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Tests of Financial Market Contagion: Evolutionary Cospectral Analysis *V.S.* Wavelet Analysis

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Abstract

This paper examines the co-movements dynamics between OCDE countries with the US and Europe. The core focus is to suggest advantageous techniques allowing the investigation with respect to time and frequency, namely evolutionary co-spectral analysis and wavelet analysis. Our study puts in evidence the existence of both long run and short-run co-movements. Both interdependence and contagion are well identified across markets; but with slight differences. Both investors and policymakers can derive worthwhile information from this research. Recognizing countries sensitivity to permanent and transitory shocks enables investors to select rational investment strategies. Similarly, policymakers can make safe crisis management policies.

Keywords: contagion, interdependence, stock markets index, evolutionary co-spectral analysis, wavelet analysis.

1. Introduction

The linkages and co-movements of financial markets have been an issue of great interest in a large body of finance literature for more than half a century. Two main reasons may explain this pronounced interest. First, with the expansion of international trade and commerce, financial globalization has been marked by a surge in capital flows across countries. Investors seek to construct international rather than national portfolios. In the meantime, modern financial theory, mainly the capital asset pricing model and arbitrage pricing theory, supports the profound impact of the correlation pattern of risk and return magnitudes and thus of asset allocation and diversification strategies (Sharpe 1964; Lintner 1965; Ross 1976). Investors are thus incited to study and comprehend the potential linkages and correlations between international financial markets in order to construct optimal portfolios (Forbes and Rigobon, 2002; Bessler and Yang 2003; Kiviaho et al. 2012). The literature on portfolio selection presumes that an optimal portfolio enables investors to incur low risks and derive high returns (Markowitz 1952; Merton 1972).

Second, the wave of financial and economic turmoil and the multiplication of political events have raised widespread concern about the co-movements of financial markets around the world. Since the 1929 Wall Street crash, the latter have been much more significant during turbulent periods and times of crises (King and Wahwani 1990; Lee and Kim 1993; Bancel and Mittoo 2011; Zaki et *al*. 2011; Ranta 2013). The need for analyzing and understanding financial markets co-movements and interrelations arises from their potential implications for goods and services markets across countries and over time. This is a consequence of not only the advanced process of globalization and liberalization (Madaleno and Pinho 2012) but also of the high integration of financial and goods markets (Gallegati 2012). Numerous studies have offered several recommendations and recurring suggestions to enhance the modeling of international financial market co-movements. The ultimate objective is to better understand these interrelations and predict their implications for both investors and policy makers. Investors are concerned with their implications for asset allocation, portfolio composition, and investment strategies, while policy makers look for the best policy responses and hedging strategies during crises.

Although one can think of a host of issues concerning international stock markets co-movements and linkages, there is no consensus regarding the definition and the measurement of cross-market interrelations (Gallegati 2012; Ranta 2013). Interdependence and contagion are the two main kinds of linkages among various markets: (Gallegati 2012). While interdependence refers to a simple correlation between markets movements, contagion is commonly defined as an increase in the correlation between them after crises (Forbes and Rigobon 2002). Pericoli and Sbracia (2003) present five representative definitions of contagion. According to them, contagion can be defined as a significant increase in either the probability of a crisis in one country or co-movements of prices and quantities across markets, conditional on a crisis occurring elsewhere. The three other classifications provided by these authors are related to the timing of contagion. Contagion can occur either when volatility spills over from the crisis country to the financial markets of other countries, or when the transmission channel differs, after a shock in one market, or even when co-movements cannot be explained by fundamentals. Furthermore, contagion can be either “fundamentals-based” or “pure” (Dornbush et al. 2000; Kaminsky and Reinhart 2000). Fundamentals-based contagion, often labeled as spillover, refers to transmission mechanisms and shocks propagation among markets that result from real interdependence and global integration in both quiet and turbulent periods (Calvo and Reinhart 1996). Pure contagion stresses excessive co-movements rather than the normal ones that are expected to occur after controlling for

fundamental factors (Eichengreen et al. 1996; Bae et al. 2003). Pure contagion is usually induced by investor behavior and bias, including overconfidence, the illusion of control, herding, and so on. Beyond such a classification, not only contagion but also interdependence can induce co-movements during crises.

The study focuses on both the interdependence and pure contagion that occurred from 1990-2013 between, on the one hand, the broad-based market return indexes of either the U.S. or Europe, and, on the other hand, fourteen OECD countries: Australia, Austria, Belgium, Denmark, France, Finland, Germany, Greece, Italy, Netherland, Norway, Portugal, Spain, and Sweden. Markets indexes are drawn from the Morgan Stanley Capital International indexes (MSCI) and consist of stocks that broadly represent stock composition in the different countries and regions.

To provide further evidence, we suggest a new methodology that usefully discriminates between contagion and interdependence, based on frequency domain analysis. We propose two advantageous frequencies approaches, the ESA and the wavelet analysis. The ESA does not impose any restrictions or require pre-treatment of the data (as it is the case of the VAR model, for instance, which requires the series to be stationary or co-integration techniques that can be applied only to time series data integrated in the first order). Second, it does not have an "end-point problem": no future information is used, implied, or required, as in band-pass or trend projection methods. In addition, the ESA gives a robust frequency representation of non-stationary process. Finally, frequency analysis provides worthwhile information about the time horizon of the interdependence between the two series: the analysis stresses whether the variables under investigation present short, medium, or long-term interdependence. This additional information allows the understanding of any contagion (high frequency) or interdependence (low frequency). Similar to the ESA, the wavelet analysis offers various benefits. First, this filtering method provides an interesting alternative to time series and frequency domain methods because it transforms the time series into different frequency components with a resolution matched to its scale. This is useful when dealing with non-stationary data that exhibit changing frequencies over time, as in the case of financial market data. Second, the wavelet analysis is suitable to macroeconomic and/or financial data in their time-scale components. Third, it provides an alternative representation of the variability and the association structure of certain stochastic processes on a scale-by-scale basis. The multi-resolution decomposition property of the wavelet transform can be used to identify separately contagion and interdependence by associating each to its corresponding frequency component.

Our analysis offers many important results. We have proved that the long-term co-movement, related to the interdependence between stock markets indexes, increased during the periods of financial turmoil (the beginning of the 1990s and the subprime crisis in 2007). This interdependence differs among countries. We have also shown that some countries, such as Germany and the Netherlands, were less sensitive to permanent shocks than other countries, including-Greece, Portugal, and Australia. Our results include another important that concerns short-term co-movements. The short-term co-movement assesses the contagion character of various stock markets indexes. We have identified the contagious character of all studied markets with some national differences. Our empirical findings demonstrate that Greece, Spain, Portugal, and Italy were more sensitive to transitory shocks than the U.S. and Europe. This study has significant economic implications. Indeed, the distinction between excessive and normal co-movements is a key issue from a portfolio diversification perspective, especially during periods of high volatility. Investors can derive important information about countries that are less sensitive to permanent shocks and those that are less sensitive to transitory shocks.

The study contributes to the research topic through at least three channels. First, most studies mainly focus on contagion, and pay little, if any, attention to interdependence, whereas this study sheds light on both sides of the interrelations in international markets. Second, the research suggests two advantageous frequency approaches, namely, the ESA and the wavelet analysis, in order to escape the limitations of the Fourier transformation and conventional time series in co-movement analysis. Third, the period of the study spans more than two decades, from 1990 to 2013. Such a long period includes numerous economic crises and financial crashes that allow us to draw relevant conclusions regarding co-movements of most dynamic international financial markets.

The remainder of this paper is structured as follows. Section 2 briefly reviews the related literature. Section 3 provides data and describes the study's empirical methodology. The results are summarized and discussed in Section 4. A conclusion follows and points to directions for future research.

2. Literature review

Despite the considerable of research on co-movement of international markets, only a few studies have identified the linkages of interdependence and contagion among various markets (Gallegati 2012). Further, no consensus exists on exactly what interdependence and contagion are (Pericoli and Sbracia 2003). Some researchers argue that scale dimension is relevant in the analysis of co-movements structure and presume that contagion is a temporary increase of short time-scale co-movements (Rua and Nunes 2009; Graham and Nikkinen 2011; Graham et al. 2012; Ranta 2013). Others examine contagion along shock propagation and particularly through some specific transmission mechanisms, rather than through changes in linkages (Ranta 2013). It is, therefore, worthwhile to discriminate between excessive and normal co-movements across financial markets, when identifying fundamental contagion from other transmission mechanisms. Distinguishing between excessive and normal co-movements is also a crucial issue from a portfolio diversification perspective, especially during periods of high volatility. This distinction has profound implications for investor asset allocation strategies and policy-maker crisis management. In fact, the usefulness of hedging strategies may substantially differ over normal and turbulent periods along with the variation of the correlation patterns of financial time series. International diversification may be markedly less valuable in embarrassing moments than in calm situations, whereas its benefits are particularly needed to compensate for potential losses (Gallegati 2012).

Most studies have applied a variety of traditional time-domain approaches to measure co-movement correlations, including testing for changes in correlation coefficients (King and Wadhwani 1990; Lee and Kim 1993; Calvo and Reinhart 1996), ARCH and GARCH models (1994; Hamao et al. 1990; Billio and Caporin 2010), cointegrating relationships (Longin and Solnik, 1995;), probit/logit models (Eichengreenet al. 1996; Kaminsky and Reinhart 2000), regime switching (Gallo and Otranto 2008), the factor model (Corsetti et al. 2005), and the copula approach (Rodriguez 2007). However, these techniques may face problems and can even produce biased results when testing for fundamental contagion and differentiating it from other transmission mechanisms. They do not provide suitable proxies for the influence of macroeconomic fundamentals, and finding good proxies is hard. Bodart and Candelon (2009) and Orlov (2009) are the most recent studies, which have tested for contagion and identified it from interdependence with distinct frequency ranges (high and low frequencies, respectively). They have applied new techniques, namely, ESA and the wavelet analysis, to avoid restriction of stationary series, unlike traditional tools of frequency analysis, such as the

Fourier approach. These techniques have appealed to several researchers and are achieving greater popularity in financial time series analysis that are related to return spillovers, stock returns and inflation, exchange rates, etc. (Fernandez 2005; Kim and In 2005; Nikkinen et al. 2011). However, a few studies have recently applied these techniques to test for financial market contagion and have distinguished it from normal interdependence (Bodart and Candelon 2009; Orlov 2009; Rua and Nunes, 2009; Graham and Nikkinen 2011; Gallegati 2012; Graham et al. 2012; Kiviaho et al. 2012; Madaleno and Pinho 2012; Loh 2013; Ranta 2013).

Most of the research dealing with financial market co-movements has been conducted in developed markets (King and Wadhwani 1990; Lin et al. 1994; Longin and Solnik 1995; Wu and Su 1998; Engested and Tanggaard 2004; Ozdemir and Cakan 2007; Rua and Nunes 2009; Gallegati 2012; Ranta 2013). Wu and Su (1998) focus on the U.S., the U.K., Japan, and Hong Kong stock markets and find that display a significant dynamic relation. Ozdemir and Cakan (2007) investigate the same sample but replace Hong Kong by France and highlight similar results. Rua and Nunes (2009) show that the co-movements of the German, Japanese, British, and American financial markets vary with respect to time and frequency. Recently, several studies have focused on contagion and financial crisis analysis. Gallegati (2012) checks whether contagion occurred during the US subprime crisis of 2007 in G7 countries (the U.S., Canada, Japan, the U.K., France, Germany, and Italy), along with Brazil and Hong Kong. Ranta (2013) investigates contagion among the major world markets—those of Germany, Great Britain, the U.S., and Japan—during the last 25 years and finds that co-movements increased during the major crises.

In contrast, there is a little empirical evidence on emerging markets, including Central and Eastern European markets and European frontier markets (Berger et al. 2011; Zaki et al. 2011; Graham et al. 2012; Wang and Shih 2013). Berger et al. (2011) highlight the significant diversification global potential for frontier markets because of their low integration with the world market. Wang and Shih (2013) focus on emerging European markets and examine time-varying world and regional integration in these economies. Zaki et al. (2011) investigate UAE banks during moments of financial distress and find that these banks are resilient and are not affected by macroeconomic factors. Graham et al. (2012) provide evidence of the low integration of 22 emerging stock markets and argue that co-movement between the U.S. and emerging markets change in time and across frequencies.

Several studies have recently compared the cross-dynamics of developed and emerging markets (Bekaert and Harvey 1995; Collins and Biekpe 2002; Lee 2004; Sharkasi et al. 2005; Chambet and Gibson 2008; Graham and Nikkinen 2011; Kiviaho et al. 2012; Madaleno and Pinho 2012; Loh 2013). Collins and Biekpe (2002) provide evidence of very low contagion across African financial markets and their peers worldwide. Lee (2004) investigates international transmission effects of the U.S., Germany, and Japan and two emerging markets, that of Egypt and Turkey, and argues that the movements of the developed markets influenced their developing peers but not vice versa.

Sharkasi et al. (2005) report the existence of intra-European market co-movements with the U.S. market, whereas the latter influences Asian markets, which in turn affect European ones. Graham and Nikkinen (2011) compare the cross-dynamics of the Finnish stock market with its developed and emerging peers. Kiviaho et al. (2012) analyze the co-movement of European frontier markets with the U.S. market and the three largest developed markets in Europe (the U.K., Germany, and France) and suggests that co-movement is stronger at lower frequencies and increases during the turbulent period of the global financial crisis of 2008/2009. Madaleno and Pinho (2012) explore financial market linkage in the U.S., the U.K., Japan, and Brazil and stress that financial crises often caused a significant delay in

international transmissions. Loh (2013) emphasizes the consistent co-movement between most of the Asia-Pacific stock markets and those of Europe and the U.S. in the long term.

3. Empirical methodology and data

This study suggests investigating co-movements between broad-based market indexes of the US and Europe on one hand, and on the other hand fourteen OECD countries, namely Australia, Austria, Belgium, Denmark, France, Finland, Germany, Greece, Italy, Netherland, Norway, Portugal, Spain and Sweden during 1990-2013. A frequency approach should be followed. However, to deal systematically with the limits of conventional time-domain techniques, such as the problems of non-stationary of time series, we employ expedient approaches for non-stationary time series.[1] To overcome the limitations of the Fourier transformation and conventional time series analysis in measuring the co-movement or correlation between stock markets indices, we propose two different frequency approaches: the ESA, defined by Priestley and Tong (1973), and the wavelet analysis.

In this section, we start our analysis by presenting the main characteristics of the theory and the estimation of ESA. Then, we focus on the wavelet analysis.

3.1 Evolutionary co-spectral approach: theory and estimation

3.1.1 Theory of the evolutionary co-spectral approach

According to Priestley (1965), a non-stationary discrete[2] process or a continuous[3] process can be written as equation (1). Priestley and Tong (1973) extend the theory of the evolutionary spectral analysis of Priestley (1965–1966), presented in detail by Ftiti (2010), to the case of a bivariate non-stationary process. In this sub-section, we summarize this theory. Consider, for example, a bivariate continuous parameter process $(x(t), y(t))$ in which each component is an oscillatory process. Each component can be written as follows:

$$X_t = \int_{-\pi}^{\pi} A_X(w,t)\ e^{iwt}\ dZ_x(w) \qquad (1)$$

$$Y_t = \int_{-\pi}^{\pi} A_Y(w,t)\ e^{iwt}\ dZ_y(w) \qquad (2)$$

With

[1]The Fourier transformation requires that the studied time series is periodic and assumed not to evolve over time. For example, Fan and Gençay (2010, p. 1307) state, "The Fourier approach is appealing when working with stationary time series. However, restricting ourselves to stationary time series is not appealing, since most economic/financial time series exhibit quite complicated patterns over time (e.g., trends, abrupt changes, and volatility clustering). In fact, if the frequency components are not stationary such that they may appear, disappear, and then reappear over time, traditional spectral tools may miss such frequency components. Wavelet filters provide a natural platform to deal with the time-varying characteristics found in most real-world time series, and thus the assumption of stationarity may be avoided. The wavelet transform intelligently adapts itself to capture features across a wide range of frequencies and thus is able to capture events that are local in time. This makes the wavelet transform an ideal tool for studying non-stationary time series."

[2]A discrete process corresponds to a process of which the value of T is countable. Indeed, a time series is considered as a discrete process.

[3]A continuous process is a process used to describe the physical signal.

$$E[dZ_x(w_1)dZ_x^*(w_2)] = E[dZ_y(w_1)dZ_y^*(w_2)] = E[dZ_x(w_1)dZ_y^*(w_2)] = 0$$

And for $w_1 = w_2$

$$E[|dZ_x(w_1)|^2] = d\mu xx(w_1);\ E[|dZ_y(w_1)|^2] = d\mu yy(w_1)\ ;$$

$$E[dZ_x(w_1)dZ_y^*(w_1)] = d\mu xy(w_1)$$

with $[.]^*$ denoting the conjugate function of $[.]$.
Let F_x, F_y denote respectively the families of oscillatory functions as $\{\varphi_{t,x}(w_1) = A_{t,x}(w_1)e^{iw_1t}\}, \{\varphi_{t,y}(w_1) = A_{t,y}(w_1)e^{iw_1t}\}$. Priestley and Tong (1973) define the evolutionary power cross-spectrum at time t with respect to the families $F_x, F_y, dH_{t,xy}$ by

$$dH_{t,xy}(w) = A_{t,x}(w)A_{t,x}^*d\mu_{xy}(w) \qquad (3)$$

Further, if x_t, y_t is a bivariate stationary process, so that F_x and F_y may be chosen to be the family of complex exponentials; namely, $F_x = F_y = \{e^{iwt}\}$, $dH_{t,xy}$ reduces to the classical definition of the cross-spectrum. Thus, for each t, we may write:

$$dH_{t,xy}(w) = E\left[A_{t,x}(w)dZ_xA_{t,y}^*dZ_y^*(w)\right] \qquad (4)$$

Priestley and Tong (1973) extend the above relation to the case of a non-stationary bivariate process, where the amplitudes are time-dependent; correspondingly, the cross-spectrum is also time-dependent. Clearly, $dH_{t,xy}(w)$ is complex-valued, and, by virtue of the Cauchy–Schwarz equality, we obtain:

$$\left|dH_{t,xy}\right|^2 \le dH_{t,xx}\,dH_{t,yy}\ ;\ \ for\ all\ t\ and\ w \qquad (5)$$

If the measure μ_{xy} is absolutely continuous with respect to the Lebesgue measure, we can write, for each t:

$$dH_{t,xy} = h_{t,xy}$$

where $h_{t,xy}$ may then be termed the evolutionary auto-spectral density function.

3.1.2. Estimation of the evolutionary co-spectral approach

In the above analysis, we have presented a summary of the evolutionary cross-spectral density theory. For this theory, Priestley and Tong (1973) show a way to estimate the evolutionary cross-spectral density function, which we developed here, consisting on an extension from the estimation of the evolutionary spectral density function in the univariate case, such as developed by Priestley (1965–1966). We analyze two series of stock market indexes of Europe and the U.S. Therefore, we detail hereafter the procedure to estimate the evolutionary cross-spectral density function.

According to Priestley and Tong (1973), if a non-stationary discrete bivariate process $[x(t), y(t)]$ has the Gramer representation for each $x(t), y(t)$ and respects the following condition $-\pi < w < \pi$, we thus have

$$X_t = \int_{-\pi}^{\pi} A_{t,x}\ e^{iwt}\ dZ_x(w) \quad and \qquad Y_t = \int_{-\pi}^{\pi} A_Y\ e^{iwt}\ dZ_y(w)$$

These equations respect all conditions of Eq.1 and Eq.2 cited above. The estimation of the evolutionary cross-spectral density function needs two filters. For the discrete univariate

process, Priestley (1966) gives two relevant windows. These are relevant filters, and they have been tested by several researchers such as Ahamada and Boutahar (2002), Ftiti (2010), and Bouchouicha and Ftiti (2012). For the discrete bivariate process, Priestley and Tong (1973) adopt the same choice that:

$$g_u = \begin{cases} \frac{1}{2\sqrt{h\pi}} \; if \; |u| \leq h \\ 0 \end{cases} \qquad W_v = \begin{cases} \frac{1}{T} \; if \; |v| \leq \frac{T}{2} \\ 0 \; if \; |v| > \frac{T}{2} \end{cases}$$

Then, the estimation of the evolutionary auto-spectral density function is as follows:

$$\hat{h}_{t,XY} = \sum_{v \in Z} W_i'(v) U_x(w, t-v) U_y(w, t-v)$$

With

$$U_x(w,t) = \sum_{u \in Z} g(u) X(t-u) e^{iw(t-u)} du$$

$$U_y(w,t) = \sum_{u \in Z} g(u) Y(t-u) e^{iw(t-u)} du$$

In this study, we let $h = 7$ and $T = 20$. We make the same choice[4] as do Artis et al. (1992), Priestley (1995), Ahamada and Boutahar (2002), Ahamada and Ben Aissa (2004), Essaadi and Boutahar (2008), and Ftiti and Essaadi (2008).

According to Priestley (1988), if we have $E(\hat{h}(w)) \approx h_t(w)$, $Var\left(\hat{h}(w)\right)$ decreases when T increases $\forall (t_1, t_2), \forall (w_1, w_2), cov(\hat{h}_{t1}(w_1), \hat{h}_{t1}(w_2)) = 0$, if at least one of the conditions (i) or (ii) is satisfied.

(i): $|w \pm w'|$ are wide enough such that $|w_2 \pm w_2| \gg$ to the band width $|H(w)|^2$.

(ii): $|S - t|$ is broader than the function of $[w(u)]$.

In order to respect conditions (i) and (ii), we choose$[t_i]$ and $[w_i]$ as follows:

$t_i = [18 + 20i]_{i=1}^{I}$ where $I = \left|\frac{T}{20}\right|$ and T is the sample size.

$$w_j = \left\{\frac{\pi}{20}(1 + 3(j-1))\right\}_{j=1}^{7}$$

To respect the (*ii*) condition, we check instability in these frequencies: $\frac{\pi}{20}, \frac{5\pi}{20}, \frac{7\pi}{20}, \frac{10\pi}{20}, \frac{13\pi}{20}, \frac{16\pi}{20}, \frac{19\pi}{20}$.

We finally have a co-spectral density function in seven frequencies. However, we retain only two frequencies, thus reflecting, respectively, the short term (high frequencies; $\frac{10\pi}{20}$), medium term (medium frequency; $\frac{4\pi}{20}$) and long-term (low frequency, $\frac{\pi}{20}$). Indeed, the first frequency $\frac{\pi}{20}$ traduces the medium-term interdependence (traducing the contagion character; low frequency), and the frequency $\frac{4\pi}{20}$ traduces the medium-term interdependence. The shift from

[4] This choice of values is justified by the fact that they respect the conditions (i) and (ii)

the frequency domain to the time domain occurs through the following formula, $\frac{2\pi}{\lambda}$, where λ is the frequency.

In our study, we have chosen to investigate the following frequencies: the frequency $\frac{\pi}{20}$ corresponds to $\frac{2\pi}{\frac{\pi}{20}}$ months, which = 3 years and one quarter, whereas $\frac{2\pi}{\frac{10\pi}{20}}$ refers to the 10 month time frame, and the frequency $\frac{10\pi}{20}$, referring to 1 year time frame.

3.1.3 The coherence function from the evolutionary co-spectral approach

According to Priestley and Tong (1973), the evolutionary cross-spectral density function may be written as:

$$h_{t,XY}(w) = C_{t,XY} - iQ_{t,XY}(w) \qquad (11)$$

$$C_{t,XY} = \mathcal{R}\{h_{XY}(w_j, t)\}$$

$$Q_{t,XY} = Im\{h_{XY}(w_j, t)\}, \qquad (12)$$

where the real-valued functions $C_{t,XY}$ and $Q_{t,XY}$ are termed the evolutionary co-spectrum and the evolutionary quadrature spectrum, respectively. If the measures $\mu_{xx}(w)$ and $\mu_{yy}(w)$ are absolutely continuous, Priestley and Tong (1973) similarly define the evolutionary auto-spectral density functions, $h_{xx}(w_j, t)$, $h_{yy}(w_j, t)$.[5] The coherency function is defined by the following expression:

$$C_{t,XY} = \frac{|h_{t,XY}(w)|}{\{h_{t,XX}(w)h_{t,YY}(w)\}^{\frac{1}{2}}} = \frac{|\mathcal{E}[dZ_Y(w)\, dZ_y^*]|}{[\mathcal{E}|dZ_X(w)|^2]^{\frac{1}{2}}\ \mathcal{E}|dZ_Y(w)|^2]^{\frac{1}{2}}} \qquad (13)$$

Priestley and Tong (1973) interpret $C_{t,XY}(w)$ as the modulus of the correlation coefficient between $dZ_x(w), dZ_Y(w)$ or, more generally, as a measure of the linear relationship between the corresponding components at frequency w in the processes $\{Y(t)\}$ and $\{X(t)\}$.

The estimation of the coherency function is based on the estimation of the cross-spectral density function between two processes, $\{Y(t)\}$ and $\{X(t)\}$, and the estimation of the auto-spectral density function of each process. Thus, the estimation coherency is written as:

$$C_{t,XY}(w) = \frac{|\hat{h}_{t,XY}|}{\{\hat{h}_{t,XX}\hat{h}_{t,YY}\}^{1/2}} \qquad (14)$$

3.2 Wavelet analysis: theory and estimation

The choice of wavelets involves several considerations, such as real versus complex wavelets, continuous or discrete wavelets, and orthogonal versus redundant decompositions. Discrete wavelets are advantageous because of their fast implementation. Their advantages are related to the data decomposition of many variables at the same time. In addition, they use a limited number of translated and dilated versions of the mother wavelet and do not generate redundant information (Benhmad 2012). However, the number of scales and the time invariant property strongly depend on the data length. The continuous wavelets yield a

[5]For more detail, see Ftiti (2010).

redundant decomposition, but they have properties that may be more suitable to noise than those of other decomposition techniques.[6] Continuous and complex wavelets are the best choices for analyzing the phase interactions between two time series. Among the continuous wavelets the ‘‘Mexican hat’’ and the ‘‘Morlet wavelet’’ are very much popular. The choice of the continuous wavelet has been found to influence the time and the scale resolution of the decomposition. While the Morlet wavelet is very well localized in scales and in frequencies, the Mexican hat wavelet has a good time localization (it can isolate a single bump), but gives a poor frequency localization. Thus, we apply the Morlet wavelet.

The conventional Fourier transformation involves the application of sine and cosine base functions for the transformation of a series into an orthonormal set of trigonometric components. These sine and cosine base functions are characterized by infinite energy and finite power; hence, they eliminate the time dependency of any signal. The Fourier transformation, thus, provides no information about the time evolution of signal's spectral characteristics. Circumventing the limitation of the Fourier transformation, the windowed Fourier transformation (WFT) has been suggested. It involves the application of the Fourier transformation within a short-time window that remains constant across frequencies. Therefore, the WFT treats a signal under fixed time-frequency window with constant intervals in the time and frequency domains, ignoring adequate resolution for all frequencies (Rua 2010).

Contrasted to it, the wavelet transformation adjusts the time resolution to the frequency; narrows down the window width with high frequencies and widens when dealing with low frequencies. It makes use of local base functions that can be translated and stretched into both time and frequency. Moreover, the wavelets are characterized by finite energy such that they grow and die out within a period.

Mathematically, the wavelets are defined as:

$$\Psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\Psi\left(\frac{t-\tau}{s}\right), \qquad (15)$$

where τ is the translation parameter; s is the dilation parameter; $\frac{1}{\sqrt{s}}$ is a normalization factor; and $\Psi_{\tau,s}(t)$ are elementary functions obtained by decomposition of a time series through wavelet transformation and are derived from a time-localized mother wavelet $\psi(t)$ (For example, see, Percival and Walden, 2002).

The convolution of the continuous wavelet transformation (CWT) of a time $x(t)$series with respect to $\Psi(t)$ is given by:

$$W_x(\tau,s) = \int_{-\infty}^{+\infty} x(t)\,\Psi^*_{\tau,s}(t)dt = \frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} x(t)\,\Psi\left(\frac{t-\tau}{s}\right)dt, \qquad (16)$$

where, * denotes the complex conjugate. To recover the original series $x(t)$ from its wavelet transformation, the inverse wavelet transformation is usefully represented as:

[6] Discrete wavelet transformations (DWT) and maximal overlap discrete wavelet transformations (MODWT) are also available choices. However, DWT requests that data be of the power 2 and has other limitations that are overcome in the MODWT. For the MODWT, most software allows the decomposition of data at max up to level 12. Thus, if someone is interested beyond level 12 decomposition, he or she needs to work hard, and "it is also difficult to exactly identify the point of time in which a variable leads or lags; which makes tough task for policy analysts to understand the exact underlying lead-lag phenomenon between the variables" Tiwari (2013). Further, the variation in the time series data that we may obtain by utilizing any method of discrete wavelet transformation at each scale can be more easily obtained with continuous transformation.

$$x(t) = \frac{1}{C_\psi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} W_x(\tau, s) \Psi_{\tau,s}(t) \frac{d\tau\, ds}{s^2} \quad (17)$$

Moreover, several interesting quantities can be captured within the wavelet domain. The measure of wavelet power spectrum that captures the relative contribution at each time and at each scale of the time series' variance is defined as $|W_x(\tau, s)|^2$. The total variance of the series can be obtained taking integration across τ and s is given by:

$$\sigma_x^2 = \frac{1}{C_\psi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |W_x(\tau, s)|^2 \frac{d\tau\, ds}{s^2} \quad (18)$$

Further, the measure of cross-wavelet spectrum that captures the covariance between two time series [say, $x(t)$ and $y(t)$] in the time-frequency space [$W_x(\tau, s)$ and $W_y(\tau, s)$] is given as:

$$W_{xy}(\tau, s) = W_x(\tau, s)\, W_y^*(\tau, s) \quad (19)$$

In line with equations 13 and 14 presented below and in following Croux et al. (2001), the cross-wavelet spectrum can be decomposed into real and imaginary components defined as:

$$\rho_{xy}(\tau, s) = \frac{\Re W_{xy}(\tau, s)}{\sqrt{|W_x(\tau, s)|^2 |W_y(\tau, s)|^2}}, \quad (20)$$

where $\Re$ measures the contemporaneous variance and corresponds to the real part of the cross-wavelet spectrum. The wavelet $\rho_{xy}(\tau, s)$ quantifies the co-movement in the time-frequency space and identifies the time-frequency period over which the co-movement is higher. Basically, the cross-wavelet spectrum acts as a contemporaneous correlation coefficient around each moment in time and for each frequency (Rua 2010).

The marked feature of the cross-wavelet spectrum lies with its ability to provide information about the co-movement, both at frequency and over time. Moreover, an assessment of the contour plot of the wavelet cross spectrum permits the identification of the time-frequency regions over which the two series commove, as well as the features of the time and frequency variation of the co-movement. The suggested wavelet-based measure, hence, enriches the analysis of the co-movement between a set of variables.

3.3 Data

To study the dependence and/or contagion of either the U.S. or Europe and the main OECD countries (fourteen), we use monthly data for stock market indices, computed as the log first difference of price index. Table 1 represents our principal descriptive statistics. It reveals the divergence between the level and the volatility of stock market indexes. For this reason, we take all data in logarithmic form.

To select the sample, we have adopted two criteria: i) we retain the main industrial OECD countries; and ii) some OECD financial markets. The final sample consists of Australia, Austria, Belgium, Denmark, France, Finland, Germany, Greece, the Netherlands, Norway, Portugal, Spain, Sweden, and Italy.

The stock market returns data for our selected countries are drawn from Morgan Stanley Capital International indexes (MSCI). These broad-based market return indexes are value weighted and calculated with dividend reinvestment. The MSCI international indexes are composed of stocks that broadly represent stock composition in the different countries and

regions. The data range from 31/01/1990 to 31/07/2013 and have been extracted from MSCI database.

The time horizon depends on data availability and includes, in addition to the major economic crises, such as the monetary and financial crises of Asia and Latin America and so on. This design allows the derivation of important conclusions regarding the links of the dynamics of stock market returns.

Note that for the evolutionary spectral estimation necessity, we lose ten observations at the beginning and at the end. Therefore, the estimated evolutionary co-spectral density functions of stock markets for our countries' samples term from 31/10/1990 to 31/09/2012. For the wavelet analysis, estimations cover 31/01/1990 to 31/07/2013. For the evolutionary co-spectral density functions, the authors used the MATLAB code developed by Ftiti (2010) and for the wavelet approaches, they employed the MATLAB code, developed by Grinsted et al. (2004).[7] In addition, we have used the MATKAB software to create all graphics.

Table 1 Descriptive statistics of stock markets indexes sample.

Countries	Mean	Min	Max	Standard Deviation	Skewness	Kurtosis
Australia	204.90	79.87	487.17	107.47	0.76	2.11
Austria	1287.85	618.19	3623.77	719.49	1.80	5.22
Belgium	1126.13	533.26	2455.72	450.54	1.01	3.51
Denmark	2686.15	708.46	6591.79	1762.74	0.74	2.13
Finland	385.94	37.86	1239.75	266.84	0.86	3.44
France	1154.94	439.81	2350.36	474.5	0.38	2.43
Germany	1209.60	498.51	2520.74	483.73	0.43	2.49
Greece	384.07	60.50	1040.20	224.28	0.95	3.14
Italy	335.21	138.50	686.83	130.41	0.79	2.82
Norway	1706.75	570.97	4614.01	990.69	1.02	2.98
Netherland	1602.56	569.53	3036.033	601.13	-0.05	2.30
Portugal	112.03	50.06	242.49	42.18	0.88	3.45
Spain	357.04	104.27	894.64	190.70	0.67	2.82
Sweden	3653.51	770.87	7781.41	2084.81	033	1.84
USA	308.39	93.02	531.49	121.36	-0.35	1.82
Europe	219.89	87.76	438.86	85.04	0.23	2.39

4. Results and discussion

4.1 Results of the evolutionary co-spectral analysis

We first use the ESA approach proposed by Priestley and Tong (1973) in order to analyze the dynamics of co-movement with respect to time and frequencies. We apply the methodology proposed in section 3.1 to estimate the equation (14) for the U.S. market and for that of Europe's main OECD countries.

[7] We are grateful to Grinsted and his coauthors for making their codes available at http://www.pol.ac.uk/home/research/waveletcoherence/, which was utilized in the present study. The Matlab code for Rua (2010) was developed by Tiwari and Olayeni (2013). All codes are available upon request to the corresponding author or Tiwari.

Figure 1 presents several graphics that describe the pattern of the co-movement measured through ESA for all countries. The right hand side displays the graphics of co-movement of each country with the U.S., while the left hand side is devoted to co-movement with Europe.

Fig.1 The dynamic co-movement of some OECD stock markets based on evolutionary spectral analysis during the period 1990-2010

The results, based on ESA, show several interesting findings. There is, at first, a difference between the co-movement in the short-term, the medium-term, and the long-term. Concerning the long-term co-movement, our results show that we have three kinds of countries. The first kind experiences a very higher coherence level during financial crisis (on average more than 50%), such as the case of Sweden, Finland,[8] Greece, Portugal, and Spain.[9] However, during quiet periods, their coherence level is around 25%. The second kind of country includes Australia, Austria, Belgium, Denmark, Norway, France, and Italy. Their coherence function has the same behavior as the first kind of country during financial crisis. However, this interdependence vanishes in quiet periods.[10] The last kind of country consists of Germany and Netherland. These countries have the lowest coherence level, during financial crash- around 30%- whereas the rest of countries exhibit more than 50%. In the quiet period, this interdependence disappears (less than 10%). In other words, Germany and Netherland have a very low sensitivity to the US and Europe financial shocks.

Concerning the short-term coherence function (high frequency=contagion), figure 1 notes a great difference pattern from the long-term co-movement between stock markets indexes of, one hand, player countries and, on the other hand, the US and Europe. The results of the short-term coherence show several interesting findings. First, we observe a higher level in the short-term especially in period of financial turmoil compared to the long-term pattern; in other words, the contagion character is identified through our analysis. Secondly, we note that our sample countries are more contagious to Europe shocks than the US ones. Third, empirical findings highlight three different co-movement patterns. The first one includes Australia, Austria France, Finland, and Sweden. For this kind, the coherence between stock market return index and USA and Europe stock market return indexes is very higher (around 60%-70%) during all financial turmoil such as in 1991 (War Gulf), 1997 (Asian crisis), 2000 (USA housing bubble), 2007 (Subprime crisis), and 2010 (sovereign debt crisis). The second kind includes Portugal, Spain, Italy, and Greece. These countries have the same peaks as the first kind but with a higher level (around 70%-80%). The third kind of country includes Belgium, Demark, Germany, the Netherlands, and Norway. This group has less sensitivity to financial turmoil. Indeed, we do not observe a higher coherence, other than at beginning of 1990s, the Asian crisis (1997), and the sovereign debt crisis (2010).

The above empirical results emphasize that the contagion character of OECD countries. Nevertheless, the contagion degree varies among them. The second group is more sensitive to European and American shocks than the first one, whereas the third group has a lower sensitivity than the other groups.

To summarize, the ESA approach enables the identification of contagion and interdependence among the stock market indexes of industrial OECD countries and those of the U.S. and Europe. The ESA offers us the opportunity to study the coherence function across both

[8]Sweden and Finland had experiences a large banking crisis respectively (1990-1993) and (1991-1993).
[9]For more details, see for example, 1(o), and 1 (p).
[10]For more details see for example, 1(u), and 1 (t).

frequencies and over time. Our results seem very important and significant. Indeed, we show that Sweden, Finland, Greece, Portugal, and Spain have a long-term dependence on Europe and the U.S. This result indicates the relative openness of the stock market of these countries in terms of foreign participation and international portfolio flows. Moreover, second group's (Australia, Austria, Belgium, Denmark, Norway, France, and Italy) long-term dependence on the U.S. and Europe increased only in periods of financial turmoil. This result is explained by the fact that the fundamentals that drive the co-movement pattern are affected during periods of financial turmoil. Loosely speaking, we prove that the long-term co-movement, traducing the nature of interdependence, was specifically raised at the beginning of the 1990s and during the subprime crisis in 2007. This result shows that the period of great moderation at the beginning of 1990s was accompanied by financial crises (the War Gulf, the Swedish crisis, the Finnish crisis, etc.). This interdependence differs by country. The empirical results reveal that some countries (for example, Germany and the Netherlands) are less sensitive to permanent shocks than are other countries (Greece, Portugal, Australia, and so on).

Our results have another important meaning, one that concerns short-term co-movements. The short-term co-movement assesses the contagion character of different studied markets. We identify the contagious character of all markets with some national differences. For instance, our results show that Greece, Spain, Portugal, and Italy are more sensitive (contagion) to the American and European transitory shocks. This result is significant. It justifies the actual difficult economic and financial situations of these countries. In addition, our result show that Germany, Netherland, Denmark, and Norway are not contaminated (no contagion) to American and European transitory shocks during financial turmoil.

All these results have important economic implications, especially, from asset allocation, risk management, and portfolio diversification. Indeed, distinguishing between excessive and normal co-movements, especially during periods of high volatility, is worthwhile and has profound implications for both the asset allocation strategies for investors and the crisis management policy of policymakers. The usefulness of hedging strategies may substantially differ over normal and turbulent periods, along with the variation of the correlation pattern of financial time series. International diversification may be markedly less valuable in embarrassing moments than in calm situations, whereas its benefits are particularly needed to compensate potential losses. Investors may derive important information about countries that are less sensitive to permanent shocks and others that are less sensitive to transitory shocks. From a portfolio management perspective, there are enhanced benefits, at least statistically, from combining Europe and market equities with global equities in the short-term relative to the long term. From a practical viewpoint, however, it is important for fund managers to ascertain if the noted changes in market co-movements are driven by fundamentals and therefore likely to be permanent, or if they are temporary.

4.2 Results of Wavelet analysis

In the empirical applications of CWT analysis, most of studies use the Morlet wavelet. This is explained by many reasons. It yields a high frequency resolution because it is very well localized in scales and in frequencies. We carry out the robustness check on the choice of the wavelet function and adopt the Paul wavelet for this purpose. Both the Morlet and the Paul wavelets are complex functions, unlike the Gaussian wavelet that is a real function. The choice of the Paul wavelet ensures that any difference is not wrongly adduced to the complex nature of the functions. Rather, it puts them on the same footing to ensure a robust

comparison of the results. The plots of the robustness results based on the Paul wavelet are reported in Figure 4 in the appendix**.** Broadly speaking, the conclusions previous conclusions are still tenable, although there are pockets of differences in the distribution of the significant islands. One possible explanation lies in the method rather than in the data. As noted in Torrence and Compo (1998, p.76), the Paul wavelet can give poorer frequency localization.

First, we assess the co-movement of the return series of stock indexes between either the U.S. or Europe and the main OECD stock markets. We begin by plotting the wavelet power spectrum of the return series of stock indices of all the countries in Figure 2. The plots of the wavelet power spectrum of the return series of stock indexes put in evidence the common signals (in terms of common islands) i.e., red within the thick black contour. Moreover, these signals, i.e., red within the thick black contour, reveal a high degree of volatility and jump across frequencies and over time.

Fig.2: Wavelet representation of Stock markets returns of our selected countries during the period 1990-2010.

Second, we apply the wavelet-based measure of cohesion proposed by Rua (2010) in order to understand the dynamics of co-movement with respect to time and frequency. We estimate the equation (20) for major stock market players, the U.S., and Europe. The empirical results are exhibited in Figure 3.[11] The vertical axis measures the frequency (expressed in terms of periods such that period=1/frequency), while the horizontal axis is linked to time in terms of years. The color scale measures cohesion and ranges of from -1 to +1. The deep blue and the deep red respectively reveal perfect negative and the perfect positive cohesion. The identification of time and frequency varying coherence is achieved by inspecting the contour plot. One can, therefore, identify both frequency bands (on the vertical axis) and time intervals (on the horizontal axis), where the uncertainty indices are synchronized.[12]

Fig.3. The cohesion of some OECD stock markets and those of the U.S and Europe during the period 1990-2010

The empirical results roughly affirm the findings of a spectral based approach. They first emphasize the high degree of cohesion at higher frequencies (i.e., one month to one year scale), intermediate frequencies (i.e., 1~2 years scale), and lower frequencies (2~8 years scale), that is, the short-term, intermediate and long-term year-scales when the cohesion of ~~a~~ country's stock index return is calculated with that of either the U.S. or Europe. Moreover, the results stress that that our sample countries are more contagious to European shocks than to those of the U.S. Moreover, there is a high degree of positive cohesion between our sample countries and Europe and the U.S., especially after 2005 (i.e., the period of the subprime crisis of 2007 and the sovereign debt crisis of 2010) and on intermediate and higher year-scales. The empirical results finally show either a zero or a negative cohesion of the sample countries with both the U.S. and Europe before the Asian crisis (1997). In order to gain a better understanding of the results of the wavelet analysis and compare these with those of the ESA analysis, we classify the co-movement into the short-term, medium-term and long term.

[11]This is because three dimensions (time, frequency and cohesion) cannot be presented on a two dimensional figure.

[12] It should be noted that for policy purposes, the results for higher frequencies should be treated with caution because they are typically noisy and, as such, rather uninformative. However, this limit is resolved in the ESA, where we have an unbiased estimation of the short-term frequency.

Thereafter for each classification we group countries into high cohesion, moderate cohesion, and low cohesion groups during the entire study period.
Let us first discuss the short-term case, i.e., case of the one month to one year scale. In this case, the first kind of country experiences a very higher degree of positive correlation (close to one) when analyzed with Europe,[13] especially after 2000; such countries include Italy, France, Germany, the Netherlands, and Spain, whereas Austria and Sweden, experienced the same after 2005 (i.e., the period of the subprime crisis of 2007 and the sovereign debt crisis of 2010). The second kind of country, which experienced a positive correlation (i.e., $0.4 < r < 0.8$) during the period under study (barring some exceptions) includes Australia, Austria, Belgium, Denmark, Finland, Germany, Norway, Portugal, Spain, and Sweden. The last kind of nation, which have experienced a reasonably high negative correlation in about 1992-1994, consists of Italy, Austria, Finland, Greece, and Sweden.
Our general observations show that there is not much difference between intermediate frequencies (i.e., 1~2 years scale) and lower frequencies (2~8 years scale), barring some exceptions at some particular frequencies for some countries; thus, we can analyze this set in a single setting. In this case, first kind of country experiences a very higher degree of positive correlation (close to one) when analyzed with Europe after 2000, as with Italy, France, Denmark, Germany, the Netherlands, Portugal, Spain; however, Austria, Belgium, and Sweden experienced the same correlation after 2005. The second kind of country that experienced a positive correlation (i.e., $0.4 < r < 0.8$) during the period under study (barring some exceptions) includes Australia, Austria, Belgium, Denmark, Finland, and Greece, and Norway (at the 1~4 year scale after 2005 and at the 4~8 year scale during entire study period). The last kind of country, which experienced a mild negative or zero correlation in about 1992-1994, consists of France, Belgium, Greece, Norway, and Australia.

4.2.1 Robustness checking (Paul wavelet)

To test the robustness of the findings obtained from wavelet analysis, we used Paul as an alternative and placed the results in Figure 4.
The results of the Paul wavelet analysis, in general, confirm the findings obtained from the Morlet wavelet based analysis. For example, if we look at the figures carefully, we find that, irrespective of time-scales, France, Denmark, Germany, the Netherlands, Portugal, Spain, and Sweden exhibit a very high degree of positive correlation with Europe, which reaches close to one, especially after 2005. However, Italy, Finland, Greece, Norway, Portugal, Spain exhibit negative or zero correlations on the one month to one year scale, but Austria, Belgium, Greece, Norway, and Australia exhibit the same on the intermediate or long-term scales.

Fig.4. The cohesion between some OECD stock markets with the U.S and Europe (Paul wavelet) during the period 1990-2010

[13] One should note that if we analyse the correlation with the U.S., we find similar results as with Europe in terms of country classification. The only difference is that if a country is less correlated Europe it may be very less (or may be negatively) correlated with the U.S., and if a country is very highly correlated with Europe, it may be highly correlated with USA.

5. Conclusion

Distinguishing between interdependence and contagion is a crucial issue from a portfolio diversification perspective. It has profound implications for both the asset allocation strategies of investors and the crisis management policies of policymakers. In fact, the usefulness of hedging strategies may substantially differ over normal and turbulent periods, along with the variation of the correlation patterns of financial time series. International diversification may be markedly less valuable in embarrassing moments than in calm situations, whereas its benefits are particularly needed to compensate for potential losses.

This study investigated the co-movements of the U.S. and Europe and fourteen OECD countries from 1990-2013 and addressed the following questions: First, is there a co-movement across stock market indexes; and second, is does this co-movement hold in the short-term or the long-term? The study contributes to the research topic by applying two sophisticated and advantageous approaches, namely, the ESA and wavelet analysis. These techniques not only allow one to distinguish between interdependence and contagion, but they above all assess the co-movement between time series with respect to time and frequency. We demonstrated that long-term co-movement, related to the interdependence of stock markets indexes, was specifically raised during the beginning of the 1990s and during the subprime crisis of 2007. Such a result highlights the influence of the period of great moderation at the beginning of the 1990s, which was accompanied by some financial crises, such as the War Gulf, the Swedish crisis, the Finnish crisis, and so on. This interdependence differs across countries. We showed that some countries, such as like Germany and the Netherlands, were less sensitive to permanent shocks than other countries, including Greece, Portugal, and Australia. Our results have another important finding concerning short-term co-movements. The short-term co-movement assesses the contagion character of various stock markets indexes. We identified the contagious character of all the markets studies, with some differences across countries. The empirical findings highlight that Greece, Spain, Portugal, and Italy, for example, are more sensitive to transitory shocks than the U.S. and Europe.

This study has significant economic implications. Indeed, distinguishing between excessive and normal co-movements is a crucial issue from a portfolio diversification perspective, especially during periods of high volatility. Investors can derive important information about countries that are less sensitive to permanent shocks and those that are less sensitive to transitory one; it helps them develop rational investment strategies and select optimal portfolios. Similarly, policymakers can draw worthwhile hedging strategies and adopt rational crisis management policies.

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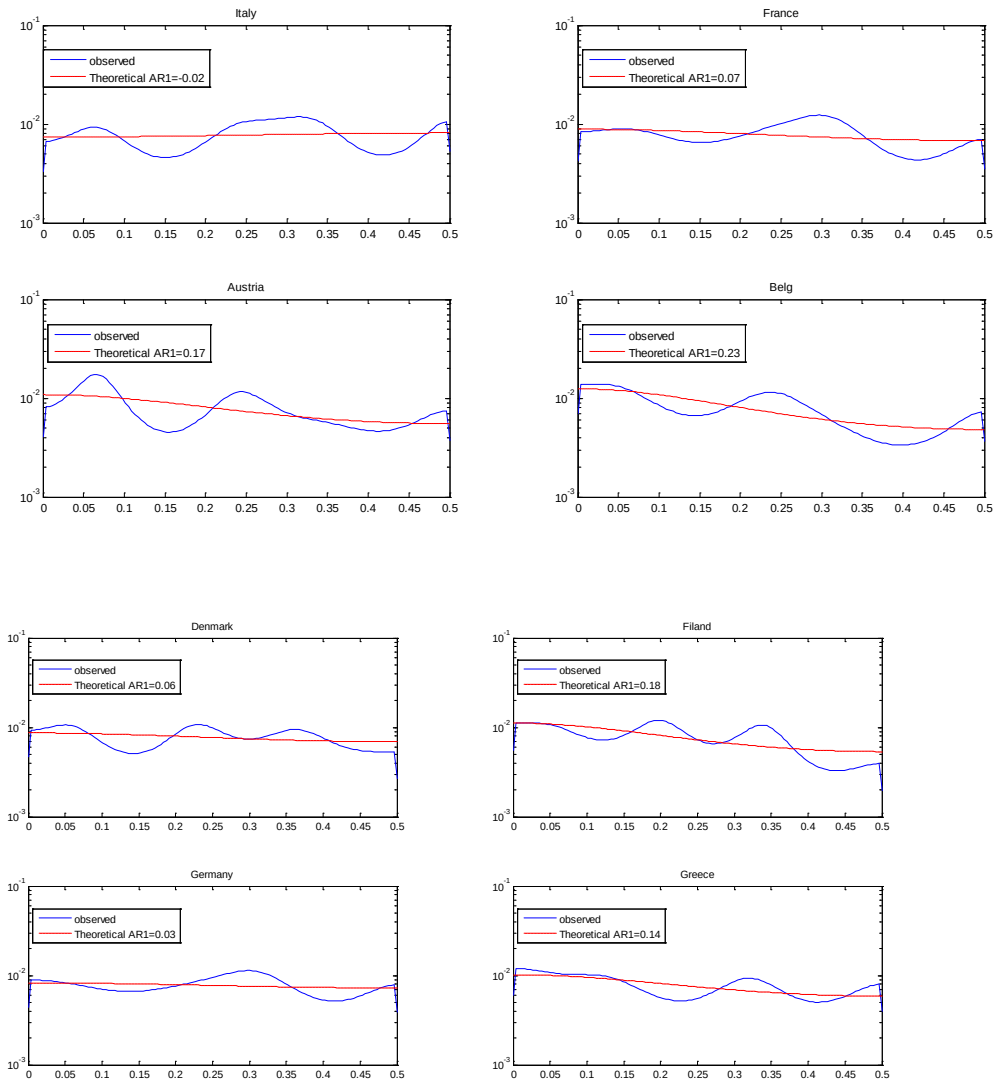
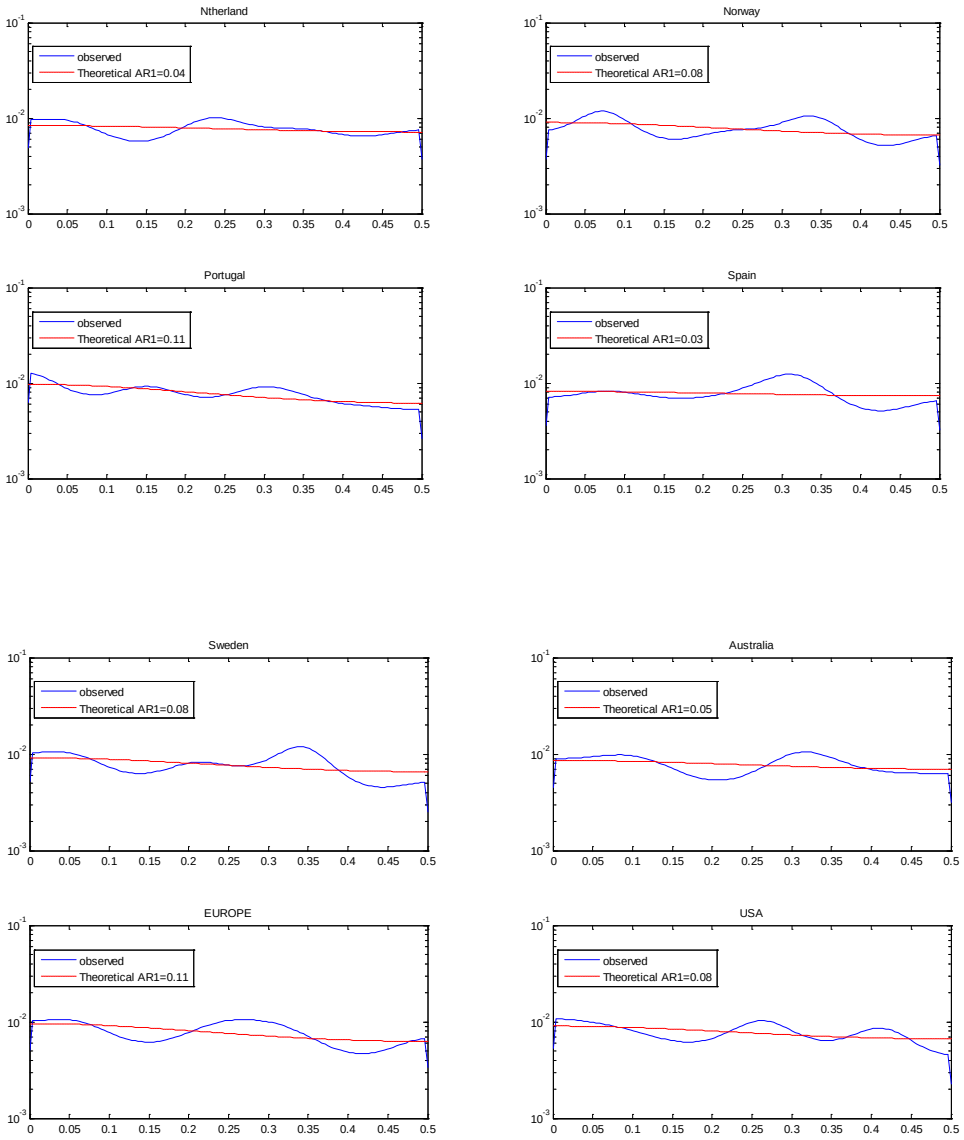
Appendix

Significance level and background noise of the distribution

The statistical significance of the wavelet power spectrum of the observed time series can be assessed relative to the null hypothesis that the signal generating the process is stationary. Since the series in this study cannot be said to be stationary at level, stationarity is induced for analysis. This transformation ensures that the observed time series is normal and can be modeled as a Gaussian AR (1) process. We assume that the null hypothesis for the power spectrum is normally distributed as an AR (1) process. This assumption affects the acceptance of null hypothesis for the power spectrum of each time series or for the co-spectrum of any two time series, as well as their coherence. The color of the noise is important for both the spectrum and the co-spectrum, while the wavelet coherence is insensitive to the choice of color. Figure A1 shows that red noise is an appropriate background to test against, the theoretical AR1 spectrum for power decay closely matches the observed spectrum.

Fig. A1: Plots of observed and theoretical AR1 spectrum for stock market returns





In what follows, we choose to work with the red noise process, given that the observed data were stationarity, but we also investigate the implication of red noise for the null hypothesis. The following simple AR (1) model will serve to illustrate the difference between white and red noise:

$$y_t = m + \alpha y_{t-1} + \varepsilon_t \quad , \tag{1}$$

where $y_0 = 0$, m is a constant, α is the autocorrelation coefficient, and $\varepsilon_t \sim N(0, \sigma^2)$. The white noise model is implied by setting $m = 0$ and $\alpha = 0$ (that is, $y_t = \varepsilon_t$) while the red noise results by setting $m = 0$ and $\alpha \to 1$. For the red noise, the Fourier power spectrum is given by:

$$P_k = \frac{1-\alpha^2}{1+\alpha^2 - 2\alpha\cos\left(\frac{2\pi k}{N}\right)} \quad , \tag{2}$$

where we see that $P_k = 1$ for white noise. Although Torrence and Compo (1998) show how to access the statistical significance of wavelet power against the null hypothesis that the data generating process is given by AR(0) or AR(1) stationary process with a certain background Fourier power spectrum, for more general processes one has to rely on Monte-Carlo simulations. Torrence and Compo (1998) compute the white noise and red noise wavelet power spectra, from which they derived, under the null, the corresponding distribution for the local wavelet power spectrum at each time *m* and for scale*s* as follows:

$$\frac{\left|W_m^x(s)\right|^2}{\sigma_X^2} \sim \frac{1}{2} P_k \chi_v^2, \tag{3}$$

where v is equal to 1 for real and 2 for complex wavelets. According to Torrence and Compo (1998), if two time-series have theoretical Fourier spectra P_k^X and P_k^Y as defined in equation (3), and are both χ^2 distributed, then the cross-wavelet distribution is given by (Torrence and Compo, 1998, p. 76)

$$\frac{\left|W_m^x(s)\tilde{W}_m^y(s)\right|}{\sigma_X \sigma_Y} \sim \frac{Z_v(p)}{v}\sqrt{P_k^X P_k^Y}, \tag{4}$$

where $Z_v(p)$ is the confidence level associated with the probability *p* for a probability density function defined by the square root of the product of two χ^2 distributions. In another context, Priestley (1981, p. 703) derives the asymptotic distribution of the *estimated* cross-amplitude power and shows that the asymptotic distribution depends on the coherence. In particular, he shows the variance of the estimated cross-amplitude power at frequency ω is:

$$\mathrm{var}\left\{\hat{\alpha}_{xy}(\omega)\right\} \sim \frac{C_W}{2N}\alpha_{xy}^2(\omega)\left\{1 + \frac{1}{\left|C_{xy}(\omega)\right|^2}\right\} \tag{5}$$

This result is an important demonstration of the relationship between the variability of the cross-amplitude estimate and the coherence of the series. It shows that at all frequencies where coherence is low, the estimate of the cross-amplitude may have an extremely large variance (Priestley, 1981, p. 703). We observe that this analogy may well be true of wavelet cross spectrum as well. Aside from this insight into the noted relationship, this conclusion has no damaging implication for the distribution in Equation (4) or for our results. To test the statistical significance of results, we make use of the Monte Carlo simulation approach. We specifically make use of ARMA (1, 0) background noise to imitate the red noise. Again, we must mention that wavelet coherence is insensitive to noise color and that the choice of background color may not affect the result reported for coherence.