	.08	.11	1.35	.06	-0.75	.27	.04
Multiple 10	6.09%	86.19%	.19	6.27%	-1.13%	-1.99%	-	-.60	63.09	1	.08	.09	1.28	.04	-0.49	.26	.03
Multiple 11	6.20%	87.82%	.23	6.98%	-1.26%	-2.24%	-	-.89	65.53	1	.09	.08	1.26	.04	-0.38	.29	.03
Multiple 13	6.41%	90.79%	.33	8.28%	-1.54%	-2.71%	-	-1.56	67.86	2	.10	.07	1.23	.03	-0.24	.31	.03

Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950. Returns and Volatility are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The Skewness and kurtosis P-statistics are computed using Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Eling and Schuhmacher (2007) and related literature for other performance measures.

Table 10: One-year Based DARE and Conditional TIPP Cushioned Portfolio Strategy Characteristics on the Dow Jones Index from 1950 to 2008

	Return (mean annual)	Return (% of risky asset one)	Beta	Volatility	VaR99%	ES99%	Max Drawdown	Skewness	*Kurtosis*	Violating Floor	Sharpe	Sortino	Omega	Kappa	Relative perf/ ES99%	Cond. Sharpe Ratio	Treynor Ratio
Risky Asset	7.06%	100.00%	1.00	14.40%	-2.33%	-3.26%	-	-1.08	35.63	-	.10	.05	1.10	.03	-	.46	.02
DARE Multiple	5.84%	82.69%	.05	3.30%	-0.58%	-0.97%	-	-1.35	53.80	0	.08	.16	1.50	.08	-1.26	.29	.06
H Multiple	5.73%	81.16%	.05	3.10%	-0.54%	-0.92%	-	-0.32	65.02	0	.06	.17	1.57	.08	-1.45	.19	.03
RM Multiple	6.10%	86.35%	.09	4.43%	-0.80%	-1.30%	-	-3.30	113.45	0	.12	.12	1.37	.05	-.74	.41	.06
Normal Multiple	6.10%	86.40%	.09	4.38%	-0.77%	-1.33%	-	-3.60	158.17	1	.12	.12	1.40	.05	-.72	.41	.06
GARCH(1,1) Multiple	6.03%	85.37%	.09	4.35%	-0.80%	-1.26%	-	-5.88	247.70	0	.11	.12	1.38	.05	-.82	.37	.05
SAV Multiple	5.87%	83.12%	.06	3.56%	-0.64%	-1.06%	-	-1.10	43.02	0	.09	.15	1.47	.07	-1.13	.29	.05
AS Multiple	5.93%	84.03%	.07	3.81%	-0.69%	-1.13%	-	-1.37	49.49	0	.10	.14	1.44	.07	-1.00	.33	.05
IGARCH(1,1) Multiple	5.80%	82.08%	.06	3.48%	-0.64%	-1.02%	-	-0.93	40.23	0	.07	.15	1.47	.07	-1.24	.23	.04
A Multiple	5.73%	81.15%	.04	2.89%	-0.48%	-0.85%	-	0.42	77.06	0	.06	.19	1.64	.09	-1.56	.20	.04

Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950. Returns and Volatility are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The Skewness and kurtosis P-statistics are computed using Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Eling and Schuhmacher (2007) and related literature for other performance measures.

Table 11: Two-year Based DARE and Unconditional TIPP Cushioned Portfolio Strategy Characteristics on the Dow Jones Index from 1950 to 2008

	Return (mean annual)	Return (% of risky asset one)	Beta	Volatility	VaR99%	ES99%	Max Drawdown	Skewness	*Kurtosis*	Violating Floor	Sharpe	Sortino	Omega	Kappa	Relative perf/ ES99%	Cond. Sharpe Ratio	Treynor Ratio
Risky Asset	7.06%	100.00%	1.00	14.40%	-2.33%	-3.26%	-	-1.08	35.63	-	.10	.05	1.10	.03	-	.46	.02
DARE Multiple	6.19%	87.72%	.58	6.84%	-1.20%	-2.21%	-	-3.41	107.07	0	.09	.08	1.27	.03	-.39	.29	.01
Multiple 1	5.64%	79.81%	.46	1.12%	-0.18%	-0.28%	-	-.36	47.05	0	.07	.53	2.92	.24	-5.15	.27	.00
Multiple 2	5.68%	80.42%	.47	2.16%	-0.37%	-0.58%	-	-1.29	56.08	0	.05	.25	1.77	.11	-2.37	.20	.00
Multiple 3	5.70%	80.73%	.49	3.20%	-0.56%	-0.92%	-	-2.00	65.38	0	.04	.16	1.49	.07	-1.48	.15	.00
Multiple 4	5.71%	80.89%	.51	4.32%	-0.77%	-1.30%	-	-2.57	73.79	0	.04	.11	1.36	.05	-1.03	.12	.00
Multiple 5	5.73%	81.08%	.54	5.56%	-1.01%	-1.75%	-	-3.08	81.15	1	.03	.09	1.29	.04	-.76	.09	.00
Multiple 6	5.76%	81.58%	.59	6.97%	-1.28%	-2.26%	-	-3.66	94.80	1	.03	.07	1.24	.03	-.57	.09	.00
Multiple 7	5.98%	84.70%	.64	8.23%	-1.53%	-2.72%	-	-3.72	98.09	1	.05	.06	1.22	.03	-.40	.15	.01
Multiple 8	6.18%	87.48%	.68	9.19%	-1.69%	-3.05%	-	-3.59	92.97	1	.07	.06	1.20	.03	-.29	.20	.01
Multiple 10	6.38%	90.29%	.78	11.05%	-2.05%	-3.60%	-	-5.64	192.24	1	.07	.05	1.18	.02	-.19	.23	.01
Multiple 11	6.22%	88.06%	.85	12.20%	-2.24%	-3.85%	-	-10.78	516.28	1	.05	.04	1.17	.02	-.22	.17	.01
Multiple 13	6.31%	89.31%	.93	13.37%	-2.48%	-4.14%	-	-12.13	612.93	2	.06	.04	1.16	.01	-.18	.18	.01

Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950. Returns and Volatility are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The Skewness and kurtosis P-statistics are computed using Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Eling and Schuhmacher (2007) and related literature for other performance measures.

Table 12: Two-year Based DARE and Conditional TIPP Cushioned Portfolio Strategy Characteristics on the Dow Jones Index from 1950 to 2008

	Return (mean annual)	Return (% of risky asset one)	Beta	Volatility	VaR99%	ES99%	Max Drawdown	Skewness	*Kurtosis*	Violating Floor	Sharpe	Sortino	Omega	Kappa	Relative perf/ ES99%	Cond. Sharpe Ratio	Treynor Ratio
Risky Asset	7.06%	100.00%	1.00	14.40%	-2.33%	-3.26%	-	-1.08	35.63	-	.10	.05	1.10	.03	-	.46	.02
DARE Multiple	6.19%	87.72%	.58	6.84%	-1.20%	-2.21%	-	-3.41	107.07	0	.09	.08	1.27	.03	-.39	.29	.01
H Multiple	5.78%	81.79%	.56	6.31%	-1.06%	-2.07%	-	-4.58	149.59	0	.03	.08	1.29	.03	-.62	.10	.00
RM Multiple	6.99%	99.03%	.68	9.14%	-1.74%	-2.97%	-	-3.13	73.67	0	.16	.07	1.22	.03	-.02	.48	.02
Normal Multiple	6.77%	95.88%	.65	8.63%	-1.66%	-2.89%	-	-3.19	81.54	1	.14	.07	1.24	.03	-.10	.42	.02
GARCH(1,1) Multiple	6.66%	94.27%	.65	8.57%	-1.65%	-2.79%	-	-3.06	70.42	0	.13	.07	1.22	.03	-.15	.39	.02
SAV Multiple	6.16%	87.24%	.59	7.02%	-1.23%	-2.25%	-	-3.29	98.70	0	.09	.08	1.26	.03	-.40	.27	.01
AS Multiple	6.08%	86.17%	.59	7.15%	-1.25%	-2.24%	-	-2.91	89.46	0	.07	.07	1.24	.03	-.44	.23	.01
IGARCH(1,1) Multiple	6.09%	86.19%	.59	6.98%	-1.23%	-2.23%	-	-3.23	100.20	0	.08	.08	1.26	.03	-.44	.24	.01
A Multiple	5.65%	79.99%	.54	5.60%	-.98%	-1.84%	-	-3.70	117.73	0	.02	.09	1.32	.04	-.77	.05	.00

Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950. Returns and Volatility are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The Skewness and kurtosis P-statistics are computed using Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Eling and Schuhmacher (2007) and related literature for other performance measures.

Table 13: Three-year Based DARE and Unconditional TIPP Cushioned Portfolio Strategy Characteristics on the Dow Jones Index from 1950 to 2008

	Return (mean annual)	Return (% of risky asset one)	Beta	Volatility	VaR99%	ES99%	Max Drawdown	Skewness	*Kurtosis*	Violating Floor	Sharpe	Sortino	Omega	Kappa	Relative perf/ ES99%	Cond. Sharpe Ratio	Treynor Ratio
Risky Asset	7.06%	100.00%	1.00	14.40%	-2.33%	-3.26%	-	-1.08	35.63	-	.10	.05	1.10	.03	-	.46	.02
DARE Multiple	6.16%	87.23%	.69	9.30%	-1.69%	-2.87%	-	-3.23	91.00	0	.06	.06	1.18	.03	-.31	.21	.01
Multiple 1	5.63%	79.74%	.47	1.64%	-.26%	-.40%	-	-2.41	124.18	0	.04	.33	2.08	.13	-3.60	.18	.00
Multiple 2	5.65%	80.05%	.49	3.25%	-.55%	-.84%	-	-4.52	205.81	0	.03	.15	1.46	.06	-1.67	.11	.00
Multiple 3	5.64%	79.85%	.52	4.92%	-.83%	-1.33%	-	-6.73	312.98	0	.02	.10	1.29	.04	-1.07	.06	.00
Multiple 4	5.61%	79.44%	.58	6.73%	-1.11%	-1.90%	-	-8.55	411.56	0	.01	.07	1.22	.03	-.76	.03	.00
Multiple 5	5.62%	79.56%	.66	8.86%	-1.47%	-2.59%	-	-10.04	497.15	1	.01	.05	1.18	.02	-.56	.02	.00
Multiple 6	5.48%	77.63%	.78	11.12%	-1.82%	-3.18%	-	-16.38	957.51	1	-.01	.04	1.15	.01	-.50	-.02	.00
Multiple 7	5.78%	81.79%	.85	12.31%	-2.05%	-3.55%	-	-14.81	834.77	1	.02	.04	1.15	.01	-.36	.06	.00
Multiple 8	6.15%	87.03%	.91	13.14%	-2.28%	-3.82%	-	-12.39	649.13	1	.04	.04	1.14	.01	-.24	.15	.01
Multiple 10	6.69%	94.71%	.99	14.25%	-2.47%	-4.12%	-	-9.78	472.65	1	.08	.04	1.14	.02	-.09	.27	.01
Multiple 11	6.86%	97.19%	1.03	14.79%	-2.61%	-4.26%	-	-8.83	414.23	1	.09	.04	1.14	.02	-.05	.31	.01
Multiple 13	7.28%	103.03%	1.16	16.42%	-2.96%	-4.82%	-	-6.36	288.99	2	.10	.04	1.14	.02	.04	.36	.01

Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950. Returns and Volatility are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The Skewness and kurtosis P-statistics are computed using Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Eling and Schuhmacher (2007) and related literature for other performance measures.

Table 14: Three-year Based DARE and Conditional TIPP Cushioned Portfolio Strategy Characteristics on the Dow Jones Index from 1950 to 2008

	Return (mean annual)	Return (% of risky asset one)	Beta	Volatility	VaR99%	ES99%	Max Drawdown	Skewness	*Kurtosis*	Violating Floor	Sharpe	Sortino	Omega	Kappa	Relative perf/ ES99%	Cond. Sharpe Ratio	Treynor Ratio
Risky Asset	7.06%	100.00%	1.00	14.40%	-2.33%	-3.26%	-	-1.08	35.63	-	.10	.05	1.10	.03	-	.46	.02
DARE Multiple	6.16%	87.23%	.69	9.30%	-1.69%	-2.87%	-	-3.23	91.00	0	.06	.06	1.18	.03	-.31	.21	.01
H Multiple	5.88%	83.32%	.67	8.90%	-1.50%	-2.77%	-	-6.91	277.52	0	.04	.06	1.19	.02	-.43	.12	.00
RM Multiple	6.96%	98.50%	.86	12.45%	-2.23%	-3.71%	-	-6.62	257.65	0	.11	.05	1.16	.02	-.03	.38	.02
Normal Multiple	6.59%	93.36%	.84	12.15%	-2.00%	-3.52%	-	-15.30	879.68	1	.08	.05	1.17	.01	-.13	.29	.01
GARCH(1,1) Multiple	6.60%	93.43%	.85	12.21%	-2.11%	-3.63%	-	-9.78	456.71	0	.08	.05	1.16	.02	-.13	.29	.01
SAV Multiple	6.23%	88.26%	.72	10.09%	-1.90%	-3.19%	-	-2.22	59.07	0	.07	.06	1.17	.03	-.26	.21	.01
AS Multiple	6.06%	85.83%	.71	9.76%	-1.89%	-2.98%	-	-1.66	37.85	0	.05	.06	1.16	.03	-.34	.17	.01
IGARCH(1,1) Multiple	5.93%	83.93%	.71	9.80%	-1.83%	-3.05%	-	-3.45	102.44	0	.04	.05	1.17	.02	-.37	.12	.01
A Multiple	5.80%	82.21%	.62	7.87%	-1.34%	-2.41%	-	-5.75	218.05	0	.03	.06	1.21	.02	-.52	.10	.00

Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950. Returns and Volatility are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The Skewness and kurtosis P-statistics are computed using Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Eling and Schuhmacher (2007) and related literature for other performance measures.

Table 15: Five-year Based DARE and Unconditional TIPP Cushioned Portfolio Strategy Characteristics on the Dow Jones Index from 1950 to 2008

	Return (mean annual)	Return (% of risky asset one)	Beta	Volatility	VaR99%	ES99%	Max Drawdown	Skewness	*Kurtosis*	Violating Floor	Sharpe	Sortino	Omega	Kappa	Relative perf/ ES99%	Cond. Sharpe Ratio	Treynor Ratio
Risky Asset	7.06%	100.00%	1.00	14.40%	-2.33%	-3.26%	-	-1.08	35.63	-	.10	.05	1.10	.03	-	.46	.02
DARE Multiple	7.42%	105.05%	.79	11.33%	-2.02%	-3.20%	-	-.68	22.72	0	.16	.06	1.16	.03	.11	.58	.02
Multiple 1	5.91%	83.73%	.47	2.31%	-.39%	-0.58%	-	.23	15.37	0	.15	.25	1.67	.15	-1.99	.61	.01
Multiple 2	6.29%	89.02%	.52	4.59%	-.80%	-1.22%	-	-.02	17.21	0	.16	.13	1.32	.07	-.64	.60	.01
Multiple 3	6.69%	94.74%	.59	6.94%	-1.17%	-1.92%	-	-.15	21.97	0	.16	.09	1.23	.05	-.19	.59	.02
Multiple 4	7.00%	99.13%	.67	9.07%	-1.55%	-2.56%	-	-.36	23.69	0	.16	.07	1.19	.04	-.02	.56	.02
Multiple 5	7.16%	101.38%	.75	10.63%	-1.89%	-2.98%	-	-.67	23.60	1	.15	.06	1.16	.03	.03	.54	.02
Multiple 6	7.32%	103.61%	.83	11.90%	-2.21%	-3.35%	-	-.74	21.41	1	.15	.06	1.15	.03	.08	.52	.02
Multiple 7	7.40%	104.83%	.88	12.66%	-2.33%	-3.52%	-	-.69	20.13	1	.15	.06	1.14	.03	.10	.52	.02
Multiple 8	7.35%	104.09%	.91	13.15%	-2.40%	-3.59%	-	-.72	20.31	1	.14	.05	1.13	.03	.08	.50	.02
Multiple 10	7.24%	102.58%	.97	14.05%	-2.62%	-3.76%	-	-.68	17.79	1	.12	.05	1.12	.03	.05	.45	.02
Multiple 11	7.19%	101.79%	1.00	14.46%	-2.71%	-3.83%	-	-.64	16.53	1	.11	.05	1.12	.03	.03	.43	.02
Multiple 13	7.14%	101.14%	1.07	15.29%	-2.90%	-3.97%	-	-.57	14.53	2	.10	.05	1.11	.03	.02	.40	.01

Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950. Returns and Volatility are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The Skewness and kurtosis P-statistics are computed using Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Eling and Schuhmacher (2007) and related literature for other performance measures.

Table 16: Five-year Based DARE and Conditional TIPP Cushioned Portfolio Strategy Characteristics on the Dow Jones Index from 1950 to 2008

	Return (mean annual)	Return (% of risky asset one)	Beta	Volatility	VaR99%	ES99%	Max Drawdown	Skewness	*Kurtosis*	Violating Floor	Sharpe	Sortino	Omega	Kappa	Relative perf/ ES99%	Cond. Sharpe Ratio	Treynor Ratio
Risky Asset	7.06%	100.00%	1.00	14.40%	-2.33%	-3.26%	-	-1.08	35.63	-	.10	.05	1.10	.03	-	.46	.02
DARE Multiple	7.42%	105.05%	.79	11.33%	-2.02%	-3.20%	-	-.68	22.72	0	.16	.06	1.16	.03	.11	.58	.02
H Multiple	7.60%	107.70%	.75	10.48%	-1.85%	-3.03%	-	-.78	29.31	0	.20	.07	1.18	.04	.18	.67	.03
RM Multiple	7.57%	107.20%	.94	13.62%	-2.48%	-3.71%	-	-.78	19.60	0	.15	.05	1.13	.03	.14	.54	.02
Normal Multiple	8.04%	113.86%	.87	12.54%	-2.30%	-3.47%	-	-.53	18.52	1	.20	.06	1.15	.04	.28	.71	.03
GARCH(1,1) Multiple	7.54%	106.79%	.90	12.95%	-2.37%	-3.57%	-	-.78	21.05	0	.15	.06	1.14	.03	.13	.55	.02
SAV Multiple	7.13%	100.98%	.79	11.33%	-2.02%	-3.22%	-	-.79	23.96	0	.14	.06	1.15	.03	.02	.49	.02
AS Multiple	6.51%	92.25%	.82	11.84%	-2.21%	-3.16%	-	-.74	18.56	0	.08	.05	1.13	.03	-.17	.30	.01
IGARCH(1,1) Multiple	6.75%	95.59%	.81	11.52%	-2.08%	-3.29%	-	-.78	22.91	0	.10	.06	1.14	.03	-.09	.36	.01
A Multiple	7.56%	107.01%	.75	9.79%	-1.66%	-2.85%	-	-.89	36.16	0	.20	.07	1.20	.04	.17	.70	.03

Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950. Returns and Volatility are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The Skewness and kurtosis P-statistics are computed using Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Eling and Schuhmacher (2007) and related literature for other performance measures.

Table 17: Ten-year Based DARE and Unconditional TIPP Cushioned Portfolio Strategy Characteristics on the Dow Jones Index from 1950 to 2008

	Return (mean annual)	Return (% of risky asset one)	Beta	Volatility	VaR99%	ES99%	Max Drawdown	Skewness	*Kurtosis*	Violating Floor	Sharpe	Sortino	Omega	Kappa	Relative perf/ ES99%	Cond. Sharpe Ratio	Treynor Ratio
Risky Asset	7.06%	100.00%	1.00	14.40%	-2.33%	-3.26%	-	-1.08	35.63	-	.10	.05	1.10	.03	-	.46	.02
DARE Multiple	7.21%	102.13%	1.43	19.31%	-3.55%	-5.41%	-	-2.39	66.98	0	.09	.04	1.10	.02	.03	.31	.01
Multiple 1	5.79%	81.94%	0.52	4.60%	-0.82%	-1.25%	-	-2.96	110.19	0	.05	.11	1.31	.05	-1.02	.18	.00
Multiple 2	5.78%	81.86%	0.71	9.86%	-1.79%	-2.84%	-	-3.80	138.09	0	.02	.05	1.15	.02	-.45	.08	.00
Multiple 3	5.42%	76.81%	1.08	15.40%	-2.93%	-4.61%	-	-4.47	163.98	0	-.01	.04	1.10	.01	-.36	-.03	.00
Multiple 4	5.93%	84.00%	1.39	18.93%	-3.40%	-5.50%	-	-4.46	163.26	0	.02	.03	1.10	.01	-.21	.07	.00
Multiple 5	6.87%	97.34%	1.59	20.84%	-3.84%	-5.92%	-	-3.36	118.34	1	.06	.04	1.10	.02	-.03	.22	.01
Multiple 6	7.52%	106.54%	1.65	21.40%	-3.88%	-5.99%	-	-3.00	108.15	1	.09	.04	1.10	.02	.08	.33	.01
Multiple 7	7.60%	107.57%	1.68	21.64%	-3.88%	-6.00%	-	-2.89	103.61	1	.09	.04	1.10	.02	.09	.34	.01
Multiple 8	7.60%	107.69%	1.71	21.89%	-3.89%	-6.00%	-	-2.80	99.14	1	.09	.04	1.10	.02	.09	.34	.01
Multiple 10	7.58%	107.29%	1.75	22.29%	-3.95%	-6.03%	-	-2.66	92.28	1	.09	.04	1.10	.02	.09	.33	.01
Multiple 11	7.55%	106.98%	1.77	22.48%	-3.98%	-6.04%	-	-2.60	89.28	1	.09	.04	1.10	.02	.08	.33	.01
Multiple 13	7.53%	106.65%	1.82	22.89%	-4.04%	-6.07%	-	-2.46	83.27	2	.09	.04	1.10	.02	.08	.32	.01

Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950. Returns and Volatility are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The Skewness and kurtosis P-statistics are computed using Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Eling and Schuhmacher (2007) and related literature for other performance measures.

Table 18: Ten-year Based DARE and Conditional TIPP Cushioned Portfolio Strategy Characteristics on the Dow Jones Index from 1950 to 2008

	Return (mean annual)	Return (% of risky asset one)	Beta	Volatility	VaR99%	ES99%	Max Drawdown	Skewness	*Kurtosis*	Violating Floor	Sharpe	Sortino	Omega	Kappa	Relative perf/ ES99%	Cond. Sharpe Ratio	Treynor Ratio
Risky Asset	7.06%	100.00%	1.00	14.40%	-2.33%	-3.26%	-	-1.08	35.63	-	.10	.05	1.10	.03	-	.46	.02
DARE Multiple	7.21%	102.13%	1.43	19.31%	-3.55%	-5.41%	-	-2.39	66.98	0	.09	.04	1.10	.02	.03	.31	.01
H Multiple	6.83%	96.70%	1.39	18.88%	-3.50%	-5.39%	-	-4.48	162.79	0	.07	.04	1.11	.02	-.04	.24	.01
RM Multiple	7.53%	106.70%	1.63	21.17%	-3.81%	-5.72%	-	-3.24	105.71	0	.09	.04	1.10	.02	.08	.35	.01
Normal Multiple	7.50%	106.19%	1.67	21.55%	-3.95%	-6.00%	-	-3.02	104.64	1	.09	.04	1.10	.02	.07	.32	.01
GARCH(1,1) Multiple	7.09%	100.35%	1.61	20.99%	-3.84%	-5.74%	-	-3.38	109.38	0	.07	.04	1.10	.02	.00	.27	.01
SAV Multiple	7.05%	99.83%	1.40	19.01%	-3.59%	-5.28%	-	-1.16	22.97	0	.08	.04	1.10	.02	.00	.28	.01
AS Multiple	6.30%	89.22%	1.41	19.16%	-3.51%	-5.24%	-	-.91	16.83	0	.04	.04	1.09	.02	-.15	.14	.01
IGARCH(1,1) Multiple	6.65%	94.12%	1.41	19.15%	-3.59%	-5.30%	-	-1.93	48.67	0	.06	.04	1.10	.02	-.08	.20	.01
A Multiple	6.46%	91.46%	1.30	18.00%	-3.30%	-5.18%	-	-5.05	194.30	0	.05	.04	1.11	.02	-.12	.17	.01

Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950. Returns and Volatility are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The Skewness and kurtosis P-statistics are computed using Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Eling and Schuhmacher (2007) and related literature for other performance measures.

Table 19: Fifteen-year Based DARE and Unconditional TIPP Cushioned Portfolio Strategy Characteristics on the Dow Jones Index from 1950 to 2008

	Return (mean annual)	Return (% of risky asset one)	Beta	Volatility	VaR99%	ES99%	Max Drawdown	Skewness	*Kurtosis*	Violating Floor	Sharpe	Sortino	Omega	Kappa	Relative perf/ ES99%	Cond. Sharpe Ratio	Treynor Ratio
Risky Asset	7.06%	100.00%	1.00	14.40%	-2.33%	-3.26%	-	-1.08	35.63	-	.10	.05	1.10	.03	-	.46	.02
DARE Multiple	7.57%	107.25%	1.29	17.89%	-3.02%	-4.55%	-	-2.77	85.94	0	.11	.04	1.10	.02	.11	.44	.02
Multiple 1	6.19%	87.71%	0.54	5.40%	-0.91%	-1.33%	-	-2.18	76.97	0	.12	.10	1.25	.05	-.65	.48	.01
Multiple 2	6.65%	94.19%	0.76	10.79%	-1.80%	-2.77%	-	-3.41	123.77	0	.10	.06	1.14	.02	-.15	.39	.01
Multiple 3	6.50%	92.00%	1.08	15.43%	-2.55%	-3.99%	-	-5.09	202.16	0	.06	.04	1.10	.02	-.14	.23	.01
Multiple 4	6.91%	97.87%	1.37	18.74%	-3.14%	-4.89%	-	-4.37	165.62	0	.07	.04	1.10	.02	-.03	.28	.01
Multiple 5	7.64%	108.19%	1.50	20.02%	-3.40%	-5.26%	-	-3.45	134.07	1	.10	.04	1.10	.02	.11	.40	.01
Multiple 6	7.75%	109.76%	1.54	20.33%	-3.45%	-5.31%	-	-3.28	126.53	1	.11	.04	1.10	.02	.13	.41	.01
Multiple 7	7.75%	109.76%	1.56	20.58%	-3.55%	-5.36%	-	-3.19	120.85	1	.11	.04	1.10	.02	.13	.41	.01
Multiple 8	7.73%	109.52%	1.59	20.88%	-3.60%	-5.43%	-	-3.07	114.44	1	.10	.04	1.10	.02	.12	.40	.01
Multiple 10	7.73%	109.41%	1.68	21.69%	-3.73%	-5.64%	-	-2.84	99.94	1	.10	.04	1.10	.02	.12	.38	.01
Multiple 11	7.70%	109.12%	1.76	22.34%	-3.84%	-5.78%	-	-2.62	89.71	1	.10	.04	1.10	.02	.11	.37	.01
Multiple 13	7.66%	108.52%	1.93	23.79%	-4.10%	-6.08%	-	-2.19	71.65	2	.09	.04	1.09	.02	.10	.35	.01

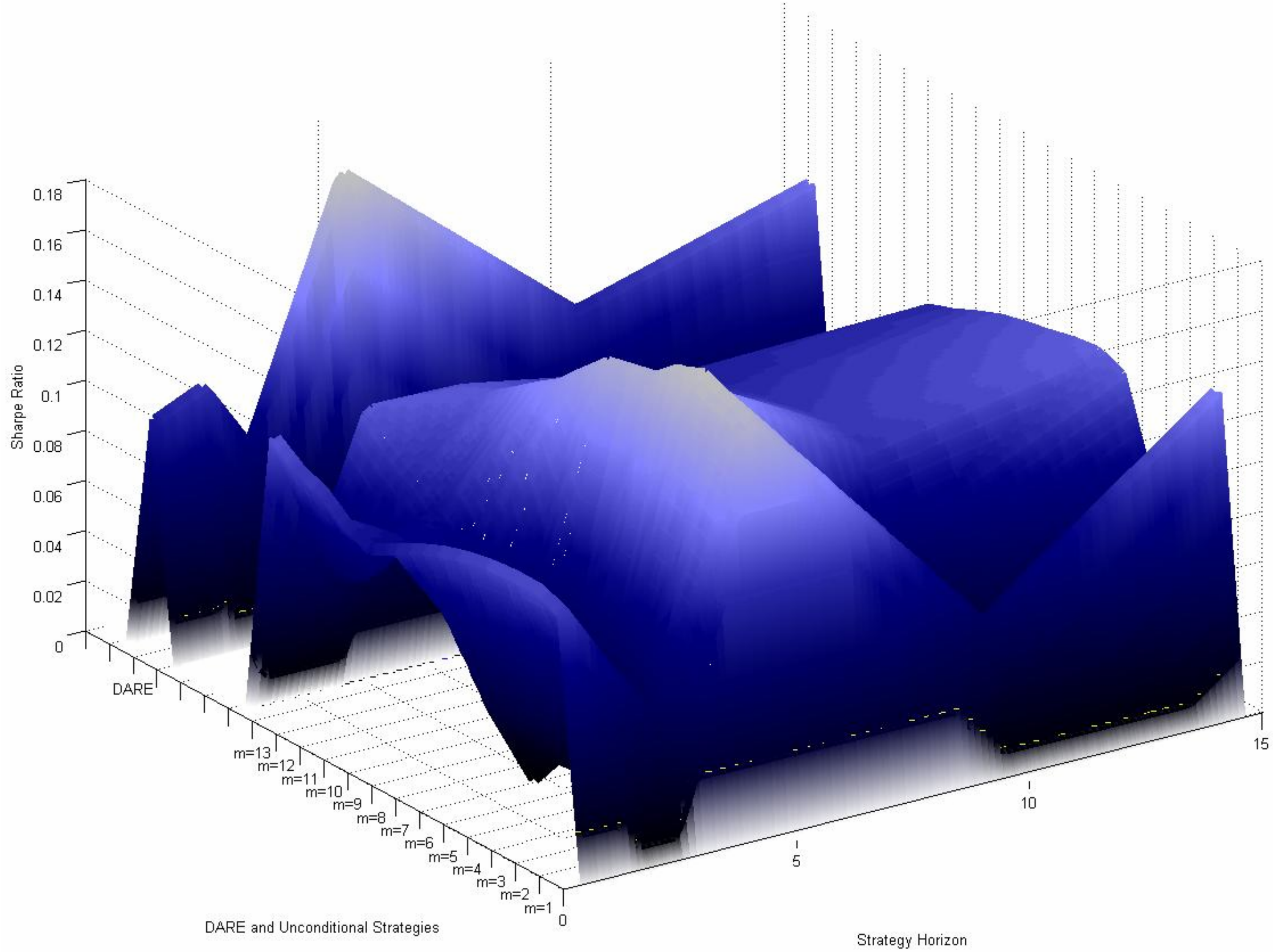
Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950. Returns and Volatility are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The Skewness and kurtosis P-statistics are computed using Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Eling and Schuhmacher (2007) and related literature for other performance measures.

Table 20: Fifteen-year Based DARE and Conditional TIPP Cushioned Portfolio Strategy Characteristics on the Dow Jones Index from 1950 to 2008

	Return (mean annual)	Return (% of risky asset one)	Beta	Volatility	VaR99%	ES99%	Max Drawdown	Skewness	*Kurtosis*	Violating Floor	Sharpe	Sortino	Omega	Kappa	Relative perf/ ES99%	Cond. Sharpe Ratio	Treynor Ratio
Risky Asset	7.06%	100.00%	1.00	14.40%	-2.33%	-3.26%	-	-1.08	35.63	-	.10	.05	1.10	.03	-	.46	.02
DARE Multiple	7.57%	107.25%	1.29	17.89%	-3.02%	-4.55%	-	-2.77	85.94	0	.11	.04	1.10	.02	.11	.44	.02
H Multiple	7.50%	106.28%	1.27	17.68%	-3.04%	-4.58%	-	-5.09	201.52	0	.11	.04	1.11	.02	.10	.42	.02
RM Multiple	7.65%	108.28%	1.46	19.56%	-3.32%	-4.93%	-	-3.83	137.59	0	.11	.04	1.10	.02	.12	.42	.01
Normal Multiple	7.78%	110.13%	1.56	20.57%	-3.52%	-5.43%	-	-3.27	121.03	1	.11	.04	1.10	.02	.13	.41	.01
GARCH(1,1) Multiple	7.24%	102.53%	1.45	19.49%	-3.34%	-5.00%	-	-3.95	139.91	0	.09	.04	1.10	.02	.04	.34	.01
SAV Multiple	7.89%	111.72%	1.30	17.97%	-3.16%	-4.56%	-	-1.01	22.11	0	.13	.05	1.10	.03	.18	.51	.02
AS Multiple	6.89%	97.56%	1.31	18.06%	-3.14%	-4.51%	-	-.74	13.64	0	.07	.04	1.09	.02	-.04	.29	.01
IGARCH(1,1) Multiple	7.24%	102.57%	1.31	18.03%	-3.13%	-4.54%	-	-2.05	57.31	0	.09	.04	1.10	.02	.04	.37	.01
A Multiple	7.43%	105.23%	1.25	17.41%	-2.99%	-4.60%	-	-5.28	215.48	0	.11	.04	1.11	.02	.08	.41	.01

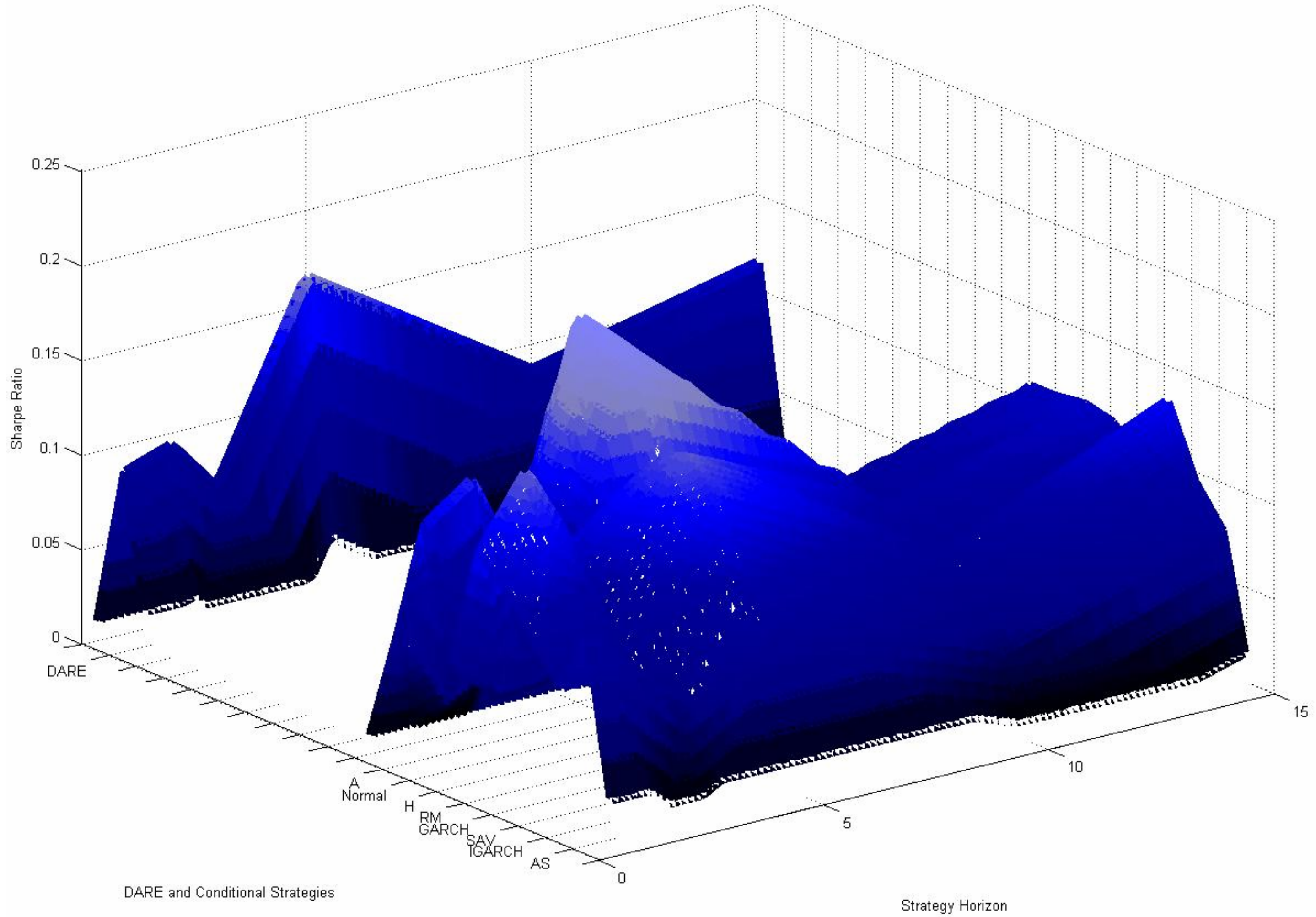
Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950. Returns and Volatility are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The Skewness and kurtosis P-statistics are computed using Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Eling and Schuhmacher (2007) and related literature for other performance measures.

Figure 7: Sharpe Ratio of DARE and Unconditional TIPP Cushioned Portfolio Strategy on a One to Fifteen-year Basis



Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950.

<u>Figure 8</u>: Sharpe Ratio of DARE and Conditional TIPP Cushioned Portfolio Strategy on a One to Fifteen-year Basis



Source: *Bloomberg*, daily data, Daily Returns of the Dow Jones Index from 01/02/1950 to 09/30/2008; computation by the authors. TIPP Strategies are presented from 1950.