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Disentangling economic recessions and depressions*

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Abstract

We propose a novel nonparametric method to distinguish between recessions vs. “depressions” and expansions vs. “booms” in aggregate economic activity. Four depression and boom episodes are identified in the NBER business cycle between 1919 and 2009, including the Great Depression and the industrial boom during the Second World War. In contrast to conventional wisdom, our results suggest that the 2007-09 Great Recession does not qualify as an economic depression. Multinomial logistic regressions show that stock market returns, real output growth, and inflation rates exhibit predictive power towards extraordinary business cycle fluctuations. Surprisingly, whereas the slope of the yield curve is a leading indicator of recessions, this property disappears for economic depressions.

JEL classification: C14, C35, E32

Keywords: Business cycles, Depression, Multinomial logistic regression, Outlier

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1 Introduction

The business cycle is traditionally modeled as a sequence of recessions and expansions in aggregate economic activity. This binary approach characterizes the regime switching models of, *inter alia*, Hamilton (1989) and Potter (1995), as well as the turning-points methodology of Bry and Boschan (1971), King and Plosser (1994) and Harding and Pagan (2002). Moreover, the prediction of business cycles has typically also been approached via binary models, see, e.g., Estrella and Mishkin (1998) and Kauppi and Saikkonen (2008). However, some recessions and expansions have been much stronger than others – e.g., in terms of duration or output contraction – throughout the history of the U.S. business cycle. This notion is also reflected by a semantic difference between recessions vs. “depressions” and expansions vs. “booms” often made by policymakers and the popular press. One possibility is that such extreme episodes are solely tail realizations from the same data generating process (DGP) as ordinary recessions or expansions, which would imply that a two-regime approach delivers correctly specified business cycle models. Alternatively, depressions and booms may be considered as “outliers” arising from another DGP than ordinary business cycle phases.¹ Neglecting this multi-phase character of the business cycle may distort model specifications of the true DGP.

The objective of this paper is therefore to investigate the possibility that the DGP of the business cycle is characterized by more than two regimes. We propose a novel non-parametric outlier detection framework to distinguish depression and boom episodes from ordinary recessions and expansions based on the occurrence of outliers in some popular business cycle characteristics. The considered characteristics include the duration, amplitude, cumulated movements, and excess cumulated movements of each phase suggested by Harding and Pagan (2002). We classify a phase as depression or boom if at least one of its characteristics exhibits an outlier for that particular time frame relative to other historical episodes. Our outlier detection algorithm results in a four-regime classification of the U.S. business cycle. This enables us to refine some of the conclusions drawn from previous recession prediction exercises conducted within the two-regime framework.

¹A multitude of outlier definitions co-exist. We adopt the definition that outliers are observations that do not arise from the same DGP as the majority of observations in the data set, conform to Barnett and Lewis (1994).

The amount of U.S. economic recessions and expansions over the past century is relatively small and the statistical distribution of business cycle characteristics is unknown. We therefore propose a distribution-free outlier detection test that performs well for small sample sizes. Our method involves bootstrapping the phase characteristics and computing the difference between the arithmetic mean and the trimmed mean of each bootstrap sample, which yields a measure of central tendency termed “mean-trimmed mean”. Singh and Xie (2003) show that the mean-trimmed mean displays a multimodal histogram if the original sample contains one or more outliers. We employ the bootstrap-based test of Silverman (1981) to assess the modality of the histogram. If the latter is multimodal, we sequentially shrink the sample by removing the most extreme observations until we end up with a unimodal histogram that is free of outliers. The omitted observations correspond to the outlying business cycle characteristics. Applying this statistical procedure, we identify four extraordinary phases of the U.S. business cycle over the time horizon 1919 to 2009, that coincide with economically meaningful episodes, like the Great Depression and the industrial production boom during the Second World War. However, despite all comparisons between the 1930s’ Great Depression and the 2007-09 economic contraction, our results suggest that the Great Recession does not qualify as depression in comparison with previous historical episodes.

There is a growing interest on severe economic contractions in the macroeconomic literature (see, e.g., Kehoe and Prescott, 2002). Large and infrequent economic slumps may command a bolder set of policy prescriptions than ordinary recessions, as argued by, e.g., Eggertsson and Krugman (2012). At the same time, rare “economic disasters”, such as depressions or wars, have been recently shown to play an important role in determining asset risk premia, see e.g., Barro (2006, 2009), Gabaix (2012), and Wachter (2013). A handful of papers have tried to incorporate such severe episodes into non-linear regime-switching time series models with more than two possible business cycle states, including Tiao and Tsay (1994), Sichel (1994), and Cakmakli et al. (2013). Meanwhile, substantial effort has been devoted to testing for the actual number of regimes within Markov-switching models, see, e.g., Cho and White (2007), Carter and Steigerwald (2012), and Carrasco et al. (2013). To the best of our knowledge, this is the first paper to consider the identification of rare and severe recessions by means of nonparametric outlier detection

techniques. Previous studies that deal with outliers in macroeconomic time series include, e.g., Balke and Fomby (1994) and Giordani et al. (2007), but these are parametric in nature.

From the preceding literature it is well-known that financial and macroeconomic variables, such as stock market returns, real output growth, and – especially – the slope of the yield curve (known as the term spread) exhibit some predictive power for future recessions and expansions across certain time horizons (see, e.g., Harvey, 1988; Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1998; Birchenhall et al., 1999; Hamilton and Kim, 2002; Kauppi and Saikkonen, 2008; Rudebusch and Williams, 2009; Christiansen, 2013).² The question arises to what extent these leading indicator properties carry over to the four-regime business cycle classification that results from applying our outlier detection test.³ The predictive ability of financial and macroeconomic variables is of potential importance for policymakers who would like to assess whether a given change in a variable signals an impending recession or a rare economic disaster, like a depression. Thus, one would like to determine whether the traditional leading indicators of recessions and expansions exhibit a different information content towards depressions and booms. In fact, upon applying a multinomial logistic regression model to the four-regime business cycle, we find that the term spread preserves its predictive power towards recessions but its leading property vanishes for economic depressions. However, we are able to show that variables like past real output growth, inflation, and stock market returns help to predict extraordinary business cycle fluctuations.

The remainder of the paper is organized as follows. Section 2 starts with a definition of business cycles and their characteristics. Subsequently, we introduce the nonparametric outlier test. Finally, we report estimated cycle characteristics across the NBER business cycle between 1919 and 2009, and we present the results from the outlier detection procedure. Section 3 provides a short theoretical digression on multinomial logit regressions. Next, the section compares empirical results of binomial with multinomial logit specifications and univariate with multivariate logit specifications, respectively. Finally, Section

²An excellent summary on the yield curve as a leading indicator is offered on the New York Fed Internet site: http://www.newyorkfed.org/research/capital_markets/ycfaq.html.

³Importantly, throughout the paper “prediction” refers to *in-sample* predictive ability, since out-of-sample forecasts of depressions and booms would not make sense given the scant amount of genuine depression and boom phases within the historical sample.

4 provides a summary and conclusions.

2 The four-regime business cycle

2.1 Measuring business cycles

Classical studies define business cycle fluctuations by means of turning points (peaks and troughs) in the level of real economic activity (see, e.g., Burns and Mitchell, 1946; Bry and Boschan, 1971; King and Plosser, 1994; Harding and Pagan, 2002). A complete cycle in (the log of) real output y_t consists of a recession phase from a peak to the subsequent trough and an expansion phase from a trough to the subsequent peak. The National Bureau of Economic Research (NBER) publishes turning point dates which have become a consensus chronology of the U.S. business cycle. We use the NBER peaks and troughs in economic activity to construct a binary variable S_t^{bin} that either reflects a recession phase ($S_t^{bin} = 1$) or an expansion phase ($S_t^{bin} = 0$).

To assess the relative strength of recessions and expansions, we focus on the tail behavior of the unconditional frequency distribution of four commonly used business cycle characteristics: the duration (D), amplitude (A), cumulative movements (C) and excess cumulative movements (E) computed for each business cycle phase (formal definitions are presented in the appendix). A number of studies have calculated these measures to characterize the cyclical behavior of real economic activity (see, e.g., Harding and Pagan, 2002; Camacho et al., 2008). Moreover, Bordo and Haubrich (2010) have also employed these characteristics to analyze cycles in money, credit, and output, while others have used the same measures to investigate financial cycles (see, e.g., Pagan and Sossonov, 2003; Claessens et al., 2012).

2.2 Testing for depressions and booms

In order to distinguish depressions and booms from ordinary business cycle phases, we apply an outlier testing procedure to the previously described business cycle characteristics. A given recession (expansion) episode is classified as an economic depression (boom) if at least one of its four characteristics is an outlier for that period. More precisely, a

depression (boom) is identified as follows. Provided that n recessions (expansions) are observed, we have a sample for each recession (expansion) characteristic denoted by $X_1, \dots, X_n$ (X_i stands for either D_i , A_i , C_i , or E_i). The observation X_i that corresponds to phase $i = 1, \dots, n$ can be seen as a random draw from an unknown cumulative distribution function $F_X(\cdot)$. Phase i is a depression (boom) if X_i is an outlier with respect to the distribution $F_X(\cdot)$, i.e., X_i is *not* distributed according to $F_X(\cdot)$.

We propose a distribution-free test for the null hypothesis that the sample $X_1, \dots, X_n$ is free of outliers, which combines two established results from nonparametric statistics. First, Singh and Xie (2003) introduce a “Bootlier Plot” to graphically detect the presence of outliers in a data set. The Bootlier Plot is the bootstrap density plot of the sample mean-trimmed mean statistics and its multimodality reflects the presence of outliers in the sample. Second, Silverman (1981) proposes a distribution-free test for the unimodality of a probability density function. Hence, we apply Silverman’s test to Bootlier Plots of phase characteristics in order to detect outliers in these characteristics. If the null hypothesis of unimodality is rejected for the full sample, we proceed with ordering the observations $X_1, \dots, X_n$ into ascending order, sequentially dropping observations from the tails of the ordered sample, and applying Silverman’s test on Bootlier Plots of these shrinking subsamples. We proceed with the iterative omission of the most extreme observations until the (subsample) Bootlier Plot becomes unimodal. The deleted observations can be identified as outliers.

Let us now discuss this nonparametric outlier detection procedure in somewhat more detail. The Bootlier Plot is obtained as follows. Let $X_1^b, \dots, X_n^b$ ($b = 1, 2, \dots, B$) denote bootstrapped samples of size n based on the original sample $X_1, \dots, X_n$ of a particular business cycle characteristic. The resampling is repeated $B = 10,000$ times. The mean-trimmed mean (MTM) statistic is defined as the difference between the arithmetic mean and the k -trimmed mean of the b th bootstrap sample:

$$MTM^b = \frac{1}{n} \sum_{i=1}^n X_i^b - \frac{1}{n-2k} \sum_{i=k+1}^{n-k} X_{(i)}^b, \quad (1)$$

where $X_{(i)}^b$ are the ascending order statistics and k is a trimming value. Upon assuming that $F_X(\cdot)$ exhibits finite first and second moments, the central limit theorem applies

and the pdf of the mean-trimmed mean $f_{MTM}(\cdot)$ converges asymptotically to a standard normal distribution in the absence of outliers. Singh and Xie (2003) show that if the original sample $X_1, \dots, X_n$ contains at least one outlier, $f_{MTM}(\cdot)$ can be expressed as a multimodal mixture of normal densities. The separation between the normal mixing components arises because only some of the bootstrap samples contain the outliers. Consequently, $f_{MTM}(\cdot)$ exhibits one mode associated with the distribution $F_X(\cdot)$ and at least another mode corresponding to one or more outliers.

Let us consider for example the durations of NBER business cycle recessions. Figure 1 reports the Bootlier Plots for recession durations for the full sample and for the subsample excluding the duration of the Great Depression phase (40 months). The multimodal plot implies that the Great Depression's duration is an outlier of the empirical distribution, which suggests that the 1929-33 downturn was a genuine depression episode. Upon removing this data point from the historical sample of durations, the subsample Bootlier Plot becomes unimodal, which indicates that there are no other outliers left in the empirical distribution of recession durations.

[Insert Figure 1 here]

In order to formally evaluate the hypothesis of unimodality of $f_{MTM}(\cdot)$ (absence of outliers in $X_1, \dots, X_n$) against the alternative hypothesis of multimodality (presence of one or more outliers), we apply the test proposed by Silverman (1981). The test requires calculating the kernel density estimate of the density function $f_{MTM}(\cdot)$. For the MTM statistics the kernel density estimate at any point x can be expressed as:

$$\hat{f}(x, h) = \frac{1}{Bh} \sum_{b=1}^B K\left(\frac{x - MTM^b}{h}\right), \quad (2)$$

where h is a bandwidth and $K(\cdot)$ is a kernel function chosen to be the standard normal density function. For a large class of kernel functions including the standard normal, the number of modes of $\hat{f}(x, h)$ decreases as the bandwidth h is increased. Thus, a sufficiently large h must exist, for which the kernel density $\hat{f}(x, h)$ has a single mode in the interior of the given closed interval $\mathfrak{I}$. The narrowest bandwidth for which the kernel density estimate is unimodal is sometimes called the “critical bandwidth”. Intuitively,

the critical bandwidth of a multimodal density is larger than that of a unimodal density, because a larger bandwidth is required to smooth out the multiple modes. Based on this idea, Silverman (1981) develops a nonparametric test for the unimodality of a pdf using the critical bandwidth as test statistic.

To implement Silverman’s test, we start with estimating the density of the bootstrap MTM statistics $MTM^1, \dots, MTM^B$ using the kernel density estimator in Equation (2). Next, we estimate the critical bandwidth, i.e., the narrowest bandwidth for which the kernel density is unimodal. Let us denote the critical bandwidth as $\hat{h}_{crit}$ and the kernel density estimated with this bandwidth as $\hat{f}(\cdot, \hat{h}_{crit})$. In order to determine the small sample distribution of the critical bandwidth $\hat{h}_{crit}$, one must draw a large number of bootstrap samples from the density $\hat{f}(\cdot, \hat{h}_{crit})$.⁴ For each draw, the density of the bootstrapped MTM statistics is again estimated using the kernel density estimator defined in Equation (2). Let $\hat{h}_{crit}^*$ denote the critical bandwidth of the kernel density obtained for one bootstrap draw. The null hypothesis of unimodality (no outliers in $X_1, \dots, X_n$) is rejected if $Pr(\hat{h}_{crit}^* \leq \hat{h}_{crit}) \geq 1 - \alpha$, where α is the nominal size. If unimodality of the full-sample Bootlier Plot is rejected, we calculate Bootlier Plots and perform Silverman’s test on bootstrapped subsamples by eliminating the most extreme observations from the tails of the full sample. We continue shrinking the subsample until the null of unimodality can no longer be rejected.⁵

Further details on the nonparametric outlier detection procedure, including its finite sample behavior, are reported in a companion paper (see Candelson and Metiu, 2013). The latter paper, *inter alia*, shows that the size and power properties of the outlier test are satisfactory in small samples like the ones encountered in this paper.

2.3 Depressions and booms in the U.S. business cycle

We compute the duration, amplitude, cumulated movements, and excess cumulated movements of the U.S. business cycle using the NBER turning points. Figure 2 plots the log

⁴We implement Silverman’s test with 1,000 bootstrap draws.

⁵By definition, outliers must be located in the upper or lower tails of the ascending order statistics $X_{(1)}, X_{(2)}, \dots, X_{(n-1)}, X_{(n)}$. We sequentially cancel the most extreme observations by considering the subsamples: $(X_{(1)}, \dots, X_{(n-1)})$, $(X_{(2)}, \dots, X_{(n)})$, $(X_{(1)}, \dots, X_{(n-2)})$, $(X_{(2)}, \dots, X_{(n-1)})$, $(X_{(3)}, \dots, X_{(n)})$, $(X_{(1)}, \dots, X_{(n-3)})$, etc. We stop this process once unimodality can no longer be rejected.

of monthly U.S. real industrial production output between March 1919 and December 2010 (base year 2007) with NBER recessions shaded in grey. Table 1 reports values of the characteristics computed for each recession and expansion phase. While on average recessions last about 10 months, the longest recession lasted 40 months from the peak in August 1929 till the trough in March 1933. The severity of recessions also varies significantly across different episodes: the average peak-to-trough amplitude of a recession is -17.6%, while the difference between the amplitude of the most severe (August 1929 - March 1933) and the mildest (March 2001 - November 2001) recession is 68.6 percentage points. Similar patterns emerge for expansions: the difference between the shortest and longest expansion is roughly 9 years, while the trough-to-peak output gains vary between 5.9% and 99.5% over the analyzed period.⁶ The varying strength of contractions and expansions suggests that one may gain more insight into the nature of the business cycle by further refining its traditional two-regime classification.

[Insert Table 1 here]

[Insert Figure 2 here]

Figure 3 shows the kernel density estimates of the Bootlier Plots corresponding to each of the four business cycle characteristics.⁷ Recall that multimodality of the Bootlier Plot indicates the presence of one or more outlying observations. The Bootlier Plots of the excess measure of expansions and of the duration, amplitude and cumulated movements of recessions exhibit more than one mode, and outlier detection tests performed on the basis of these plots lead to p-values smaller than 1%, indicating a rejection of the null hypothesis of unimodality (no outliers). We iteratively run outlier detection tests to determine the subsamples that are free of outliers. Business cycle phases with outlying characteristics are shaded in grey in Table 1. We detect six outliers via our iterative procedure, one in the excess cumulated movements of expansions, one in recession durations, one in recession amplitudes, and three in the cumulated movements of recessions. Table 1 reports the

⁶Earlier evidence for the asymmetries of U.S. business cycles has been provided by, e.g., Neftci (1984), DeLong and Summers (1986), McQueen and Thorley (1993), McKay and Reis (2008), and Morley and Piger (2012).

⁷We compute the MTM statistics with a two-sided trimming value of $k = 2$ recommended by Singh and Xie (2003), who show that in the presence of an outlier the separation between the modes of the bootstrap density $f_{MTM}(\cdot)$ is approximately proportional to $1/k$ independent of the sample size.

p-values of the outlier detection test for the subsamples with unimodal Bootlier Plots. As Figure 3 shows, each Bootlier Plot becomes unimodal once the outliers are removed from the samples.

[Insert Figure 3 here]

The outlying phase characteristics correspond to four extraordinary episodes in the history of the U.S. business cycle. The expansion with an outlying excess measure is relabeled as a boom, while the three recessions with outlying characteristics are classified as depressions. The first depression occurred between January 1920 - July 1921. Romer (1988) argues that this deflationary downturn may have been triggered by an aggregate supply shock due to an upsurge in postwar production of agricultural goods and primary commodities which caused a drastic decline in wholesale prices. The second depression is the August 1929 - March 1933 collapse in real economic activity known as the Great Depression. Figure 4 (a) plots log industrial production from peak to trough. Our results reveal that the Great Depression corresponds to the longest, sharpest, and most abrupt decline in real output, with an amplitude of -72.1% and a cumulative loss of 1742.7% born over 40 months. These values are between three to four standard deviations larger than for ordinary recessions. The vast literature on the Great Depression typically describes an adverse interplay between a contraction in aggregate production, systemic banking crises, and tight monetary policy (see e.g., Bernanke, 1983). The third depression that we detect is dated between May 1937 and June 1938. Velde (2009) argues that the 1937-38 episode is a prime example of a double-dip recession emerging from premature policy tightening in the aftermath of a recession. Despite comparisons often made with the Great Depression, the recent Great Recession is not classified as a depression by our statistical procedure, although it is undoubtedly the longest (15 months) and most severe postwar recession, with the largest amplitude (-18.5%) and cumulative output loss (147.7%).⁸

[Insert Figure 4 here]

⁸Notice that the reason that the Great Recession is not classified as a depression is not because it is dwarfed by the three prewar depressions, as the outlier detection procedure does not directly compare the Great Recession with the identified depression episodes. Instead whether the Great Recession is a depression solely depends on its characteristics compared to other recessions, since the sequential omission of the depression characteristics in the testing algorithm is equivalent to ultimately considering a subsample that does no longer contain the three depression periods. Thus, if the Great Recession was a depression, it should lead to a multimodal Bootlier Plot for this latter subsample.

The June 1938 - February 1945 wartime expansion constitutes the only genuine boom phase. Figure 4 (b) shows log industrial production from trough to peak. The grey area in the plot corresponds to the measure of excess cumulative movements. This measure reflects the departure of the real output series from a triangular path for which the transition between two consecutive turning points would be linear, it thus conveys information about the shape of the business cycle phase.⁹ Judged by the excess characteristic, the WWII boom exhibits by far the largest excess: 11.7% compared to a mean of 2.5% and standard deviation of 3.06%. The concave shape of the upswing reflects a sharp surge in aggregate production related to the arms industry, the expansion of productive capacity through government-owned, privately operated capital, and an increase in labor force participation in durable goods manufacturing during World War II (see Braun and McGrattan, 1993). As the impetus of the economic boom abates by the end of the war, the economy approaches its peak at a rather subdued pace, giving rise to the strong concavity observed in the figure.

3 Predicting the four-regime business cycle

3.1 Multinomial logistic regression

We examine by means of multinomial logistic regressions whether macroeconomic and financial variables convey leading information on the four-regime U.S. business cycle. Based on the outlier detection results discussed in the preceding sections, we replace the binomial cycle variable with a multinomial variable S_t^{mul} that can take the values 0, 1, 2, and 3, corresponding to the expansion ($S_t^{mul} = 0$), recession ($S_t^{mul} = 1$), boom ($S_t^{mul} = 2$), and depression ($S_t^{mul} = 3$) regimes of the business cycle. Let I_t represent the information set which contains the past history of an L -dimensional vector of exogenous variables, $\mathbf{x}_t$. In a four-state multinomial logit model, the probability that the business cycle is in regime $j = 1, 2, 3$ at time $t + m$ conditional on I_t obeys a logistic distribution

⁹More precisely, the excess cumulated movements provides an approximation to the second derivative of the time series. If $E_i > 0$, the business cycle phase exhibits a concave shape, i.e., the slope of y_t changes abruptly at the beginning of the phase but the changes in slope become more subdued as the phase approaches its turning point. Conversely if $E_i < 0$, the shape is convex, and the slope changes more gradually at early stages of the phase (see Camacho et al., 2008).

function:

$$Pr(S_{t+m}^{mul} = j|I_t) = \frac{\exp(\beta'_j \mathbf{x}_t)}{1 + \sum_{h=1}^3 \exp(\beta'_h \mathbf{x}_t)}, \quad (3)$$

where $m = 3, 6, \dots, 24$ stands for the prediction horizon (expressed in months). The model is identified by imposing the condition that the expansion regime ($j = 0$) is the reference state and all coefficients are expressed relative to this regime. Hence, using the fact that the state probabilities must sum to unity, the conditional probability of the expansion regime is given by:

$$Pr(S_{t+m}^{mul} = 0|I_t) = \frac{1}{1 + \sum_{h=1}^3 \exp(\beta'_h \mathbf{x}_t)}. \quad (4)$$

The multinomial logit model nests the binomial model which has been used in much of the preceding literature (e.g., Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1998; Kauppi and Saikkonen, 2008), with $j = 1$ and binomial dependent variable ($S_t^{bin} = 0, 1$) corresponding to the NBER expansions and recessions.

Estimation is done by maximum likelihood optimization.¹⁰ The log-likelihood for a time series consisting of T observations is given by:

$$\log(L(\beta)) = \sum_{t=1}^T \sum_{j=0}^3 \mathbf{1}(S_{t+m}^{mul} = j) \log(Pr(S_{t+m}^{mul} = j|I_t)), \quad (5)$$

where $\mathbf{1}(\cdot)$ is the indicator function. The model's goodness-of-fit can be measured using McFadden (1974)'s pseudo- R^2 , which is given by:

$$\text{Pseudo-}R^2 = 1 - \left(\frac{\log(L_U)}{\log(L_R)} \right), \quad (6)$$

where $\log(L_U)$ is the log-likelihood of the unrestricted (full) model, while $\log(L_R)$ is the log-likelihood of the restricted (intercept only) model. For time horizons of $m > 1$, the prediction horizon exceeds the data frequency which creates serially correlated

¹⁰In the business cycle literature probit models are more commonly employed than logit models to predict recessions (e.g., Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1998; Kauppi and Saikkonen, 2008; Christiansen, 2013). However, this approach is less preferable in the multinomial specification because evaluating the state probabilities involves calculating high-dimensional integrals of the multivariate normal distribution, which may make the computation of the maximum likelihood estimates infeasible (see McCulloch and Rossi, 1994). Therefore, we opt for the logistic approach. Nevertheless, binary probit and logit outcomes for our data were found to lie very close to each other, which suggests that it does not matter whether one assumes a normal density or a logistic density for sake of maximum likelihood optimization.

logit disturbance terms by construction, i.e., the overlapping data problem. Just as in linear regressions models, Newey and West (1987) standard errors that are robust to heteroskedasticity and serial correlation can remedy for this problem (see Estrella and Mishkin, 1998).

We are interested in the effect of a *ceteris paribus* change in one of the right-hand-side (RHS) variables on the response probability defined in Equation (3), i.e., the so-called marginal effects.¹¹ The l th marginal effect gives the change in the probability that the business cycle is in state j at time $t + m$ in response to a one unit increase of the l th RHS variable from its mean value ($l = 1, \dots, L$):

$$\frac{\partial Pr(S_{t+m}^{mul} = j | I_t)}{\partial x_{l,t}} = Pr(S_{t+m}^{mul} = j | I_t) \left(\beta_{j,l} - \frac{\sum_{h=1}^3 \beta_{h,l} \exp(\beta'_h \mathbf{x}_t)}{1 + \sum_{h=1}^3 \exp(\beta'_h \mathbf{x}_t)} \right), \quad (7)$$

where $\beta_{j,l}$ is the l th element of β_h .

Within this multinomial framework, we would like to assess to what extent macroeconomic and financial variables that traditionally exhibit leading indicator properties for recessions and expansions have a different information content for depressions and booms. Anderson (1984) defines a pair of regimes as “indistinguishable” if the RHS variables $\mathbf{x}_t$ deliver the same prediction for both regimes. If so, the two regimes can be merged into a single regime. More precisely, if $\beta_{1,l}$ are the coefficients that correspond to the explanatory variables $x_{l,t}$ in the recession regime ($S_t^{mul} = 1$) relative to the reference (expansion) state ($S_t^{mul} = 0$), and $\beta_{3,l}$ are the coefficients of the depression regime ($S_t^{mul} = 3$) relative to the reference state ($S_t^{mul} = 0$), then the null hypothesis that recessions and depressions are indistinguishable with respect to $\mathbf{x}_t$ is $\beta_{1,l} - \beta_{3,l} = 0$ for $l = 1, \dots, L$, which can be tested using conventional likelihood-ratio (LR) and Wald statistics.

¹¹The majority of related business cycle studies only report coefficient levels β in Equation (3) because the sign of the coefficients and the marginal effects are identical in the binary framework. However, in the multinomial model, an explanatory variable’s marginal effect depends on all coefficients which implies that it does not necessarily exhibit the same sign as the coefficient on the corresponding independent variable.

3.2 Logistic regression results

Potential leading indicator variables are selected in line with previous empirical literature on business cycle prediction (see, e.g., the seminal paper by Estrella and Mishkin, 1998). Taking into account that our RHS variables need to be available from 1919 onwards, we end up with the term spread ($SPREAD$), the S&P 500 stock index returns ($\Delta \log SP500$), the inflation rate ($\Delta \log PPI$), and the growth rate of log industrial production (Δy_t), all at the monthly frequency.¹² Data are taken from the St. Louis Fed and from Goyal and Welch (2008). In line with the business cycle data, the sample covers the period from March 1919 until June 2009. Figure 5 shows the data together with the NBER recessions. The figure already provides some casual evidence that the business cycle and the macroeconomic and financial variables are related. The term spread seems to display counter-cyclical dynamics. Although stock returns, inflation, and output growth do not exhibit a clear-cut pattern around recessions, they tend to decline during the three depressions identified.

[Insert Figure 5 here]

We distinguish between binomial and multinomial models with a single RHS variable (univariate model) or multiple RHS variables (multivariate model). Tables 2-8 report estimation results for these four possible logit model specifications. The tables have a comparable structure and information content. Each table considers prediction time horizons of $m = 3$ up to $m = 24$ months and reports marginal effects together with robust standard errors and accompanying Z -statistics. Since all regressors are expressed in percentages, the marginal effects can be interpreted as changes in the m -month ahead probability of a recession, depression, expansion or boom, in response to a one-percent rise of a RHS variable. The tables also report log-likelihoods and pseudo R-squared values.¹³

¹²The term spread is given by the difference between 10-year U.S. Treasury bonds and 3-month Treasury bills. Moreover, our inflation variable is the growth rate of the producer price index (PPI), which is a potential leading indicator for real economic activity because it measures the change in selling prices received by domestic producers. We also ran a robustness check with CPI inflation and our results were nearly identical.

¹³The considered models are estimated over the entire sample period. We do not perform out-of-sample prediction exercises for the extraordinary phases of the business cycle because the number of in-sample depression and booms (four in total) is too low to make a credible out-of-sample prediction assessment.

Table 2 reports estimation results for univariate binomial logistic regressions as a benchmark for comparison with the multinomial results and a robustness check of earlier binomial business cycle studies. In line with previous evidence, we find that the slope of the yield curve is an accurate predictor of future recessions and this for all considered time horizons. Rising term spreads reduce future recession likelihoods in a statistically significant way. Different theoretical explanations have been launched for this empirical observation. The most popular ones are related to the stance of monetary policy and the information content on expectations regarding future economic prospects reflected in the term spread. Current monetary policy can have a simultaneous impact on both the yield curve and future real activity. For example, an expansionary monetary policy can jointly induce a decline in the short rate – leading to a steeper yield curve – and stimulate future economic activity. The Rational Expectations Hypothesis (REH) of the term structure of interest rates can also contribute to understanding the empirical relation between the yield curve and the business cycle. According to the REH, an upward (downward) sloping yield curve indicates that future short-term interest rates are expected to rise (fall). Hence, given that short-term interest rates are typically pro-cyclical, a positive (negative) term spread signals a future business cycle expansion (recession).

We also consider S&P 500 stock index returns in the univariate logit model. The Efficient Markets Hypothesis implies that current stock prices equal the present value of the expected future dividend stream, which in turn reflects expectations about future real economic activity. Thus, in line with what one would expect, rising stock index returns significantly reduce recession likelihoods for time horizons up to one year ahead.

Turning to the significance of the macroeconomic variables in the univariate logit regressions, rising real output growth significantly reduces the probability of future recessions for time horizons up to three quarters ahead. The significant outcomes reflect the temporal persistence of economic activity. However, also notice that the sign of the coefficient reverses for a time horizon of two years ahead, which is suggestive of a boom-bust cycle in the data. Finally, the predictive horizon of inflation tends to be relatively short (up to six months). Interestingly, a marginal rise in inflation leads to a significant drop in the recession probability up to two quarters in the future. This result is in line with,

e.g., Estrella and Hardouvelis (1991) but it contradicts certain structural macroeconomic studies, e.g., Smets and Wouters (2007) who find a negative correlation between current inflation and future output growth.

Summarizing the univariate binomial model outcomes, both financial and macro variables impact the recession likelihoods, but the term spread is by far the best (in-sample) predictor of future real economic activity. First of all, the term spread is the only variable that exhibits significant predictive power over all time horizons. Moreover, the absolute values of the marginal effects in the term-spread regressions and the pseudo- R^2 exceed the other variables' marginal effects and pseudo- R^2 values for the majority of considered time horizons. Finally, the term spread is the only leading indicator of which (the absolute value of) the marginal effects and the pseudo- R^2 exhibit an inverted U-shape (reaching a maximum value for the annual time horizon); all other marginal effects and pseudo- R^2 's monotonically decline with the time horizon. Hence, the slope of the yield curve seems to have an "optimal" predictive horizon for recessions that lies between three and five quarters.

[Insert Table 3 here]

We also run the multivariate version of the binomial logit regression as a robustness check, see Table 3. The outcomes of the univariate regressions are generally confirmed. The term spread continues to have predictive power for future recessions over the whole forecast horizon and its marginal effects are significant up to two years into the future. Similarly, stock market returns and real output growth preserve their predictive power for horizons of three quarters and one year, respectively. The marginal inflation effect is no longer significant for the 6-months horizon. The (univariate and multivariate) binomial logistic regression results largely confirm the preceding literature on binary business cycle prediction (e.g., Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1998; Hamilton and Kim, 2002; Kauppi and Saikkonen, 2008).

Turning to the multinomial logistic regressions, univariate results are reported in Tables 4-7 for the term spread, S&P 500 returns, the inflation rate, and real output growth, while multivariate results are summarized in Table 8. The marginal effects across business cycle phases are reported which add up to 0 for identification purposes. We also report

testing outcomes (LR and Wald) for the null hypothesis that the RHS variables exhibit coefficients that are equal across a regime pair (recession vs. depression or expansion vs. boom). If the null hypothesis cannot be rejected, the two regimes are indistinguishable and can be combined into one regime, otherwise the RHS variables provide different predictions across the two regimes (see Anderson, 1984).

[Insert Table 4 here]

[Insert Table 5 here]

[Insert Table 6 here]

[Insert Table 7 here]

The marginal effects (in absolute value) of ordinary recessions and expansions are somewhat smaller in the multinomial regressions compared to the binomial results but remain statistically significant for the same variables and time horizons. Hence, the binary results are robust to adding regimes to the business cycle. Strikingly, however, the term spread fails to predict depressions despite the fact that it preserves predictive power for recessions and expansions. At the same time, pseudo- R^2 and marginal recession effects are largest for the term spread across different time horizons. Moreover, the term spread is the only early warning indicator whose marginal effects and pseudo- R^2 exhibit an inverted U-shape (reaching a maximum value around the annual time horizon); whereas marginal effects and pseudo- R^2 's for other macro and financial variables monotonically decline with the time horizon, just as in the binomial case.

In contrast to the term spread, the depression probability significantly diminishes when output growth, S&P 500 returns or inflation rise. Specifically, the univariate outcomes show that output growth and stock returns exhibit predictive power for depressions up to the 1-year time horizon. Remarkably, inflation seems to be the most important leading indicator for economic depressions in terms of marginal effects and across time horizons (up to 24 months). Moreover, the inflation rate has a significantly different information content for recessions and depressions for the entire forecast horizon of two years (see the outcomes of the LR and Wald tests). A *ceteris paribus* one percent rise in inflation leads to a significant increase in the recession probability between 9 and 24 months ahead, while it leads to a significant drop in the depression probability at vir-

tually every forecast horizon. The apparently different impact of a rise in inflation on the occurrence of recession vs. depression regimes is somewhat puzzling but the existing literature gives some hints to what may be behind this phenomenon. Applying a New Keynesian model on postwar macroeconomic data, Smets and Wouters (2007) show that inflation is primarily driven by price and wage mark-up shocks inducing a negative correlation between current inflation and future output. Recessions are predominantly situated after the Second World War, and thus the (multinomial) inflation impact on recession likelihoods is in accordance with the Smets and Wouters findings. At the same time, all three (prewar) depressions were deflationary episodes, see e.g. Figure 5 (c). This may explain the significantly negative effect of a rise in inflation on the depression probability. Thus, the inflation results illustrate that the impact of a financial or macro variable can be regime-dependent.

As a robustness exercise, we report estimates of a multinomial logit regression that includes all four explanatory variables in Table 8. The variables' marginal effects preserve the same signs and degrees of statistical significance. Moreover, LR and Wald tests that compare coefficients across regimes lead to the same conclusions as the test results obtained from univariate logit regressions.

[Insert Table 8 here]

Figure 6 complements the regression results with graphs of the predicted probabilities of a recession and depression 3-months in the future for the univariate multinomial logit models. These probability plots provide a qualitative complement to goodness-of-fit measures like the pseudo- R^2 .

[Insert Figure 6 here]

One clearly observes that, while the term spread has virtually no predictive power for depressions, the other three variables do a better job at predicting depressions, which is consistent with the earlier reported results. The plots also confirm that the leading indicators clearly differentiate between ordinary and extraordinary business cycle phases. The peaks in the predicted recession probability coincide with the recessions shaded in light grey, while the peaks of depression probabilities are associated with depressions shaded

in dark grey. The distinction between recessions and depressions further increases in the multivariate model, see Figure 7. The central message from the figure is that depression probabilities exceed recession probabilities during depressions: they are higher than 80% in all three cases. Thus, the multinomial model successfully disentangles economic recessions and depressions and considering a multivariate specification adds to that effect.¹⁴

[Insert Figure 7 here]

4 Conclusions

The business cycle is traditionally approached as a sequence of recessions and expansions in aggregate real economic activity. However, from a historical perspective it is well-known that some recessions have been much longer, deeper or more abrupt than others. Thus, the question arises whether the binary characterization of the business cycle is not overly simplistic, see also the semantic distinction often made by policymakers or the popular press between economic recessions vs. depressions and expansions vs. booms. Meanwhile, vast contractions of real output have stimulated interest in rare economic disasters among economists. The binary framework does not allow for considering such rare and severe events by construction.

Questioning the validity of the binary approach constitutes the starting point of this paper. We distinguish depression and boom episodes from ordinary recessions and expansions by means of a novel nonparametric framework. Using industrial production data and the peak and trough dates of the NBER business cycle between 1919 and 2009, we compute the duration, amplitude, cumulated movements, and excess cumulated movements of each recession and expansion phase. A nonparametric outlier detection test is applied to judge whether depressions and booms occur in the U.S. business cycle based on the magnitude of the above mentioned business cycle characteristics. The outlier test enables one to distinguish tail events in cycle characteristics arising from the same probability distribution from outliers generated by a different underlying process. The presence

¹⁴Interestingly, the indicators signal an impending depression with a probability of about 80% in March 2009, which suggests that the U.S. economy was evolving towards a depression regime at that time. However, the depression probability drops to approximately 10% by June 2009; hence, the economy avoided slipping into depression.

of outlying business cycle phases justifies the specification of an econometric model that is richer in dynamics than the binary recession/expansion approach. The re-mapping of the binary NBER business cycle into a four-regime cycle constitutes the first contribution of the paper. We identify four extraordinary episodes: three recessions are relabeled as economic depressions and one expansion is classified as a period of economic boom. Interestingly, the 2007-09 Great Recession is not classified as a depression by our outlier test procedure.

In the second part of the paper, we analyze by means of multinomial logistic regressions whether the four-regime business cycle can be predicted using macroeconomic and financial variables. The considered predictors include the term spread, stock index returns, real output growth, and the inflation rate. Logistic regressions are run for variables that are lagged between 3 months and 24 months relative to the business cycle variable. We further distinguish univariate models, where each variable is included separately, from multivariate models with all four variables. Our outcomes must be interpreted with due caution because the rare event character of depressions and booms enables only in-sample evaluation of the estimated models.

Several key results emerge from our logistic regression analysis. First, we find that recessions become less likely when the term spread, stock market returns, and real output growth increase, regardless of whether one considers binomial or multinomial business cycle phases. Stated otherwise, the predictive power that macroeconomic and financial variables exhibit towards recessions in a binary prediction framework carries over to the multinomial framework. Second, it is possible to determine on the basis of leading indicators whether the economy is headed towards a recession or a depression; significant information can thus be gained by refining the two-regime classification of the business cycle. Conform the preceding literature we find that the slope of the yield curve outperforms other leading indicators in predicting recessions. However, the term spread does not have anything to say about future economic depressions. In contrast, stock market returns, real output growth, and the inflation rate convey statistically relevant information for predicting economic depressions: a large drop in these variables signals a higher likelihood of a future economic depression.

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Appendix: Business cycle characteristics

Formally, we observe $i = 1, \dots, n$ business cycle phases over a given period $t = 1, \dots, T$ ($n < T$). First, the *duration* of the i th recession (expansion) measures the number of months between a peak (trough) occurring at t_1 and the next trough (peak) occurring at t_2 :

$$D_i = \sum_{t=t_1}^{t_2} Q_t,$$

where $Q_t = S_t^{bin}$ in case of a recession and $Q_t = 1 - S_t^{bin}$ in case of an expansion. Second, the *amplitude* of the i th recession (expansion) measures the change in real output y_t between a peak (trough) at t_1 to the next trough (peak) at t_2 :

$$A_i = y_{t_2} - y_{t_1}.$$

Third, the *cumulative movements* of y_t within the i th phase measure the overall cost (gain) of the recession (expansion):

$$C_i = \sum_{j=t_1+1}^{t_2} (y_j - y_{t_1}).$$

Finally, the *excess cumulative movements* measure the curvature of y_t within the phase defined as:

$$E_i = (C_i - 0.5A_i - 0.5A_iD_i)/D_i.$$

Figure 1: Bootlier Plots of NBER Recession Durations

Note: Bootlier Plots of durations for all NBER business cycle recessions (full sample) and for the subsample that excludes the duration of the Great Depression (40 months).

Figure 2: The U.S. Business Cycle

Note: Log levels of U.S. real industrial production, March 1919 - December 2010. NBER recessions are shaded in grey.

Figure 3: Bootlier Plots

Note: Kernel density estimates of the bootstrap density of MTM statistics (Bootlier Plots) of each business cycle characteristic for the full sample and for the subsample that excludes the outlying phases. First row: duration. Second row: amplitude. Third row: cumulated movements. Fourth row: excess cumulated movements.

Figure 4: Depressions and Booms of the U.S. Business Cycle

(a) The Great Depression, 1929M8-1933M3

(b) The WWII industry boom, 1938M6-1945M2

Note: Figure (a): NBER peak-to-trough depression between August 1929 and March 1933 ($E_i = -6.61 < 0$: convex recession). Figure (b): NBER trough-to-peak boom between June 1938 and February 1945 ($E_i = 11.68 > 0$: concave expansion). The excess cumulative movements (E_i statistic) are shaded in grey.

Figure 5: Time Series and the U.S. Business Cycle

(a) Term spread

(b) S&P 500 returns

(c) Inflation

(d) Real output growth

Note: Time series are expressed as monthly percentages between March 1919 and June 2009. NBER recessions are shaded in grey.

Figure 6: Predicted Probability of Recession and Depression (Univariate)

Note: Predicted probabilities estimated from the univariate multinomial logit model. First column: predicted probability of a recession (three months ahead); second column: predicted probability of a depression (three months ahead). 1st row: term spread; 2nd row: S&P 500 returns; 3rd row: inflation rate; 4th row: real output growth. Recessions are shaded in light grey and depressions are shaded in dark grey.

Figure 7: Predicted Probability of Recession and Depression (Multivariate)

Note: Predicted probabilities of a recession (Figure (a)) and depression (Figure (b)) occurring three months ahead. The probabilities are estimated from the multivariate multinomial logit model including all leading indicators ($SPREAD$, $\Delta \log SP500$, $\Delta \log PPI$, and Δy). Recessions are shaded in light grey and depressions are shaded in dark grey.

Table 1: U.S. Business Cycle Characteristics

Turning points		Duration (months)		Amplitude (%)		Cumulation (%)		Excess (%)	
Peak date	Trough date	Prev. trough to this peak	Peak to trough	Prev. trough to this peak	Peak to trough	Prev. trough to this peak	Peak to trough	Prev. trough to this peak	Peak to trough
	March 1919								
January 1920	July 1921	10	15	20.07	-37.95	97.90	-366.46	-1.25	-4.19
May 1923	July 1924	22	11	45.81	-18.92	509.09	-98.40	-0.81	1.38
October 1926	November 1927	27	10	23.21	-5.76	410.78	-21.65	3.18	1.00
August 1929	March 1933	21	40	20.87	-72.11	235.09	-1742.67	0.26	-6.61
May 1937	June 1938	50	10	72.25	-37.03	2023.92	-276.69	3.63	-7.30
February 1945	October 1945	80	5	99.52	-33.65	4964.97	-105.90	11.68	-0.99
November 1948	October 1949	37	8	18.38	-8.04	407.15	-46.65	1.56	-1.31
July 1953	May 1954	45	7	38.06	-8.85	1049.35	-62.08	3.86	-3.81
August 1957	April 1958	39	5	19.54	-12.76	519.24	-52.63	3.29	-2.87
April 1960	February 1961	24	7	19.61	-6.25	325.38	-33.23	3.34	-1.18
December 1969	November 1970	106	8	55.29	-4.15	3518.74	-12.86	5.29	0.73
November 1973	March 1975	36	13	21.05	-13.57	366.10	-61.14	-0.65	2.60
January 1980	July 1980	58	3	25.95	-6.75	943.87	-20.29	3.07	-2.26
July 1981	November 1982	12	13	5.91	-8.91	47.82	-77.69	0.78	-1.18
July 1990	March 1991	92	5	29.00	-4.08	1731.95	-15.17	4.17	-0.58
March 2001	November 2001	120	5	40.58	-3.47	2633.43	-14.91	1.48	-0.90
December 2007	June 2009	73	15	14.37	-18.52	545.13	-147.67	0.18	0.03
Outlier test p-value without outliers		0.50	0.22	0.30	0.29	0.18	0.19	0.07	0.24

Note: NBER turning point dates of the U.S. business cycle and business cycle characteristics (duration, amplitude, cumulated movements and excess cumulated movements). The outlier phase characteristics are shaded in grey. We report the p-values of Silverman's modality test for the subsamples without outliers.

Table 2: Univariate Binomial Logistic Regressions

	$Pr(S_{t+m} = 1 I_t) = F(\beta_0 + \beta_1 SPREAD_t)$							
	3 mths	6 mths	9 mths	12 mths	15 mths	18 mths	21 mths	24 mths
$\frac{\partial Pr(S_{t+m}=1)}{\partial SPREAD_t}$	-0.059 (0.010)	-0.079 (0.010)	-0.093 (0.010)	-0.094 (0.010)	-0.089 (0.010)	-0.081 (0.010)	-0.077 (0.010)	-0.069 (0.009)
Z-statistic	-5.98 ^a	-7.58 ^a	-8.86 ^a	-8.97 ^a	-8.59 ^a	-8.21 ^a	-8.09 ^a	-7.37 ^a
Pseudo R^2	0.034	0.061	0.085	0.088	0.079	0.067	0.062	0.050
Log-likelihood	-541.493	-525.523	-511.739	-506.967	-507.791	-509.912	-508.471	-509.971
	$Pr(S_{t+m} = 1 I_t) = F(\beta_0 + \beta_1 \Delta \log SP500_t)$							
	3 mths	6 mths	9 mths	12 mths	15 mths	18 mths	21 mths	24 mths
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log SP500_t}$	-0.014 (0.003)	-0.013 (0.002)	-0.012 (0.002)	-0.008 (0.002)	-0.003 (0.002)	-0.000 (0.002)	0.002 (0.002)	0.005 (0.002)
Z-statistic	-4.97 ^a	-4.72 ^a	-4.86 ^a	-3.43 ^a	-1.190	-0.080	0.970	2.37 ^b
Pseudo R^2	0.035	0.031	0.023	0.010	0.001	0.000	0.001	0.005
Log-likelihood	-541.144	-542.547	-546.257	-550.145	-550.493	-546.498	-541.378	-534.386
	$Pr(S_{t+m} = 1 I_t) = F(\beta_0 + \beta_1 \Delta \log CPI_t)$							
	3 mths	6 mths	9 mths	12 mths	15 mths	18 mths	21 mths	24 mths
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log CPI_t}$	-0.086 (0.025)	-0.050 (0.025)	-0.001 (0.024)	0.042 (0.024)	0.021 (0.023)	0.013 (0.022)	-0.007 (0.022)	-0.006 (0.021)
Z-statistic	-3.42 ^a	-2.01 ^b	-0.040	1.74 ^c	0.940	0.560	-0.290	-0.260
Pseudo R^2	0.016	0.005	0.000	0.004	0.001	0.000	0.000	0.000
Log-likelihood	-551.891	-556.854	-559.139	-553.709	-550.620	-546.311	-541.743	-537.019
	$Pr(S_{t+m} = 1 I_t) = F(\beta_0 + \beta_1 \Delta \log PPI_t)$							
	3 mths	6 mths	9 mths	12 mths	15 mths	18 mths	21 mths	24 mths
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log PPI_t}$	-0.0623 (0.014)	-0.033 (0.013)	-0.002 (0.012)	0.017 (0.012)	0.016 (0.011)	0.010 (0.011)	0.005 (0.010)	0.0001 (0.010)
Z-statistic	-4.37 ^a	-2.50 ^b	-0.14	1.46	1.43	0.94	0.53	0.09
Pseudo R^2	0.030	0.009	0.000	0.002	0.002	0.001	0.000	0.000
Log-likelihood	-544.064	-555.090	-559.128	-554.639	-550.097	-546.068	-541.691	-537.053
	$Pr(S_{t+m} = 1 I_t) = F(\beta_0 + \beta_1 \Delta y_t)$							
	3 mths	6 mths	9 mths	12 mths	15 mths	18 mths	21 mths	24 mths
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta y_t}$	-0.075 (0.011)	-0.052 (0.009)	-0.023 (0.007)	-0.003 (0.006)	0.004 (0.006)	0.008 (0.006)	0.011 (0.006)	0.014 (0.006)
Z-statistic	-6.66 ^a	-6.03 ^a	-3.30 ^a	-0.460	0.660	1.250	1.86 ^c	2.48 ^b
Pseudo R^2	0.106	0.056	0.012	0.000	0.000	0.001	0.003	0.005
Log-likelihood	-500.941	-528.805	-552.622	-555.723	-550.963	-545.739	-540.248	-534.297

Note: Estimation results of univariate binomial logistic regressions with $SPREAD_t$, $\Delta \log SP500_t$, $\Delta \log CPI_t$, $\Delta \log PPI_t$, or Δy_t on the right-hand side (m = months ahead). The marginal effect ($\partial Pr(.) / \partial x_{l,t}$) is the partial derivative of the predicted probability with respect to the RHS variable evaluated at its mean. Robust standard errors are in brackets and the superscripts ^a, ^b, and ^c denote significance at the 1%, 5% and 10% level, respectively. Identification is achieved by normalizing with the coefficients of the expansion state. We provide the model log-likelihood, and goodness-of-fit is measured by McFadden's pseudo R-squared.

Table 3: Multivariate Binomial Logistic Regression

	$Pr(S_{t+m} = 1 I_t) = F(\beta_0 + \beta_1 SPREAD_t + \beta_2 SP500_t + \beta_3 INFL_t + \beta_4 \Delta y_t)$							
	3 mths	6 mths	9 mths	12 mths	15 mths	18 mths	21 mths	24 mths
$\frac{\partial Pr(S_{t+m}=1)}{\partial SPREAD_t}$	-0.058 (0.009)	-0.077 (0.010)	-0.091 (0.010)	-0.094 (0.011)	-0.090 (0.010)	-0.083 (0.010)	-0.080 (0.010)	-0.074 (0.009)
Z-statistic	-6.21 ^a	-7.50 ^a	-8.77 ^a	-8.91 ^a	-8.63 ^a	-8.37 ^a	-8.41 ^a	-7.92 ^a
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log SP500_t}$	-0.012 (0.003)	-0.011 (0.003)	-0.011 (0.002)	-0.008 (0.002)	-0.003 (0.003)	0.000 (0.003)	0.003 (0.002)	0.006 (0.002)
Z-statistic	-4.10 ^a	-4.29 ^a	-4.94 ^a	-3.40 ^a	-0.95	0.05	1.08	2.52 ^b
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log PPI_t}$	-0.032 (0.014)	-0.008 (0.013)	0.010 (0.011)	0.017 (0.011)	0.014 (0.011)	0.007 (0.010)	0.000 (0.010)	-0.005 (0.010)
Z-statistic	-2.36 ^b	-0.66	0.98	1.64	1.31	0.64	0.04	-0.48
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta y_t}$	-0.065 (0.011)	-0.044 (0.008)	-0.019 (0.007)	0.001 (0.007)	0.008 (0.007)	0.012 (0.007)	0.016 (0.006)	0.018 (0.006)
Z-statistic	-5.83 ^a	-5.37 ^a	-2.72 ^a	0.16	1.07	1.68 ^c	2.33 ^b	2.80 ^a
Pseudo R^2	0.175	0.142	0.117	0.101	0.083	0.071	0.069	0.065
Log-likelihood	-462.707	-480.348	-493.654	-499.453	-505.234	-507.683	-504.684	-502.060

Note: Estimation results of the multivariate binomial logistic regression with $SPREAD_t$, $\Delta \log SP500_t$, $\Delta \log PPI_t$, and Δy_t on the right-hand side (m = months ahead). The marginal effect ($\partial Pr(.) / \partial x_{l,t}$) is the partial derivative of the predicted probability with respect to the RHS variable evaluated at its mean. Robust standard errors are in brackets and the superscripts ^a, ^b, and ^c denote significance at the 1%, 5% and 10% level, respectively. Identification is achieved by normalizing with the coefficients of the expansion state. We provide the model log-likelihood, and goodness-of-fit is measured by McFadden's pseudo R-squared.

Table 4: Multinomial Logistic Regression with Term Spread

	$Pr(S_{t+m} = j I_t) = F(\beta_0 + \beta_1 SPREAD_t)$							
	3 mths	6 mths	9 mths	12 mths	15 mths	18 mths	21 mths	24 mths
$\frac{\partial Pr(S_{t+m}=0)}{\partial SPREAD_t}$	0.031 (0.010)	0.047 (0.011)	0.055 (0.011)	0.056 (0.011)	0.052 (0.011)	0.047 (0.011)	0.046 (0.010)	0.039 (0.010)
Z-statistic	3.13 ^a	4.49 ^a	4.97 ^a	4.97 ^a	4.65 ^a	4.31 ^a	4.50 ^a	3.99 ^a
$\frac{\partial Pr(S_{t+m}=1)}{\partial SPREAD_t}$	-0.055 (0.008)	-0.072 (0.008)	-0.082 (0.007)	-0.082 (0.007)	-0.079 (0.007)	-0.072 (0.007)	-0.062 (0.007)	-0.049 (0.007)
Z-statistic	-7.34 ^a	-9.39 ^a	-11.05 ^a	-10.88 ^a	-10.58 ^a	-10.46 ^a	-9.04 ^a	-6.90 ^a
$\frac{\partial Pr(S_{t+m}=2)}{\partial SPREAD_t}$	0.026 (0.003)	0.027 (0.004)	0.029 (0.004)	0.030 (0.004)	0.030 (0.004)	0.030 (0.004)	0.031 (0.004)	0.031 (0.005)
Z-statistic	7.35 ^a	7.42 ^a	7.49 ^a	7.54 ^a	7.58 ^a	7.61 ^a	7.63 ^a	7.66 ^a
$\frac{\partial Pr(S_{t+m}=3)}{\partial SPREAD_t}$	-0.002 (0.007)	-0.003 (0.008)	-0.002 (0.009)	-0.004 (0.010)	-0.004 (0.009)	-0.005 (0.010)	-0.015 (0.008)	-0.021 (0.006)
Z-statistic	-0.25	-0.32	-0.22	-0.41	-0.40	-0.52	-1.92 ^c	-3.59 ^a
Pseudo R^2	0.033	0.054	0.075	0.076	0.068	0.057	0.047	0.040
Log-likelihood	-938.930	-917.079	-895.691	-890.054	-889.984	-892.180	-894.103	-892.523
LR: $Bo \equiv Ex$	12.520 ^a	12.121 ^a	12.435 ^a	13.462 ^a	14.892 ^a	16.450 ^a	17.596 ^a	19.322 ^a
Wald: $Bo \equiv Ex$	12.190 ^a	11.840 ^a	12.167 ^a	13.150 ^a	14.502 ^a	15.950 ^a	16.975 ^a	18.545 ^a
LR: $De \equiv Re$	12.603 ^a	24.255 ^a	38.657 ^a	34.557 ^a	27.860 ^a	18.881 ^a	3.428 ^c	0.216
Wald: $De \equiv Re$	12.256 ^a	22.995 ^a	35.618 ^a	32.033 ^a	26.188 ^a	18.059 ^a	3.378 ^c	0.216

Note: Estimation results of the univariate multinomial logistic regression with $SPREAD_t$ on the right-hand side (m = months ahead). The marginal effect ($\partial Pr(.) / \partial x_{l,t}$) is the partial derivative of the predicted probability with respect to the RHS variable evaluated at its mean. Robust standard errors are in brackets and the superscripts ^a, ^b, and ^c denote significance at the 1%, 5% and 10% level, respectively. Identification is achieved by normalizing with the coefficients of the expansion state. We provide the model log-likelihood, and goodness-of-fit is measured by McFadden's pseudo R-squared. We also report LR and Wald test for the null hypothesis that all coefficients (except intercepts) associated with a given pair of business cycle regimes (boom (Bo) and expansion (Ex) or depression (De) and recession (Re)) are equal (indistinguishability). The test statistics are $\chi^2(1)$ distributed under the null.

Table 5: Multinomial Logistic Regression with Stock Returns

	$Pr(S_{t+m} = j I_t) = F(\beta_0 + \beta_1 \Delta \log SP500_t)$							
	3 mths	6 mths	9 mths	12 mths	15 mths	18 mths	21 mths	24 mths
$\frac{\partial Pr(S_{t+m}=0)}{\partial \Delta \log SP500_t}$	0.014	0.014	0.013	0.010	0.005	0.002	-0.000	-0.003
	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)
Z-statistic	4.50 ^a	4.58 ^a	4.75 ^a	3.71 ^a	2.11 ^b	0.99	-0.02	-1.24
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log SP500_t}$	-0.007	-0.007	-0.006	-0.004	0.000	0.002	0.002	0.004
	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)
Z-statistic	-3.61 ^a	-4.01 ^a	-3.26 ^a	-2.56 ^a	0.20	1.02	1.37	2.80 ^a
$\frac{\partial Pr(S_{t+m}=2)}{\partial \Delta \log SP500_t}$	-0.000	-0.001	-0.002	-0.002	-0.003	-0.002	-0.002	-0.002
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Z-statistic	-0.05	-0.40	-0.95	-1.15	-1.56	-1.35	-1.15	-1.20
$\frac{\partial Pr(S_{t+m}=3)}{\partial \Delta \log SP500_t}$	-0.006	-0.006	-0.006	-0.004	-0.003	-0.002	0.000	0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)
Z-statistic	-4.75 ^a	-3.95 ^a	-4.72 ^a	-2.71 ^a	-1.83 ^c	-0.94	0.06	0.61
Pseudo R^2	0.024	0.021	0.018	0.009	0.005	0.003	0.002	0.004
Log-likelihood	-947.329	-949.444	-950.831	-954.210	-950.140	-943.983	-936.551	-925.964
LR: $Bo \equiv Ex$	0.725	1.753	3.737 ^b	3.852 ^b	4.244 ^b	2.764 ^c	1.722	1.341
Wald: $Bo \equiv Ex$	0.725	1.770	3.846 ^b	4.025 ^b	4.493 ^b	2.903 ^c	1.793	1.395
LR: $De \equiv Re$	6.787 ^a	3.745 ^b	5.856 ^b	1.935	3.384 ^c	1.940	0.180	0.143
Wald: $De \equiv Re$	6.689 ^a	3.772 ^b	5.839 ^b	1.978	3.49 ^c	1.990	0.181	0.142

Note: Estimation results of the univariate multinomial logistic regression with $\Delta \log SP500_t$ on the right-hand side (m = months ahead). The marginal effect ($\partial Pr(.) / \partial x_{l,t}$) is the partial derivative of the predicted probability with respect to the RHS variable evaluated at its mean. Robust standard errors are in brackets and the superscripts ^a, ^b, and ^c denote significance at the 1%, 5% and 10% level, respectively. Identification is achieved by normalizing with the coefficients of the expansion state. We provide the model log-likelihood, and goodness-of-fit is measured by McFadden's pseudo R-squared. We also report LR and Wald test for the null hypothesis that all coefficients (except intercepts) associated with a given pair of business cycle regimes (boom (Bo) and expansion (Ex) or depression (De) and recession (Re)) are equal (indistinguishability). The test statistics are $\chi^2(1)$ distributed under the null.

Table 6: Multinomial Logistic Regression with Inflation

	$Pr(S_{t+m} = j I_t) = F(\beta_0 + \beta_1 \Delta \log PPI_t)$							
	3 mths	6 mths	9 mths	12 mths	15 mths	18 mths	21 mths	24 mths
$\frac{\partial Pr(S_{t+m}=0)}{\partial \Delta \log PPI_t}$	0.039 (0.015)	0.018 (0.013)	-0.003 (0.012)	-0.018 (0.012)	-.020 (0.012)	-0.019 (0.012)	-0.018 (0.013)	-0.014 (0.013)
Z-statistic	2.55 ^b	1.36	-0.21	-1.44	-1.59	-1.52	-1.43	-1.13
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log PPI_t}$	-0.006 (0.015)	0.007 (0.012)	0.020 (0.009)	0.031 (0.010)	0.032 (0.010)	0.029 (0.009)	0.029 (0.009)	0.024 (0.009)
Z-statistic	-0.40	0.61	2.20 ^b	3.24 ^a	3.31 ^a	3.15 ^a	3.31 ^a	2.65 ^a
$\frac{\partial Pr(S_{t+m}=2)}{\partial \Delta \log PPI_t}$	0.007 (0.007)	0.004 (0.007)	0.000 (0.007)	-0.000 (0.007)	0.002 (0.007)	0.006 (0.008)	0.008 (0.008)	0.007 (0.007)
Z-statistic	0.99	0.55	0.06	-0.01	0.28	0.74	1.01	1.03
$\frac{\partial Pr(S_{t+m}=3)}{\partial \Delta \log PPI_t}$	-0.040 (0.007)	-0.029 (0.007)	-0.018 (0.007)	-0.013 (0.007)	-0.014 (0.007)	-0.016 (0.006)	-0.018 (0.005)	-0.018 (0.005)
Z-statistic	-5.82 ^a	-4.42 ^a	-2.63 ^a	-1.80 ^c	-2.09 ^b	-2.75 ^a	-3.73 ^a	-3.89 ^a
Pseudo R^2	0.038	0.017	0.007	0.007	0.008	0.009	0.011	0.010
Log-likelihood	-933.459	-952.812	-962.034	-955.796	-938.015	-947.039	-927.815	-920.492
LR: $Bo \equiv Ex$	0.104	0.054	0.008	0.047	0.255	0.832	1.315	1.212
Wald: $Bo \equiv Ex$	0.107	0.054	0.008	0.047	0.254	0.839	1.345	1.242
LR: $De \equiv Re$	40.986 ^a	23.785 ^a	12.862 ^a	11.534 ^a	12.745 ^a	15.016 ^a	18.858 ^a	16.693 ^a
Wald: $De \equiv Re$	30.370 ^a	21.312 ^a	13.446 ^a	12.197 ^a	13.599 ^a	16.118 ^a	20.050 ^a	17.792 ^a

Note: Estimation results of the univariate multinomial logistic regression with $\Delta \log PPI_t$ on the right-hand side (m = months ahead). The marginal effect ($\partial Pr(\cdot)/\partial x_{l,t}$) is the partial derivative of the predicted probability with respect to the RHS variable evaluated at its mean. Robust standard errors are in brackets and the superscripts ^a, ^b, and ^c denote significance at the 1%, 5% and 10% level, respectively. Identification is achieved by normalizing with the coefficients of the expansion state. We provide the model log-likelihood, and goodness-of-fit is measured by McFadden's pseudo R-squared. We also report LR and Wald test for the null hypothesis that all coefficients (except intercepts) associated with a given pair of business cycle regimes (boom (Bo) and expansion (Ex) or depression (De) and recession (Re)) are equal (indistinguishability). The test statistics are $\chi^2(1)$ distributed under the null.

Table 7: Multinomial Logistic Regression with Output Growth

	$Pr(S_{t+m} = j I_t) = F(\beta_0 + \beta_1 \Delta y_t)$							
	3 mths	6 mths	9 mths	12 mths	15 mths	18 mths	21 mths	24 mths
$\frac{\partial Pr(S_{t+m}=0)}{\partial \Delta y_t}$	0.055 (0.010)	0.031 (0.008)	0.010 (0.007)	-0.008 (0.007)	-0.015 (0.008)	-0.018 (0.008)	-0.022 (0.008)	-0.027 (0.009)
Z-statistic	5.21 ^a	3.89 ^a	1.590	-1.180	-1.88 ^c	-2.22 ^b	-2.66 ^a	-3.03 ^a
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta y_t}$	-0.046 (0.008)	-0.025 (0.006)	-0.007 (0.004)	0.006 (0.004)	0.008 (0.005)	0.010 (0.005)	0.012 (0.005)	0.015 (0.005)
Z-statistic	-5.53 ^a	-4.22 ^a	-1.530	1.142	1.750 ^c	1.99 ^b	2.46 ^b	3.18 ^a
$\frac{\partial Pr(S_{t+m}=2)}{\partial \Delta y_t}$	0.018 (0.004)	0.017 (0.004)	0.011 (0.005)	0.010 (0.005)	0.010 (0.005)	0.010 (0.005)	0.010 (0.005)	0.011 (0.005)
Z-statistic	4.74 ^a	4.35 ^a	2.37 ^b	2.20 ^b	2.12 ^b	2.03 ^b	2.15 ^b	2.26 ^b
$\frac{\partial Pr(S_{t+m}=3)}{\partial \Delta y_t}$	-0.027 (0.004)	-0.023 (0.004)	-0.014 (0.004)	-0.008 (0.004)	-0.004 (0.005)	-0.001 (0.005)	0.000 (0.004)	0.001 (0.004)
Z-statistic	-6.89 ^a	-6.54 ^a	-3.71 ^a	-1.84 ^c	-0.690	-0.250	0.030	0.150
Pseudo R^2	0.074	0.048	0.014	0.007	0.005	0.005	0.007	0.009
Log-likelihood	-898.841	-923.124	-955.277	-956.439	-949.483	-941.182	-931.428	-920.729
LR: $Bo \equiv Ex$	11.425 ^a	12.560 ^a	5.938 ^b	7.378 ^a	7.697 ^a	7.532 ^a	8.924 ^a	10.511 ^a
Wald: $Bo \equiv Ex$	12.304 ^a	13.326 ^a	6.405 ^b	7.899 ^a	8.216 ^a	8.027 ^a	9.509 ^a	11.191 ^a
LR: $De \equiv Re$	12.691 ^a	18.241 ^a	7.543 ^a	5.268 ^b	2.224	1.293	1.022	1.356
Wald: $De \equiv Re$	11.881 ^a	16.590 ^a	7.641 ^a	5.435 ^b	2.239	1.285	1.008	1.324

Note: Estimation results of the univariate multinomial logistic regression with Δy_t on the right-hand side (m = months ahead). The marginal effect ($\partial Pr(.) / \partial x_{l,t}$) is the partial derivative of the predicted probability with respect to the RHS variable evaluated at its mean. Robust standard errors are in brackets and the superscripts ^a, ^b, and ^c denote significance based on the Z-statistic at the 1%, 5% and 10% level, respectively. Identification is achieved by normalizing with the coefficients of the expansion state. We report the model log-likelihood, and goodness-of-fit is measured by McFadden's pseudo R-squared. Likelihood ratio (LR) and Wald tests are reported for the null hypothesis that all coefficients (except intercepts) associated with a given pair of business cycle regimes (boom (*Bo*) and expansion (*Ex*); depression (*De*) and recession (*Re*)) are equal (indistinguishability), and consequently the two regimes can be merged into one regime; significance is based on the $\chi^2(1)$ statistic.

Table 8: Multivariate Multinomial Logistic Regression

	$Pr(S_{t+m} = j I_t) = F(\beta_0 + \beta_1 SPREAD_t + \beta_2 \Delta \log SP500_t + \beta_3 \Delta \log PPI_t + \beta_4 \Delta y_t)$							
	3 mths	6 mths	9 mths	12 mths	15 mths	18 mths	21 mths	24 mths
$\frac{\partial Pr(S_{t+m}=0)}{\partial SPREAD_t}$	0.033 (0.009)	0.050 (0.010)	0.057 (0.011)	0.057 (0.011)	0.053 (0.011)	0.048 (0.011)	0.047 (0.010)	0.042 (0.010)
Z-statistic	3.47 ^a	4.67 ^a	5.16 ^a	5.07 ^a	4.80 ^a	4.45 ^a	4.59 ^a	4.32 ^a
$\frac{\partial Pr(S_{t+m}=0)}{\partial \Delta \log SP500_t}$	0.011 (0.003)	0.012 (0.003)	0.012 (0.003)	0.009 (0.002)	0.004 (0.002)	0.002 (0.002)	-0.001 (0.003)	-0.004 (0.003)
Z-statistic	3.76 ^a	4.19 ^a	4.55 ^a	3.67 ^a	1.79 ^c	0.71	-0.23	-1.45
$\frac{\partial Pr(S_{t+m}=0)}{\partial \Delta \log PPI_t}$	0.028 (0.023)	0.012 (0.012)	-0.000 (0.012)	-0.006 (0.012)	-0.006 (0.012)	-0.004 (0.012)	-0.004 (0.012)	-0.002 (0.012)
Z-statistic	2.08 ^b	0.99	0.05	-0.50	-0.50	-0.37	-0.31	-0.14
$\frac{\partial Pr(S_{t+m}=0)}{\partial \Delta y_t}$	0.045 (0.010)	0.024 (0.008)	0.005 (0.007)	-0.013 (0.007)	-0.018 (0.008)	-0.021 (0.008)	-0.025 (0.008)	-0.028 (0.009)
Z-statistic	4.35 ^a	3.07 ^a	0.74	-1.83 ^c	-2.34 ^b	-2.62 ^a	-2.97 ^a	-3.22 ^a
$\frac{\partial Pr(S_{t+m}=1)}{\partial SPREAD_t}$	-0.054 (0.007)	-0.070 (0.008)	-0.081 (0.008)	-0.082 (0.008)	-0.079 (0.008)	-0.073 (0.007)	-0.063 (0.007)	-0.051 (0.007)
Z-statistic	-7.27 ^a	-9.08 ^a	-10.82 ^a	-10.72 ^a	-10.36 ^a	-10.30 ^a	-9.05 ^a	-7.19 ^a
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log SP500_t}$	-0.007 (0.002)	-0.007 (0.002)	-0.004 (0.002)	-0.003 (0.002)	0.001 (0.002)	0.003 (0.002)	0.003 (0.002)	0.005 (0.002)
Z-statistic	-3.30 ^a	-3.78 ^a	-2.70 ^a	-2.01 ^b	0.85	1.55	1.71 ^c	2.93 ^a
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta \log PPI_t}$	0.000 (0.011)	0.008 (0.009)	0.017 (0.007)	0.021 (0.008)	0.024 (0.008)	0.022 (0.008)	0.022 (0.008)	0.018 (0.009)
Z-statistic	0.01	0.96	2.40 ^b	2.83 ^a	3.12 ^a	2.94 ^a	2.71 ^a	1.99 ^b
$\frac{\partial Pr(S_{t+m}=1)}{\partial \Delta y_t}$	-0.044 (0.008)	-0.021 (0.006)	-0.006 (0.005)	0.008 (0.004)	0.008 (0.005)	0.010 (0.005)	0.011 (0.005)	0.014 (0.005)
Z-statistic	-5.41 ^a	-3.91 ^a	-1.16	1.70 ^c	1.71 ^c	1.87 ^c	2.24 ^b	2.78 ^a

Note: See next page.

Table 8: Multivariate Multinomial Logistic Regression - *Continued*

$\frac{\partial Pr(S_{t+m}=2)}{\partial SPREAD_t}$	0.024 (0.004)	0.025 (0.004)	0.027 (0.004)	0.028 (0.004)	0.028 (0.004)	0.029 (0.004)	0.029 (0.004)	0.029 (0.004)
Z-statistic	6.70 ^a	6.69 ^a	6.64 ^a	6.63 ^a	6.67 ^a	6.77 ^a	6.79 ^a	6.82 ^a
$\frac{\partial Pr(S_{t+m}=2)}{\partial \Delta \log SPI_t}$	-0.001 (0.002)	-0.001 (0.002)	-0.002 (0.002)	-0.002 (0.002)	-0.003 (0.001)	-0.002 (0.001)	-0.002 (0.001)	-0.002 (0.001)
z-statistic	-0.44	-0.75	-1.29	-1.52	-2.00 ^b	-1.76 ^c	-1.55	-1.59
$\frac{\partial Pr(S_{t+m}=2)}{\partial \Delta \log PPI_t}$	-0.003 (0.007)	-0.006 (0.007)	-0.007 (0.007)	-0.008 (0.007)	-0.005 (0.007)	-0.001 (0.007)	0.001 (0.008)	0.000 (0.007)
Z-statistic	-0.38	-0.81	-1.04	-1.06	-0.71	-0.16	0.09	0.03
$\frac{\partial Pr(S_{t+m}=2)}{\partial \Delta y_t}$	0.016 (0.004)	0.015 (0.004)	0.010 (0.004)	0.009 (0.004)	0.008 (0.004)	0.008 (0.004)	0.008 (0.004)	0.009 (0.004)
Z-statistic	4.40 ^a	4.24 ^a	2.42 ^b	2.27 ^b	2.16 ^b	1.88 ^c	1.90 ^c	2.02 ^b
$\frac{\partial Pr(S_{t+m}=3)}{\partial SPREAD_t}$	-0.003 (0.005)	-0.003 (0.007)	-0.002 (0.009)	-0.003 (0.009)	-0.002 (0.009)	-0.003 (0.010)	-0.013 (0.008)	-0.020 (0.006)
Z-statistic	-0.50	-0.45	-0.27	-0.36	-0.27	-0.34	-1.65 ^c	-3.40 ^a
$\frac{\partial Pr(S_{t+m}=3)}{\partial \Delta \log SPI_t}$	-0.004 (0.001)	-0.004 (0.001)	-0.005 (0.001)	-0.004 (0.002)	-0.003 (0.002)	-0.002 (0.002)	0.000 (0.001)	0.001 (0.002)
Z-statistic	-2.89 ^a	-2.58 ^a	-3.86 ^a	-2.46 ^b	-1.84 ^c	-1.01	0.04	0.71
$\frac{\partial Pr(S_{t+m}=3)}{\partial \Delta \log PPI_t}$	-0.025 (0.006)	-0.015 (0.006)	-0.009 (0.007)	-0.008 (0.007)	-0.012 (0.007)	-0.017 (0.006)	-0.019 (0.006)	-0.017 (0.005)
Z-statistic	-4.05 ^a	-2.46 ^b	-1.25	-1.05	-1.77 ^c	-2.77 ^a	-3.39 ^a	-3.25 ^a
$\frac{\partial Pr(S_{t+m}=3)}{\partial \Delta y_t}$	-0.018 (0.004)	-0.018 (0.004)	-0.010 (0.004)	-0.004 (0.005)	0.001 (0.005)	0.003 (0.005)	0.005 (0.004)	0.005 (0.004)
Z-statistic	-4.69 ^a	-4.77 ^a	-2.37 ^b	-0.99	0.12	0.70	1.24	1.36
Pseudo R^2	0.140	0.121	0.107	0.097	0.086	0.074	0.066	0.062
Log-likelihood	-834.393	-852.227	-865.332	-869.637	-872.657	-876.007	-876.350	-871.475
LR: $Bo \equiv Ex$	23.996 ^a	26.089 ^a	22.290 ^a	24.612 ^a	26.575 ^a	25.996 ^a	27.228 ^a	29.928 ^a
Wald: $Bo \equiv Ex$	23.963 ^a	26.066 ^a	22.318 ^a	24.618 ^a	26.530 ^a	25.780 ^a	26.845 ^a	29.330 ^a
LR: $De \equiv Re$	52.661 ^a	49.638 ^a	53.525 ^a	49.370 ^a	46.060 ^a	37.487 ^a	22.035 ^a	16.251 ^a
Wald: $De \equiv Re$	39.953 ^a	41.496 ^a	46.066 ^a	44.043 ^a	42.218 ^a	35.203 ^a	21.731 ^a	16.782 ^a

Note: Estimation results of the multivariate multinomial logistic regression (m = months ahead). The marginal effect ($\partial Pr(.) / \partial x_{i,t}$) is the partial derivative of the predicted probability with respect to the RHS variable evaluated at its mean. Robust standard errors are in brackets and the superscripts ^a, ^b, and ^c denote significance at the 1%, 5% and 10% level, respectively. Identification is achieved by normalizing with the coefficients of the expansion state. We provide the model log-likelihood, and goodness-of-fit is measured by McFadden's pseudo R-squared. We also report LR and Wald test for the null hypothesis that all coefficients (except intercepts) associated with a given pair of business cycle regimes (boom (Bo) and expansion (Ex) or depression (De) and recession (Re)) are equal (indistinguishability). The test statistics are $\chi^2(4)$ distributed under the null.