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Wavelet based Estimation of Time-Varying Long Memory Model with Nonlinear Fractional Integration Parameter

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Abstract

In this paper, we propose a time-varying long memory model where the fractional integration parameter varies nonlinearly according to Smooth Transition Regressive (STR) model. To estimate the fractional integration parameter, we suggest a new estimation method based on wavelet approach. In particular, we consider the instantaneous least squares estimator (ILSE). We conduct some simulation experiments and provide an empirical application to modeling the dynamics of volatilities of some financial time series. The obtained results show that the model proposed offers an interesting framework to describe time-varying long range dependence of volatilities and provide evidence of regime change in persistence to shocks.

Keywords: time-varying long memory model, fractional integration parameter, STR model, wavelet, ILSE, volatilities of financial time series

JEL Classification: C13, C22, C32, G15

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1 Introduction

Long memory models have been extensively applied in empirical studies to model the dynamics of economic and financial time series[1] that exhibit long range dependence in both return and volatility. The most widely used model is the AutoRegressive Fractional Integrated Moving Average (ARFIMA) model introduced by Granger and Joyeux (1980) and Hosking (1981). This model has been used to reproduce persistence in mean or variance of some financial time series like stock market indices and exchange rates (Diebold et al. (1991), Cheung (1993), Baillie and Bollerslev (1994), Andersen and Bollerslev (1997) and Davidson (2004)). In time domain, such model is characterized by an autocorrelation function that decays at hyperbolical rate rather than geometric rate found in most stationary time series while in frequency domain, it is characterized by high power at low frequencies especially near the origin.

The previous studies suppose that the fractional integration parameter (d) is constant over time which implies that the long range dependence structure of the underlying phenomenon persists over time. This assumption seems to be restrictive due to the presence of structural breaks in fractional integration parameter (Granger and Hyung (2004), Beltratti and Morana (2008) and Baillie and Morana (2009)). In the same contexte, some authors found that the fractional integration parameter is time-varying (d_t) and that persistence to shocks varies over time (Jensen (1999a,b), Whitcher and Jensen (2000), Beran (2009) and Roueff and von Sachs (2011)). However, these authors don't specifiy the process of evolution of d_t.

Recent studies provided that the time-varying fractional integration parameter (d_t) follows several regime switching models. Among these models, Beine and Laurent (2001) found that d_t is driven by a Markov-switching model. Dufrénot et al. (2005a,b; 2008) found that d_t follows a Self Exciting Thresold Autoregressive (SETAR) model. Recently, Boutahar et al. (2008), and Aloy et al. (2011) find that d_t evolves according to Smooth Transition Regressive (STR) model (Teräsvirta (1994, 1998)).

The existence of time-varying behaviour in d_t could be explained, among others, by heterogeneity of agents in time horizons and strategies (Lux and Marchesi (2000), Kirman and Teyssière (2002), Iori (2002) and Alfarano and Lux (2002)), presence of multiple attractors or "intermittency" in volatility clustering (Gaunersdorfer et al. (2001)) and changes in financial market structure such as creation of new financial products.

In this paper, we propose a long memory model where the fractional integration parameter varies over time. In particular, we assume, following Boutahar et al. (2008) and Aloy et al. (2011), that the fractional integration parameter varies according two regimes where the transition from one regime to the other is smooth. The interesting feature of the model is that it allows for both presence of lange range dependence in series and asymmetry in degrees of persistence.

[1] Long memory models have been also used to model the dynamics of the series in other fields like hydrology and geophysics.

To estimate the time-varying fractional integration parameter, Boutahar et al. (2008) considered the arranged regression which consists on arranging the observations of endogenous and exogenous variables according to ascending order of magnitude of the observations of another variable. In this paper, we propose an alternative method based on two steps. In a first step, we focus on estimating time-varying fractional integration parameter using wavelet approach. The advantage of estimating d_t is that it allows us to implement tests to check the presence of nonlinearity and to determine the appropriate transition function and transition variable. In a second step, we propose to reproduce the dynamics of d_t while referring to STR model.

The strength of the wavelet approach lies on its capacity to simultaneously localize a process in time and scale. At high scales, the wavelet has a small centralized time support enabling it to focus on short-lived time phenomena. At low scales, the wavelet has a large support allowing it to identify long periodic behavior. By moving from low to high scales, the wavelet zooms in a process's behavior, identifying singularities, jumps and cups (Mallat and Zhong (1992), Mallat and Hwang (1992) and Mallat (1999)). In this paper, we consider the Maximum Overlap Discrete Wavelet Transform (MODWT) advanced by Percival and Walden (2000) that provides us an approximate log-linear relationship between time-varying variance of MODWT coefficients and time-varying long memory parameter d_t, then, we apply the instantaneous least squares estimator (ILSE) to obtain local estimates for time-varying fractional integration parameters.

Although, the wavelet approach has been used by several authors to estimate time-varying long memory parameters (Jensen (1999a,b) and Whitcher and Jensen (2000)), the authors don't specifiy the process of d_t. In this paper, we assume that d_t evolves nonlinearly according to STR model advanced by Teräsvirta (1994, 1998).

We apply this model to study the persistence of volatility of some financial series. We find evidence of two distinct regimes in the persistence of volatilities depending on the values of the transition variable, i.e., the lagged fractional integration parameter d_{t-1}. When the values of d_{t-1} are low, the volatilities are anti-persistent whereas when the values of d_{t-1} are high, the volatilities of some financial series are persistent.

These empirical findings have several implications for forecasting financial series, pricing assets, composing portfolio and managing risks. For example, establishing the presence of time-varing long memory behaviour in exhange rate implies that the the persistence to shocks is changing over time. This persistence of shocks may give the Central Bank's authorities additional incentives to intervene in currency markets.

The paper is organized as follows. Section 2 presents the time-varying long memory model. Section 3 proposes a wavelet based estimation method for the model. Section 4 focuses on nonlinear modeling of time-varying fractional integration parameter by STR model. Section 5 gives some simulations that shows the power of the proposed method. Section 6 contains an empirical application to some financial time series and section 7 concludes the paper.

2 Time-varying long memory model

First, we present the classical ARFIMA model. Then, we describe the time-varying ARFIMA model.

2.1 Presentation of ARFIMA model

Let X_t, $t = 1, ..., T$ denote a time series process. Following Granger and Joyeux (1980), the usual ARFIMA(p, d, q) model is given by:

$$\Phi(B)(1-B)^d X_t = \Theta(B)\varepsilon_t. \tag{1}$$

Where B is the backshift (lag) operator such that $B^i X_t = X_{t-i}$, $\Phi(B)$ and $\Theta(B)$ are polynomials in B involving autoregressive and moving average cooefficients of orders p and q respectively with their roots strictly outside the unit circle and have no common factors, d is the fractional integration parameter and ε_t is a white noise process with zero mean and variance σ_ε^2.

The fractional differencing lag operator $(1-B)^d$ is defined by the binomial expansion:

$$(1-B)^d = \sum_{i=0}^{\infty} \frac{\Gamma(i-d)}{\Gamma(i+1)-\Gamma(d)} B^i. \tag{2}$$

Where $\Gamma(.)$ denotes the gamma function.

The parameters found in $\Phi(B)$ and $\Theta(B)$ constitute the short memory parameters and affect only the short-run dynamics of the process while the fractional integration parameter d detects the long memory behaviour of the process. Various cases are possible: If $-0.5 < d < 0$, the process is anti-persistent memory. If $0 < d < 0.5$, the process is stationary long memory and possesses shocks that disappear hyperbolically. If $0.5 \leq d < 1$, the process is nonstationary but mean reverting with finite impulse response weights. When $d = 0$, the process reduces to the standard ARMA and when $d = 1$, the process becomes ARIMA and implies infinite persistence of the mean to a shock in the returns.

2.2 Presentation of TV-ARFIMA model

If we suppose that d varies over time, i.e., d_t, we obtain the so called time-varying ARFIMA(p, d, q) model that we note TV-ARFIMA(p, d, q). This long memory model is a member of non stationary class of processes known as locally stationary processes (Dahlhaus (1996) and Whitcher and Jensen (2000)) and is given by:

$$\Phi(B)(1-B)^{d_t} X_t = \Theta(B)\varepsilon_t. \tag{3}$$

Where $\Phi(B)$ and $\Theta(B)$ are stable polynomials, i.e., their roots are strictly outside the unit circle. $d_t < 0.5$ is the time-varying fractional integration parameter and ε_t is a white noise process with zero mean and variance σ_ε^2.

Let us recall briefly the commonly used definitions of time-varying long memory model in frequence and time domain:

Definion 1 (in frequency domain): X_t is a locally stationary long memory process if there exists a function $sdf\,(t,\lambda)$ called the time-varying spectral density function such that:

$$sdf\,(t,\lambda) \sim \lambda^{-2d(t)} \text{ as } \lambda \to 0^{+}. \tag{4}$$

Thus, if $d\,(t) > 0$, $sdf\,(t,\lambda)$ is smooth for frequencies close to zero, but is unbounded when $\lambda = 0$. In other words, the energy of X_t is concentrated over those frequencies associated with long-term cycles. If $d\,(t) < 0$, then $f\,(t,\lambda) = 0$ and X_t is a locally stationary series that is anti-persistent. As a result of the time-varying long memory parameter, X_t will be smoother with less variation in its amplitude during time periods where $d\,(t) > 0$, and will have large fluctuations in its values when $d\,(t) < 0$.

Definion 2 (in time domain): X_t is a locally stationary long memory process if there exists a function denoted the local autocovariance function $cov_X\,(t, g-h)$ that can be simplified as:

$$cov_X\,(t, g-h) \sim |g-h|^{2d(t)-1} \text{ as } |g-h| \to \infty. \tag{5}$$

As indicated, the slow hyperbolic decay of $cov_X\,(t, g-h)$ is the feature most often noted when discussing the dynamics of a long memory process.

Although this model indicate that the fractional integration parameter is time-varying, it do not provide information about the evolution of d_t. Following Boutahar et al. (2008) and Aloy et al. (2011), we suppose that d_t evoloves according to STR model advanced by Teräsvirta (1994, 1998):

$$d_t = d_1\,[1 - F\,(s_t;\gamma,c)] + d_2 F\,(s_t;\gamma,c)\,. \tag{6}$$

Where d_1 and d_2 are the values of the fraction integration parameter in the first and in the second regime respectively. $F\,(s_t;\gamma,c)$ is the transition function that is continuous bounded between 0 to 1 with s_t denoting the transition variable which can be one of the lagged endogenous variable $s_t = d_{t-i}$, $\forall t > i$, $t = 1, ..., T$ or an exogneous variable. The slope parameter γ measures the speed of the transition between the two extreme regimes (associated with the extreme values 0 and 1 of the transition function) which can be either positive or negative depending upon whether the logistic curve is increasing or not. The parameter c represents the threshold for the transition variable s_t which defines the underlying regimes: $s_t \leq c$ (resp. $s_t > c$) means that the underlying regime is the first (resp. the second) one.

We find in the literature two types of transition function:

- the logistic function, for the logistic STR (LSTR) model:

$$F(s_t;\gamma,c) = \left(1 + \exp\left(-\gamma(s_t - c)\right)\right)^{-1}, \tag{7}$$

- the exponential function, for the exponential STR (ESTR) model:

$$F(s_t;\gamma,c)=1-\exp\left(-\gamma(s_t-c)^2\right). \tag{8}$$

The stochastic fractional filter $(1-B)^{d_t}$ can be defined as follows:

$$(1-B)^{d(t,w)}=\sum_{i=0}^{\infty}\frac{\Gamma\left(i-d\left(t,w\right)\right)}{\Gamma\left(i+1\right)-\Gamma\left(d\left(t,w\right)\right)}B^i. \tag{9}$$

Where $d\left(t,w\right):=d_t\left(w\right)$ and Γ is the gamma function.

Model (3) can be written as an infinite moving average process in terms of ε_{t-i}:

$$X_t\equiv\sum_{i=0}^{\infty}a_i\left(s_t\right)\pi_i\varepsilon_{t-i}. \tag{10}$$

Where $a_i\left(y\right)=\frac{\Gamma(i+D(y))}{\Gamma(i+1)\Gamma(D(y))}$ and $D\left(y\right)=d_1\left[1-F(y;\gamma,c)\right]+d_2F(y;\gamma,c)$, i.e., $D\left(y\right)=d_1+(d_2-d_1)\left[1+\exp\left(-\gamma(y-c)\right)\right]^{-1}$ if the transition function is logistic and $D\left(y\right)=d_1+\left(d_2-d_1\right)\left[1-\exp\left(-\gamma(y-c)^2\right)\right]$ if the transition function is exponentiel. π_i is the solution to $\frac{\Theta(z)}{\Phi(z)}=\sum_i\pi_iz^i$.

In this paper, we suggest an estimation method by performing two steps to estimating the time-varying long memory model defined by:

$$\begin{cases}\Phi\left(B\right)\left(1-B\right)^{d_t}X_t=\Theta\left(B\right)\varepsilon_t,\\ d_t=d_1\left[1-F\left(s_t;\gamma,c\right)\right]+d_2F\left(s_t;\gamma,c\right).\end{cases} \tag{11}$$

In the first step, we estimate the fractional integration parameter d_t using a wavelet approch (Equation (3)). In the second step, the STR model is estimated on d_t (Equation 4).

3 Wavelet based estimation of time-varying fractional integration parameter

The estimation of fractional integration parameter d in fractionally integrated process $I(d)$ has been widely examined in the literature and numerous estimation methods have been devised for when the property of stationarity holds. Among these methods, we find the parametric methods which are (approximate or exact) likelihood methods in time domain or frequency domain, the semiparametric estimators which are based on spectral density and the nonparametric methods. In this paper, we adopt another estimation method based on wavelet approach. We first recall some basis of wavelets then we present the instantaneous least squares estimator that we use to estimate the fractional integration parameter d_t.

3.1 Wavelet methodology

Wavelets are mathematical tools that are widely applied for analysing time series. The starting point in such analysis is based on decomposing a time series on a scale-by-scale basis. It is analogous to standard Fourier transform, yet the complex exponentials are replaced by the wavelet functions that -via dilation and translation operations- allow a flexible time-frequency resolution and enable to describe local characteristics of a given function in a parsimonious way. Wavelets are orthonormal bases obtained by dyadically dilating and translating a pair of specially constructed functions φ and ψ which are called father wavelet and mother wavelet respectively such that:

$$\int \varphi(t)\, dt = 1, \tag{12}$$

$$\int \psi(t)\, dt = 0. \tag{13}$$

The smooth and the low-frequency part of the series are captured by the father wavelet while the detail and the high-frequency components are described by the mother wavelet. The obtained wavelet basis is:

$$\varphi_{j,k}(t) = 2^{j/2}\varphi\left(2^j t - k\right), \tag{14}$$

$$\psi_{j,k}(t) = 2^{j/2}\psi\left(2^j t - k\right). \tag{15}$$

Where $j = 1, \ldots, J$ indexes the scale and $k = 1, \ldots, 2^j$ indexes the translation. The parameter j is used as the parameter of dilation of the waves' functions. This parameter j adjusts the support of $\psi_{j,k}(t)$ in order to locally capture the characteristics of high or low frequencies. The parameter k is used to relocate the wavelets in the temporal scale. The maximum number of scales that can be considered in the analysis is limited by the number of observations ($T \geq 2^J$).

One special property of the wavelet expansion is the localization property that the coefficient of $\psi_{j,k}(t)$ reveals information content of the function at approximate location $k2^{-j}$ and frequency 2^j. Using wavelets, any function in $L^2(\mathbb{R})$ can be can expanded over the wavelet basis, uniquely, as a linear combination at arbitrary level $J_0 \in \mathbb{N}$ across different scales of the type:

$$f(t) = \sum_k c_{J_0,k}\varphi_{J_0,k}(x) + \sum_{j>J_0}\sum_k d_{j,k}\psi_{j,k}(x). \tag{16}$$

Where $\varphi_{J_0,k}$ is a scaling function with the corresponding coarse scale coefficients $c_{J_0,k}$ and $d_{j,k}$ are the detail (fine scale) coefficients given respectively by $c_{J_0,k} = \int f(x)\,\varphi_{J_0,k}(x)\,dx$ and $d_{j,k} = \int f(x)\,\psi_{j,k}(x)\,dx$. These coefficients give a measure of the contribution of the corresponding wavelet to the function.

This expression represents the decomposition of $f(t)$ into orthogonal components at different resolutions and constitutes the so called wavelet multiresolution analysis (decomposition) (MRA).

In practical applications, we invariably deal with sequences of values indexed by integers rather than functions defined over the entire real axis. In stead of actual wavelets, we use short sequences of values referred to as wavelet filters. The number of values in the sequence is called the width of the wavelet filter. Thus, the wavelet analysis considered from a filtering perspective is then well-adapted for time series analysis. For the discrete wavelet transform, the wavelet coeffcients can be calculated from the MRA scheme. The recursive MRA scheme which is implemented by a two-channel filter bank (i.e., a low-pass filter and a high-pass filter) representation of the wavelet transform, is divided into decomposition and reconstruction schemes according to the forward and inverse wavelet transform.

Daubechies (1992) has constructed a class of wavelet functions where φ is a function such that $\{\varphi(t-k),\ k \in \mathbb{Z}\}$ forms an orthonormal basis of piecewise constant functions of length one. The Daubechies wavelet has many desirable properties, its most useful property is possessing the smallest support for a given number of vanishing moments. Daubechies defined a useful class of wavelet filters namely the Daubechies compactly supported wavelet filters and distinguishes between two choices: the extremal phase filters $D(L)$ and the least asymmetric filters $LA(L)$.

A modified version of the DWT is the non-decimated or Maximal Overlap Discrete Wavelet Transform (MODWT) (Percival and Walden (2000)). The MODWT algorithm carries out the same filtering steps as the standard DWT but does not subsample (decimate by 2); therefore the number of scaling and wavelet coefficients at each level of the transform is the same as the number of sample observations. The concepts of partial MODWT and MODWT-based multiresolution analysis are defined in a similar way to those of the DWT. Percival and Walden (2000) list several properties that distinguish the MODWT from the DWT. For present purposes, it is sufficient to mention that the MODWT can handle any sample size and that the details and the smooth component of the MODWT are associated with zero phase filters. This means features in the original time series may be suitably aligned with those of the MODWT-based Multiresolution Analysis.

3.2 Instantaneous least squares estimator

The basic idea to estimate the fractional integration parameter d via a wavelet transform of the time series is the wavelet variance. Wavelet variance analysis consists in partitioning the variance of a time series into pieces that are associated to different time scales. This one substitutes the notion of variability over certain scales for the global measure of variability estimated by the sample variance tells us what scales are important contributors to the overall variability of a series.

In particular, let $X_1, X_2, \ldots, X_T$ be a time series of interest, which is assumed to be a realization of a stationary process with variance σ_X^2. If the scaling coefficients for level j are associated with averages of length 2^j, then the level j wavelet coefficients (which are differences of averages half this length) are

associated with changes at scale $\tau_j \equiv 2^{j-1}\Delta t$, where Δt is the sampling interval of X_t. Thus, the wavelet variance $V_X^2(\tau_j)$ for scale $\left(\tau_j \equiv 2^{j-1}\right)^2$ is defined as:

$$\sigma_X^2 \equiv \sum_{j=1}^{J} V_X^2(\tau_j). \tag{17}$$

For estimating d, the fractional integration parameter, via a wavelet approach many methods have been suggested. They can be summarized in three computationally efficient schemes. The first scheme is a wavelet-based approximation to the maximum likelihood estimator (MLE) of d under the assumption of multivariate Gaussianity (McCoy and Walden (1996) and Jensen (1999a, 2000)). The second make use of the fact that the relationship between the variance of the wavelet coefficients across scales is dictated by d. In this framework, we construct a least squares estimator (LSE) of d (Abry and Veitch (1998) and Jensen (1999b)). The third utilize only certains coefficients that are colocated in time, and we refer to it as an the instantaneous least squares estimator (ILSE). However, this estimator is independent upon the entire time series.

The main idea of the instantaneous least squares estimator is to use a single wavelet coefficient from each scale of resolution, i.e., we only use $\tilde{d}_{j,t_j}$ to estimate $V_X^2(\tau_j)$, where t_j is the time index of the j^{th} level MODWT coefficient associated with time t in X_t, $t = 1, \ldots, T$. The time index t_j can be meaningfully determined only if we use linear phase wavelet filter.

Formally, let the vector of dimension containing the wavelet coefficients obtained by the MODWT transform. The instantaneous least squares estimator is given by:

$$\hat{d}_{ILSE,t} = \frac{\Delta_J \sum \ln(\tau_j) Y_t(\tau_j) - \sum \ln(\tau_j) \sum Y_t(\tau_j)}{2\left(\Delta_J \sum \ln^2(\tau_j) - \left(\sum \ln(\tau_j)\right)^2\right)} + \frac{1}{2}. \tag{18}$$

Where $\Delta_J = J - J_0 + 1$ and all sums are over $j = J_0, \ldots, J$ and $Y_t(\tau_j) \equiv \ln\left(d_{j,t_j}^2\right) - psi(1/2) - \ln(2)$, with psi is the digamma function (see also a MODWT-based weighted least squares estimator developed by Percival and Walden (2000)).

4 Nonlinear modeling of time-varying fractional integration parameter

In this section, we propose to modeling the time-varying fraction integration by STR model. To do so, we first test the constancy of fractional integration parameter, then we choose the appropriate transition function and finally we estimate the model.

[2] In the present work, we consider $\Delta t = 1$.

4.1 Testing fractional integration parameter constancy

We focus on testing the constancy of fractional integration parameter ($d_t = d$), that is the model is a standard long memory model (Equation (1)) against STR specification of fractional integration parameter (Equation (6)) and the model is a time-varying long memory model (Equation (3)). For that, we apply a Lagrange Multiplier (LM) test. This test has previously been developed by Teräsvirta (1994, 1998) to test the linearity of the autoregressive model and considered by Aloy et al. (2011) to test the constancy of fractional integration parameter.

The null hypothesis of the test can be expressed as $H_0 : \gamma = 0$ or, equivalently, as $H_0^{'} : d_1 = d_2$ (that is equality of the parameters in the two regimes) against $H_1 : \gamma > 0$. However, under the null hypothesis, the model is not identified due to the nuisance parameters γ and c. Consequently, the standard LM test can not be implemented. As suggested by Luukkonen et al. (1988) and done by Aloy et al. (2011), to solve this problem, we replace the transition function $F(s_t; \gamma, c)$ by its third-order Taylor approximation around $\gamma = 0$. In the reparameterized model, the identification problem is no longer present and the constancy can be easily tested using the LM-type test. More precisely, the test refers to the following auxiliary regression:

$$\hat{\varepsilon}_t = \beta_0 + \beta_1^{'} d_t + \beta_2^{'} d_t s_t + \beta_3^{'} d_t s_t^2 + \beta_4^{'} d_t s_t^3 + \eta_t. \tag{19}$$

Where $\hat{\varepsilon}_t$ are the residuals obtained from the linear autoregressive model estimated by OLS for d_t[3].

The null hypothesis of constancy of d_t becomes $H_0^{''} : \beta_2 = \beta_3 = \beta_4 = 0$.

As in standard cases, the LM statistic is asymptotically distributed, under $H_0^{''}$, as χ^2 with one degree of freedom. A Fisher version can be used.

Teräsvirta (1994, 1998) suggests testing the null hypothesis for several candidate transition variables. If the null hypothesis is rejected for more than one transition variable, he suggests to choose the variable with the strongest rejection of linearity (i.e., with the smallest p-value).

After selecting the transition variable, the next step consists of choosing the appropriate form of the transition function.

4.2 Choosing the appropriate form for transition function

To discriminate LSTAR model from ESTAR model, we use a sequence of nested hypotheses that test for the order of the polynomial in the auxiliary regression as follows:

$H_{04} : \beta_4 = 0,$
$H_{03} : \beta_3 = 0 / \beta_4 = 0,$
$H_{02} : \beta_2 = 0 / \beta_3 = \beta_4 = 0.$

[3] $d_t = \alpha_0 + \alpha_1 d_{t-1} + \varepsilon_t$.

The rejection of H_{04} implies that the model is LSTR model. If the test of H_{03} has the smallest p-value, the model is ESTR model. If both H_{04} and H_{03} are accepted while H_{02} is rejected, we conclude that the model is LSTR model.

After choosing the appropriate form for transition function, we estimate the specified STR model by implementing a nonlinear least squares (NLS) estimation method. Under the additional assumption that the errors are normally distributed, the NLS estimates are similar to the maximum likelihood estimates. Otherwise, the NLS estimates can be interpreted as quasi maximum likelihood estimates.

5 Simulation experiments

In this section, we carry out some Monte Carlo simulation experiments in order to establish the robustness of the estimation method of time-varying fractional integration parameter d_t using wavelet approach for finite sample. For purpose of simplicity, the TV-ARFIMA model used in simulation assumes that the transition function is logistic (Equation (8)) where the transition variable is d_{t-1}, the thresold parameter $c = 0$ and $\Phi(B) = \Theta(B) = 1$, i.e., we consider the simplest TV-ARFIMA$(0, d_t, 0)$ model given by:

$$\begin{cases} (1-B)^{d_t} X_t = \varepsilon_t, \\ d_t = d_1 + (d_2 - d_1)\left[1 + \exp\left(-\gamma d_{t-1}\right)\right]^{-1}, \end{cases} \tag{20}$$

for $t = 1, \ldots, T$.

We consider 5000 replications of X_t generated by (20) with different sample sizes ($T = 250$, 500, 1000, 2000 and 4000) and various slope parameters ($\gamma = 5$, 10, 20, 40 and 80) according to $d_1 = 0.1$, $d_2 = 0.3$ and $c = 0$. In this paper, we use a non-decimated discrete wavelet transform (MODWT) and consider the $LA(12)$ wavelet filter. Compared to the extremal phase filters $D(L)$ length, the least asymmetric filters $LA(L)$ offer superior frequency localization properties given a filter of the same length. To select the value of j, we specify over $j = 1, \ldots, J$, as large as possible range of scales sustainable with the data to hand. As announced previously, we estimate first the time-varying fractional integration parameter using instantaneous least squares estimator (ILSE). Then, we apply an LSTAR model to model the dynamic of the fractional integration parameter using nonlinear least squares (NLS) method. The simulation results are reported in Table 1.1 to 1.5.

Table 1.1: Simulation results for $\gamma = 5$

T	250	500	1000	2000	4000
N	5000	5000	5000	5000	5000
d_1	0.1	0.1	0.1	0.1	0.1
d_2	0.3	0.3	0.3	0.3	0.3
γ	5	5	5	5	5
c	0	0	0	0	0
$\hat{d_1}$	0.1336	0.1316	0.1288	0.1227	0.1168
$\hat{d_2}$	0.3229	0.3238	0.3203	0.3213	0.3069
$\hat{\gamma}$	6.3662	6.2414	5.8842	5.5681	5.3015
$\hat{c}$	0.0150	0.0120	0.0132	0.0120	0.0106
RMSE of $\hat{d_1}$	0.0032	0.0031	0.0021	0.0021	0.0019
RMSE of $\hat{d_2}$	0.0047	0.0034	0.0033	0.0032	0.0032
RMSE of $\hat{\gamma}$	0.1581	0.1327	0.1281	0.1216	0.1214
RMSE of $\hat{c}$	0.0049	0.0045	0.0062	0.0072	0.0054
Variance of $\hat{d_1}$	0.0028	0.0020	0.0015	0.0013	0.0008
Variance of $\hat{d_2}$	0.0017	0.0013	0.0011	0.0008	0.0003
Variance of $\hat{\gamma}$	1.1472	0.8454	0.6193	0.4359	0.4169
Variance of $\hat{c}$	0.0019	0.0012	0.0010	0.0008	0.0003

Table 1.2: Simulation results for $\gamma = 10$

T	250	500	1000	2000	4000
N	5000	5000	5000	5000	5000
d_1	0.1	0.1	0.1	0.1	0.1
d_2	0.3	0.3	0.3	0.3	0.3
γ	10	10	10	10	10
c	0	0	0	0	0
$\hat{d_1}$	0.1682	0.1634	0.1578	0.1435	0.1379
$\hat{d_2}$	0.3316	0.3310	0.3220	0.3166	0.3129
$\hat{\gamma}$	12.0079	11.4349	10.5271	10.5630	10.8311
$\hat{c}$	0.0174	0.0164	0.0140	0.0135	0.0097
RMSE of $\hat{d_1}$	0.0028	0.0022	0.0021	0.0014	0.0014
RMSE of $\hat{d_2}$	0.0027	0.0022	0.0017	0.0017	0.0016
RMSE of $\hat{\gamma}$	0.1278	0.1294	0.1445	0.1990	0.2162
RMSE of $\hat{c}$	0.0046	0.0077	0.0084	0.0130	0.0384
Variance of $\hat{d_1}$	0.0056	0.0046	0.0031	0.0026	0.0019
Variance of $\hat{d_2}$	0.0012	0.0009	0.0007	0.0006	0.0006
Variance of $\hat{\gamma}$	1.1556	0.8222	0.5300	0.4257	0.4171
Variance of $\hat{c}$	0.0029	0.0020	0.0012	0.0011	0.0009

Table 1.3: Simulation results for $\gamma = 20$

T	250	500	1000	2000	4000
N	5000	5000	5000	5000	5000
d_1	0.1	0.1	0.1	0.1	0.1
d_2	0.3	0.3	0.3	0.3	0.3
γ	20	20	20	20	20
c	0	0	0	0	0
$\hat{d_1}$	0.1719	0.1451	0.1232	0.1100	0.1005
$\hat{d_2}$	0.3318	0.3647	0.3364	0.3275	0.3021
$\hat{\gamma}$	22.8213	21.9476	21.6931	21.0084	20.2016
$\hat{c}$	0.0167	0.0105	0.0103	0.0100	0.0098
RMSE of $\hat{d_1}$	0.0045	0.0023	0.0018	0.0040	0.0032
RMSE of $\hat{d_2}$	0.0047	0.0045	0.0041	0.0040	0.0037
RMSE of $\hat{\gamma}$	0.1993	0.2621	0.2205	0.1342	0.0987
RMSE of $\hat{c}$	0.0069	0.0059	0.0043	0.0040	0.0032
Variance of $\hat{d_1}$	0.0058	0.0052	0.0046	0.0004	0.0001
Variance of $\hat{d_2}$	0.0014	0.0008	0.0006	0.0005	0.0004
Variance of $\hat{\gamma}$	1.9259	1.1052	0.9598	0.4365	0.3008
Variance of $\hat{c}$	0.0030	0.0022	0.0018	0.0010	0.0000

Table 1.4: Simulation results for $\gamma = 40$

T	250	500	1000	2000	4000
N	5000	5000	5000	5000	5000
d_1	0.1	0.1	0.1	0.1	0.1
d_2	0.3	0.3	0.3	0.3	0.3
γ	40	40	40	40	40
c	0	0	0	0	0
$\hat{d_1}$	0.1336	0.1098	0.1037	0.1027	0.1086
$\hat{d_2}$	0.3448	0.3237	0.3195	0.3076	0.3047
$\hat{\gamma}$	41.5910	40.4062	40.2050	40.1012	40.1342
$\hat{c}$	0.0092	0.0088	0.0066	0.0065	0.0063
RMSE of $\hat{d_1}$	0.0010	0.0008	0.0006	0.0005	0.0003
RMSE of $\hat{d_2}$	0.0032	0.0028	0.0024	0.0024	0.0017
RMSE of $\hat{\gamma}$	0.0632	0.0585	0.0494	0.0320	0.0308
RMSE of $\hat{c}$	0.0032	0.0029	0.0025	0.0018	0.0018
Variance of $\hat{d_1}$	0.0030	0.0020	0.0018	0.0017	0.0013
Variance of $\hat{d_2}$	0.0012	0.0009	0.0008	0.0006	0.0005
Variance of $\hat{\gamma}$	0.8263	0.6599	0.4257	0.2121	0.1711
Variance of $\hat{c}$	0.0008	0.0007	0.0004	0.0002	0.0002

Table 1.5: Simulation results for $\gamma = 80$

T	250	500	1000	2000	4000
N	5000	5000	5000	5000	5000
d_1	0.1	0.1	0.1	0.1	0.1
d_2	0.3	0.3	0.3	0.3	0.3
γ	80	80	80	80	80
c	0	0	0	0	0
$\hat{d_1}$	0.1357	0.1255	0.1108	0.1077	0.1070
$\hat{d_2}$	0.3311	0.3189	0.3165	0.3119	0.3087
$\hat{\gamma}$	81.3227	80.4274	80.2518	80.1093	80.0687
$\hat{c}$	0.0681	0.0628	0.0508	0.0202	0.0100
RMSE of $\hat{d_1}$	0.0032	0.0004	0.0003	0.0002	0.0001
RMSE of $\hat{d_2}$	0.0023	0.0021	0.0014	0.0009	0.0009
RMSE of $\hat{\gamma}$	0.0632	0.0533	0.0462	0.0394	0.0316
RMSE of $\hat{c}$	0.0017	0.0016	0.0012	0.0012	0.0011
Variance of $\hat{d_1}$	0.0038	0.0025	0.0018	0.0017	0.0017
Variance of $\hat{d_2}$	0.0013	0.0011	0.0009	0.0008	0.0005
Variance of $\hat{\gamma}$	1.0542	0.8850	0.5920	0.2467	0.2366
Variance of $\hat{c}$	0.0018	0.0014	0.0008	0.0008	0.0008

Note: In Table 1.1 to 1.5, T is the number of observations. N is the number of replications. d_1, d_2, γ and c are the true generation values of the parameters. $\hat{d_1}$, $\hat{d_2}$, $\hat{\gamma}$ and $\hat{c}$ are the estimators of d_1, d_2, γ and c respectively. RMSE is the Root Mean Squared Error.

We see that the estimated values of $\hat{d_1}$, $\hat{d_2}$, $\hat{\gamma}$ and $\hat{c}$ are very near the values d_1, d_2, γ and c used to simulate the model indicating a small bias. In addition, we see that the performance of the tests improves with the sample size T and the slope parameter γ. Indeed, for a fixed slope parameter, when the sample size increases, the variance of $\hat{d_1}$, $\hat{d_2}$, $\hat{\gamma}$ and $\hat{c}$ and the RMSE of $\hat{d_1}$, $\hat{d_2}$, $\hat{\gamma}$ and $\hat{c}$ decrease. Similarly, for a fixed sample size, when the slope parameter increases, the variance of $\hat{d_1}$, $\hat{d_2}$, $\hat{\gamma}$ and $\hat{c}$ and the RMSE of $\hat{d_1}$, $\hat{d_2}$, $\hat{\gamma}$ and $\hat{c}$ decrease.

6 Empirical application

This section contains an empirical application of TV-ARFIMA$(0, d_t, 0)$ model to modelling the dynamcis of volatilities of financial time series. First, we describe the data then we present the empirical results.

6.1 Description of data

The financial time series consist of five daily stock market indices (CAC 40, Dow Jones, DAX 30, FTSE and Nikkei 225) and three daily exchange rates (American dollar relative to Euro, Japanese yen relative to Euro and British

pound relative to Euro) covering the period from 04/01/1999 to 25/01/2012 and yielding to 3408 observations. Theses data are obtained from Datastream.

These data are transformed in logarithm form and taken in first difference[4], so we obtain:

$$X_t = \ln(S_t) - \ln(S_{t-1}). \tag{21}$$

Where S_t is nominal stock market index (resp. nominal exchange rate), X_t is stock market return (resp. exchange rate return) and $t = 1, \ldots, T$ with T is the total number of observations.

The series of interest is the centered absolute returns R_t defined as:

$$R_t = |X_t| - \left|\bar{X}_t\right|. \tag{22}$$

Where X_t is stock market return (resp. exchange rate return) given by Equation (21). $\bar{X}_t$ is mean of stock market return (resp. mean of exchange rate return) over the whole sample.

Following Andersen and Bollerslev (1998) and Boutahar et al. (2008), we use centered absolute returns as a proxy of volatility instead of squared returns because the latter measure can be noisy.

The centered absolute returns of the series are plotted in Figure 1. We observe that the volatilities of stock market and exchange rate are not constant and there are several peaks. We observe that large volatilities tend to be followed by large changes, of either sign, and that small volatilities tend to be followed by small changes suggesting evidence of volatilities clustering. In addition, we see several peaks.

Insert Figure 1

6.2 Empirical results

In a first step, we estimate the fractional integration parameter (d) over the whole period to verify the presence of long range dependence. For that, we consider several estimators in both time domain and wavelet domain. In time domain, we consider the GPH estimator introduced by Geweke and Porter-Hudac (1983) which is theoretically valid for $0 < d < 0.5$. However, if the estimate of the memory parameter is on the verge of stationarity, we need to consider an estimator which is consistent for $d > 0.5$ as well as $0 < d < 0.5$. Further, if the number of frequencies m included in the regression is restricted such that $m = O\left(T^{4/5}\right)$, then we obtain the asymptotic normality (see Hurvich et al. (1998)). In addition, we use the Exact Local Whittle estimator developed by Shimotsu and Phillips (2005), it is a semiparametric estimator generally giving a good estimation method for the memory parameter in terms of consistency and limit distribution. This estimator is consistent and has an $N\left(0, \frac{1}{4}\right)$ limit distribution for all values of d if the optimization covers an interval of width

[4] A preliminary analysis of stationarity of the series in level shows evidence of presence of unit roots. So, we consider the series in first difference.

less than $\frac{9}{2}$ and the mean of process is known. Whereas in wavelet domain, we apply some ordinary least squares estimators of long memory parameter from a fractionally integrated process. The first estimator is a semiparametric wavelet based estimator for the Hurst parameter and is proposed by Abry and Veitch (1998). Under the general conditions and the Gaussian assumptions, this estimator is unbiased and efficient. The second estimator is developed by Jensen (1999b) based on the fact that a log linear relationship exists between the variance of the wavelet coefficients from the long memory process and its scale equal to the long memory parameter. This log-linear relationship yields a consistent ordinary least squares estimator. The third estimator is done by formulating an instantaneous estimator that is independent of size of sample, it can be used to check for departures from statistical consistency within a proposed block size. Indeed, we use only a single wavelet coefficient from each scale.

Table 2 reports the estimation results of fractional integration parameter for volatilities of stock markets indices and exchange rates using the different estimators described above.

Table 2: Estimation results of constant fractional integration parameter

	$RCAC$	RDJ	$RDAX$	$RFTSE$	$RNKEI$	$RUSD$	$RJPY$	$RGBP$
$\hat{d}_{GPH}$	0.304	0.247	0.300	0.312	0.266	0.152	0.245	0.141
$\hat{d}_{ELW}$	0.319	0.307	0.333	0.340	0.318	0.181	0.276	0.223
$\hat{d}_{LSE_AV}$	0.484	0.476	0.466	0.407	0.430	0.300	0.137	0.152
$\hat{d}_{LSE_J}$	0.363	0.368	0.369	0.359	0.340	0.214	0.301	0.228
$\hat{d}_{ILSE}$	0.411	0.413	0.427	0.411	0.347	0.267	0.331	0.309

Note: $RCAC$, RDJ, $RDAX$, $RFTSE$, $RNKEI$, $RUSD$, $RJPY$ and $RGBP$ are the centered absolute returns of the CAC 40, the Dow Jones, the DAX 30, the FTSE, the Nikkei 225, the American dollar relative to the Euro, the Japanese yen relative to the Euro and the British pound relative to the Euro respectively obtained by using Equation (22). $\hat{d}_{GPH}$ is the GPH estimator of the Geweke and Porter-Hudac (1983), $\hat{d}_{ELW}$ is the Exact Local Whittle estimator of Shimotsu and Phillips (2005), $\hat{d}_{LSE_AV}$ is the estimator of Abry and Veitch (1998), $\hat{d}_{LSE_J}$ is the estimator of Jensen (1999b) and $\hat{d}_{ILSE}$ is the mean of instantaneous least squares estimator given by $\hat{d}_{ILSE} = \frac{\sum_{t=1}^{T} \hat{d}_{ILSE,t}}{T}$, where $\hat{d}_{ILSE,t}$ is the instantaneous least squares estimator and T is the number of observations.

We see that, for all volatilities and using all estimation methods, there is strong empirical evidence of long memory behavior in the whole period since the fractional integration parameter $\hat{d}$ is positive and less than 0.5. In particular, we seen that $\hat{d}$ are higher for stock market volatilities indicating a higher degree of persistence to shocks in stock markets.

One limitation of these estimators is that they assume that the fractional integration parameter is constant and that the effet of shocks is persist over time. However, in practice the fractional integration parameter and the persistence of shocks may vary over time. For that, we apply the instantaneous least squares estimator described in Section 3.2 to estimate the fractional integration

parameter d_t at each time. The obtained time-varying fractional integration parameters of each volatility are plotted in Figure 2.

Insert Figure 2

We see clearly that $\hat{d}_t$ changes substantially over time. This result is in line with those obtained by Jensen and Whitcher (2000) and Cajueiro and Tabak (2008) who found empirical evidence of time-varying fractional integration parameter. In particular, this parameter decreases in values where intervals of increased variablility at a variety of large scales. Moreover, the large values of $\hat{d}_t$ greater than one correspond to scheduled economic information annoucements. We see negative values of $\hat{d}_t$ corresponding to prescheduled new annoucements and unexpected market crashes or political upheavals (Jensen and Whitcher (2000)).

Now, we propose to model the time-varying fractional integration parameter by STR model. To this end, we apply the LM test described in Section 4.1 to test for constancy of $\hat{d}_t$ against STAR specification. As candidate transition variables, we consider the lagged fractional integration parameter and lagged volatilities $\left(s_t = \left\{\hat{d}_{t-i},\ X_{t-i},\ i = 1, \ldots, 8\right\}\right)$. The null hypothesis of constancy of $\hat{d}_t$ is strongly rejected with $\hat{d}_{t-1}$, i.e., $\hat{d}_{t-1}$ has the smallest p-value, against STR specification (see Table 3)[5].

Nevertheless, it should be stressed here that the statistical inference based on the asymptotic approximation is not applicable given that the asymptotic distribution under the null hypothesis depends on econometric regression of explanatory variables that were previously estimated from an auxiliary equation in the first step. To remain this issue, we rely on an alternative method based on bootstrapping in order to correct the distortions of the significance levels[6]. The obtained results with $\hat{d}_{t-1}$ are reported in Table 3.

Table 3: p-values of constancy test of fractional integration parameter

	$RCAC$	RDJ	$RDAX$	$RFTSE$	$RNKEI$	$RUSD$	$RJPY$	$RGBP$
F_1	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***

Note: F_1 is the statistic of Fisher test where the null hypothesis is given by $H_0^{''}$. *** indicates the rejection of the null hypothesis at the 1% significance level.

Now, we choose the appropriate form of the transition function (logistic (Equation (7)) or exponentiel (Equation (8)). To do so, we apply the nested tests described in Section 4.2. The obtained results are reported in Table 4.

[5] In reality, we conducted the test with several transition variables including X_{t-i} and d_{t-i}, $i = 1, \ldots, 8$. We obtain the lowst p-value with d_{t-1}, so we retain this variable as transition variable. In Table 3, we do not report the obtained results for the other variables. They are available upon request.

[6] For more details about the the econometric problems posed by the use of constructed variables in an econometric regression, see Pagan (1984).

Table 4: p-values of nested tests

	$RCAC$	RDJ	$RDAX$	$RFTSE$	$RNKEI$	$RUSD$	$RJPY$	$RGBP$
F_2	0.151	0.296	0.657	0.383	0.166	0.769	0.602	0.506
F_3	0.123	0.154	0.047	0.014*	0.008***	0.0265**	0.544	0.256
F_4	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***

Note: F_2, F_3 and F_4 are the statistics of Fisher test where the null hypothesis are given by H_{02}, H_{03} and H_{04} respectively. ***, ** and * indicate the rejection of the null hypothesis at the 1%, 5% and 10% significance level respectively.

We see that d_t varies according to LSTAR model. Thus, we estimate an LSTAR model for the fractional integration parameter using nonlinear least squares (NLS) method. The obtained results are reported in Table 5.

Table 5: Estimation results of LSTAR model for time-varying fractional integration parameter

	$RCAC$	RDJ	$RDAX$	$RFTSE$	$RNKEI$	$RUSD$	$RGBP$	$RJPY$
$\hat{d}_1$	-0.311**	-0.095*	-0.409***	-0.179***	-0.117**	-0.467***	-0.783***	-0.378***
$\hat{d}_2$	0.443***	0.623***	0.438***	0.418***	0.428***	0.428***	0.592***	0.413***
$\hat{\gamma}$	3.215	5.045	3.194	3.657	5.407	3.744	2.319	4.299
$\hat{c}$	0.146	0.204	0.085	0.210	0.141	-0.049	-0.172	-0.017

Note: ***, ** and * indicate significances at the 1%, 5% and 10% significance level respectively. The standard deviations used to calculate the statistics of the test of student are based on bootstrapping to correct the distortions of the significance levels.

The empirical results show significant evidence of two regimes for fractional integration parameter depending on the size of persitence to shocks. In the first regime (lower regime), which occurs when $d_{t-1} < \hat{c}$, we see that the estimated fractional integration parameters $\hat{d}_1$ are significant and negative. Moreover, for all the volatilities except $RGBP$, we see that $-0.5 < \hat{d}_1 < 0$ indicating that stock market and exchange rate volatilities are anti-persistent memory and characterized by a short memory process in response to both scheduled releases of economic news and unexpected political upheavals.

The second regime (higher regime) happens when $d_{t-1} > \hat{c}$. We observe that the estimated fractional integration parameters $\hat{d}_2$ are significant and positive. For RDJ and $RGBP$, we notice that $0.5 < \hat{d}_2 < 1$ suggesting that these volatilities are nonstationary but mean-reverting in the sense that the shocks do not have permanent effects[7]. For $RCAC$, $RDAX$, $RFTSE$, $RNKEI$, $RUSD$ and $RJPY$, we observe that $0 < \hat{d}_2 < 0.5$ indicating that these volatilities are stationary long memory and implying that the effects of shocks tend to persist.

In both regimes, we notice the fractional integration parameter of stock market volatilities are greater than the fractional integration parameter of exchange rate volatilities implying higher degree of persistence to shocks in stock markets. Furthermore, we find that the thresold parameter $\hat{c}$ is negative for the

[7] Indeed, the limiting value of the impulse response function.

exchange rate volatilities whereas $\hat{c}$ is positive for the stock market volatilities. In addition, the speed of transition γ is higher for the stock market volatilities. This means that the switching from the lower regime to upper regime is more rapidly in stock market volatilities than exchange rate volatilities.

In spite of obtained results, we can deduce that, first, the global long memory behaviour of volatility is the sum of short memory and long memory behaviour. Second, pockets of predictability exist when the peristence of volatility is high.

These findings are similar to those obtained by Dufrénot et al. (2005) and Boutahar et al. (2008) who found significant evidence of changing patterns in persistence of volatility of stock market indexes and exchange rates.

7 Conclusion

In this paper, we develope a wavelet based estimator of time-varying long memory (TV-ARFIMA) model where the fractional integration parameter evolves nonlinearly according to Logistic Smooth Transition AutoRegressive (LSTAR) model. In particular, the wavelet based estimation proposed is executed via an instantaneous least squares estimator.

The simulation results are encouraging and show the robustness of the method proposed. We provide an empirical application of this model to modeling the volatilities of stock market indexes and exchange rates as measured by their centered absolute returns. The empirical results show evidence of time-varying of fractional integration parameter and therefore a time-varying concerning the persistence of the shocks. More precisely, the fractional integration parameter follows two regimes. The first regime corresponds to low level of fractional integration parameter, there is no evidence of long range dependence for all volatilities. The second regime corresponds to high level of fractional integration parameter, there is significant evidence of long range dependence for some volatilities.

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