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Abstract

This paper extends extant valuation models of interest rate swaps (IRS) with counterparty credit risk by accounting for wrong-way risk and OIS discounting. The proposed model extends Brigo and Pallavicini's (2007) and Ruiz et al.'s (2013) models, by capturing wrong-way risk in the CVA calculation by way of the correlation between the intensity of default of the counterparty and the market interest rate. Under the proposed no-arbitrage pricing model, cash flows are discounted using the OIS rates (mostly used by market practitioners following the 2007-2009 credit crisis), a proxy for risk-free rates. We therefore propose a unified framework that captures under one umbrella: CVA, wrong-way risk, and OIS discounting. The model parameters are estimated using real market data. Our findings indicate that it is important to account for both counterparty and wrong-way risk in IRS valuation since the two phenomena have non-negligible impacts on the CVA value. Also, using the OIS rates as risk-free discount rates, our model yields adjustment values higher than those obtained with the traditional Libor discount rates.

Keywords: Interest Rate Swap, Counterparty Credit Risk, Credit Value Adjustment (CVA), Wrong-Way Risk, Overnight Indexed Swap (OIS)

1. Introduction

The over-the-counter (OTC) market for interest rate derivatives has grown extensively over the past decades (the notional amount increased from less than USD 50 trillion in 1998 to more than USD 500 trillion at the end of May 2016)[1] and now offers a greater variety of products and maturities. Corporations and financial institutions frequently use interest rate derivatives to manage their directional exposures to interest rates movements. Interest rate swaps (IRS), used in this study, represent the biggest market share of interest rate derivatives.

The valuation of IRS has long being considered by practitioners as simple, and researchers and market participants had all agreed on the traditional standardized approach used for valuation. Since the subprime credit crisis of 2007-2009, the state of the market has radically changed and the standardized approach used by market practitioners to value interest rate derivatives has become obsolete and less reliable (Smith; 2013). Indeed, the latest subprime crisis severely affected the world economy and led to liquidity and credit crises for financial institutions and corporations. Market rates, which used to be highly correlated before the crisis, became incompatible with one another and now include different liquidity and credit spreads (e.g., Mercurio; 2009). The liquidity and credit premiums embedded in the different rates seem to have a considerable impact on market prices of financial instruments, including derivatives. Therefore, in the aftermath of the 2007-2009 subprime credit crisis, researchers and market participants have had to tackle these new valuation problems.

On the one hand, basis spreads, meaning differences in market rates for different maturities, observed on interest rates markets after the beginning of the crisis in 2007 indicate that the traditional valuation framework is less appropriate and must be revisited. To better evaluate interest rate underlying instruments, we must account for credit and liquidity risks of different tenors. Therefore, swap valuation has become increasingly

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[1] http://www.bis.org/statistics/derstats.htm; http://www.swapsinfo.org/charts/derivatives/notional-outstanding?date_start=2004-12-31&date_end=2016-06-06&gtrprod=1%2C2%2C4%2C5%2C10%2C11%2C14%2C16%2C8%2C17%2C3%2C6%2C13&type=&submit=Update+Data

complex with the new techniques introduced. We need to use more than one term structure of interest rates to determine the swap curve. Indeed, under the no-arbitrage argument, the valuation of a vanilla interest rate swap incorporates not only the differences between the posted rates such as the Libor rates, which have a credit risk component, but also an appropriate proxy for the risk-free rate used to discount the cash flows. This approach termed the dual curve, or the *Overnight Indexed Swap* (OIS)[2] discounting, radically changes the valuation approach from the traditional approach (e.g., Hull and White (2015) and Smith (2013)).

On the other hand, the existence of counterparty risk in interest rate derivatives transactions has been of interest since the beginning of the crisis. Before the crisis, many risky fixed-income securities (loans, mortgages, and corporate bonds) were issued, and hence constitute a large proportion of the demand for interest rate and credit derivatives contracts in the marketplace. At the end of 2008, AIG, the largest insurance company in the US, defaulted due to its large vulnerable portfolio of credit default swaps (CDS), bringing to the fore the lack of proper assessment of counterparty credit risk in these instruments. Managing counterparty credit risk is more than just adding an additional spread on interest rates and financial instruments prices; counterparty risk also affects the collateral and the decision-making process of a financial institution. Under the revised Basel II and the ongoing Basel III rules (BIS; 2006, 2011), banks have to revise their valuation approach to account for counterparty credit risk. The concept of *credit value adjustment* (CVA)[3] is now universally accepted and CVA is regularly computed by market participants (e.g., Brigo and Pallavicini (2007), Černý and Witzany (2015) and Hull and White (2012), among others). OTC transactions with potential counterparty credit risk now include a CVA component in the valuation calculation.

This paper proposes a new valuation approach for IRS that combines the calculation of the CVA and the use of OIS rates as a proxy for the risk-free discount rates. We build

[2] *Overnight Index Swap (OIS)* is a fixed interest rate swap against a floating rate tied to a daily overnight reference index, for example the Federal funds rate for the US market and EONIA (Euro OverNight Index Average) rate for the Euro market.

[3] *Credit Value Adjustment (CVA)* is the difference between the price of a credit vulnerable derivative product (or portfolio) and an equivalent product with no counterparty default. In other words, it is the market price of counterparty risk.

on existing frameworks, such as the ones by Brigo and Pallavicini (2007), for the calculation of CVA, and Ruiz et al.'s (2013) empirical approach to account for *wrong-way risk* (WWR).[4] Our objective is to propose a unified framework that uses the OIS discount as a proxy for risk-free discounting in the valuation of IRS with counterparty credit risk. In our framework, counterparty credit risk is captured by the CVA value and takes into account the probability of default of the counterparty, the level and volatility of the expected exposures, and the wrong-way risk.

The new proposed approach, which calculates CVA with OIS discounting for the IRS cash flows, is easy to calibrate with real data and is user-friendly. As expected, we find that it is important to account for both counterparty default and wrong-way risk in IRS valuation since the two phenomena have non-negligible impacts on the CVA value. Using the OIS rates as risk-free discount rates, our model yields adjustment values higher than those obtained with the traditional Libor discount rates.

The remainder of the paper is structured as follows. In section 2, we first provide the reader with a quick overview of the traditional IRS valuation approach used before the 2007-2009 subprime credit crisis. We then propose the new valuation framework. In section 3, we present the calibration exercises and the empirical results. We conclude in section 4.

2. The proposed valuation framework

2.1. Overview of the traditional valuation approach

The traditional approach to valuing interest rate swaps (IRS) before the 2007-2009 credit crisis was based on the construction of a unique yield curve used to build the term structure of interest rates and the discount factors. We present below a simple version of that traditional approach to value a vanilla IRS with a default-free counterpart.

We define by L_{t,T_n}, the Libor spot rate between t and T_n. This rate is the discount rate of a zero coupon bond valued at *t* with maturity T_n and is obtained as follows:

[4] *Wrong-Way Risk (WWR)* arrives when the value of a derivative contract and the probability of default of the counterparty are inadequately correlated.

$$L_{t,T_n} = \frac{1}{\delta_{t,n}}\left(\frac{1}{P_{t,T_n}} - 1\right), \tag{1}$$

where P_{t,T_n} refers to the risk-free discount factor and $\delta_{t,n}$ is the fraction of the year in the interval $[t, T_n]$. The forward Libor rate $F_{t,T_{n-1},T_n}$from T_{n-1} to T_n at time t is given by:

$$F_{t,T_{n-1},T_n} = \frac{1}{\delta_n}\left(\frac{P_{t,T_{n-1}}}{P_{t,T_n}} - 1\right). \tag{2}$$

An instrument based on the forward Libor rate is the Forward Rate Agreement (FRA). The payoff of a long position in the FRA with maturity T_n can be obtained by the difference between the spot Libor rate and the fixed rate Kas follows:

$$V_{T_n} = \delta_n\left(L_{T_{n-1},T_n} - K\right). \tag{3}$$

Hence, the value of the FRA at t is obtained by:

$$V_t = \delta_n\left(E_t\left[L_{T_{n-1},T_n}\right] - K\right)P_{t,T_n}, \tag{4}$$

where $E_t[.]$ is the conditional expectation operator at t. At the beginning of the contract, the FRA exercise rate K is the rate that sets the value of the contract at zero, i.e.:

$$K = E_t\left[L_{T_{n-1},T_n}\right] = F_{t,T_{n-1},T_n}. \tag{5}$$

Combining equations (2) and (5) yields:

$$F_{t,T_{n-1},T_n} = E_t\left[L_{T_{n-1},T_n}\right] = \frac{1}{\delta_n}\left(\frac{P_{t,T_{n-1}}}{P_{t,T_n}} - 1\right). \tag{6}$$

A vanilla IRS can be seen as a portfolio of forward rate agreements (FRAs) in which the two legs of the swap have to be equal at the initiation of the contract:

$$\underbrace{C_N \sum_{n=1}^{N} \Delta_n P_{t,T_n}}_{Cashflows\ of\ the\ fixed\ leg} = \underbrace{\sum_{n=1}^{N} \delta_n E_t\left[L_{T_{n-1},T_n}\right] P_{t,T_n}}_{Cashflows\ of\ the\ floating\ leg}, \tag{7}$$

where C_N is the nominal swap rate (the fixed coupon) of the IRS with N payments remaining at date t, Δ_n and δ_n are the proportion of the number of days for the fixed and floating legs, respectively. For simplicity, we assume that payments of the fixed and floating legs are done simultaneously, i.e., $\Delta_n = \delta_n$. Combining equations (6) and (7), we obtain:

$$C_N \sum_{n=1}^{N} \Delta_n P_{t,T_n} = \sum_{n=1}^{N} \Delta_n \left(\frac{1}{\Delta_n} \left(\frac{P_{t,T_{n-1}}}{P_{t,T_n}} - 1 \right) \right) P_{t,T_n}$$

$$= \sum_{n=1}^{N} \left(P_{t,T_{n-1}} - P_{t,T_n} \right)$$

$$= \left(P_{t,T_0} - P_{t,T_N} \right). \tag{8}$$

The right hand side of this equation can be seen as a combination of a long position in a zero-coupon bond with maturity T_0 and a short position in another zero-coupon bond with maturity T_N. The nominal swap rate is determined from equation (8) as follows:

$$C_N = \frac{P_{t,T_0} - P_{t,T_N}}{\sum_{n=1}^{N} \Delta_n P_{t,T_n}} . \tag{9}$$

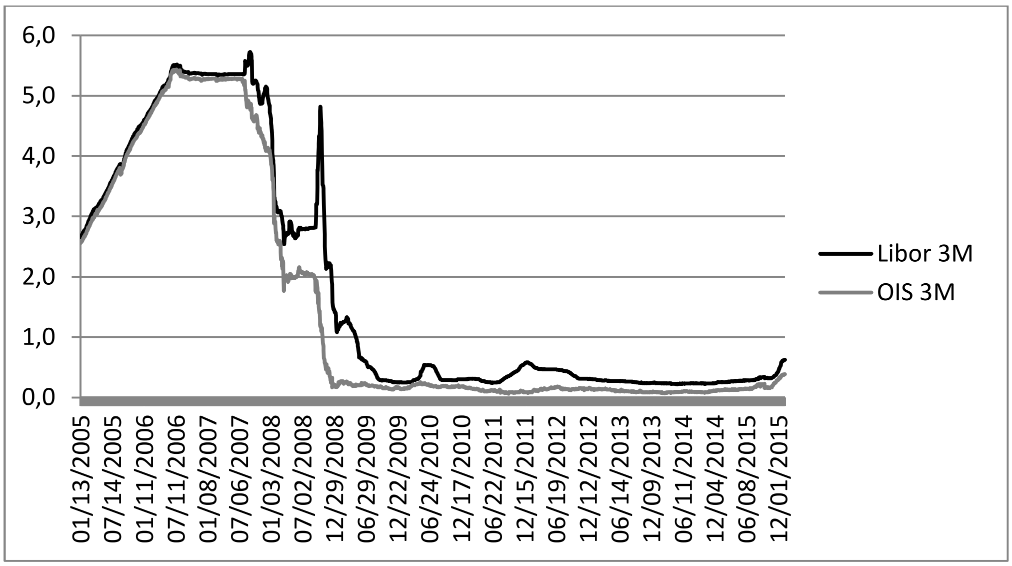
2.2. Using the OIS discount rate

Overnight Indexed Swap (OIS) is a fixed interest rate swap against a floating rate tied to a published index of a daily overnight reference rate, for example the Federal funds rates for US market and the EONIA rate in the Euro market. Being an overnight rate that refers to lending for an extremely short period of time, the OIS rate can be assumed to incorporate negligible credit or liquidity risk (Morini; 2009, p. 10). Moreover, OIS rates seem to be good predictors of market sentiment on future lending. The Libor rate is based on the interbank non-guaranteed deposit short-term rates between the big international banks (Michaud and Upper; 2008, p. 48). Libor rates are expected to provide market participants with an indication of default or liquidity risk underlying Libor market participants. Therefore, differences between the OIS rate and the Libor rate can be seen as an indication of the credit and/or liquidity risks affecting counterparties when they lend for maturities longer than one day.

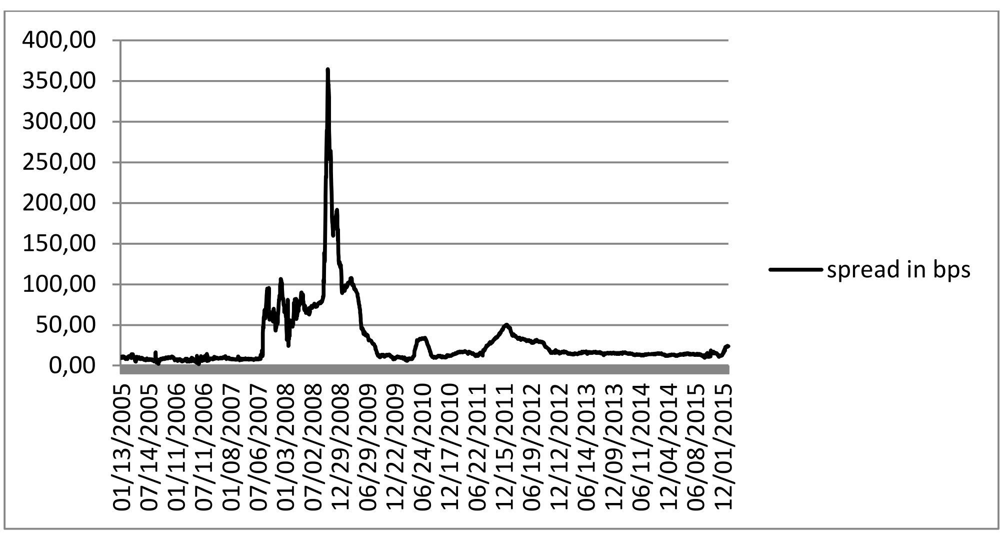
Before August 2007, market interest rates were compatible with what was described in standard manuals. As shown in Figure 1, the Libor and OIS rates were closely related and followed each other; the spread between the two was very narrow and could be ignored before 2007. After the liquidity crisis started, the spread between the two rates widened (Mercurio; 2009). The spread between the two rates reached its first maximum in September 2008, just before the US government overtook Fannie Mae and Freddie Mac and following the announcement by Lehman Brothers of a USD 4 billion in losses, which drove it to default. During the 2007-2009 subprime crisis, the spreads between the two rates were non-negligible. It is only after the crisis period, i.e., at the beginning of 2010, that the spread between the two rates came back to its historical lower normal level.

Figure 1: US dollar Libor vs OIS rate

A. US Dollar 3M Libor vs 3M OIS rate



B. US Dollar 3M Libor – 3M OIS spread



Source: Bloomberg

Given that the Libor-OIS spread was almost negligible before the 2007-2009 financial crisis, it made sense to use one of the two rates as a proxy for the risk-free rate. However, the significant widening of the Libor-OIS spread since August 2007 now justifies the use of OIS rate as the appropriate risk-free rate proxy. The OIS rate then became the preferred rate used by practitioners to construct the term structure of the risk-free interest rates (Morini; 2009). Because of this new increasing demand of OIS instruments, OIS markets have become more and more liquid over the past few years and their maturities have expanded. They are available for up to 30 years maturity, therefore one can use them to construct a complete risk-free yield curve.

Using this OIS yield curve, we can calculate the risk-free discount factors P_{OIS} as follows:

$$P_{OIS}(t, T_N) = \frac{1 - K_{OIS} \sum_{n=1}^{N-1} \delta_n P_{OIS}(t, T_n)}{1 + K_{OIS} \delta_N}, \tag{10}$$

where K_{OIS} is the OIS rate at time t with maturity T_N and δ_n represents the year fraction between t and T_n.

The new multi-curve framework requires a second discount factor linked to the forward curve. Unlike the OIS discount factor (P_{OIS}) given above, we use the Libor rates and equations (1) and (6) given above to obtain this second discount factor and its associated forward rates (FRA). Hence, given the following two increasing time vectors $T = \{T_0, \ldots, T_N\}$ and $S = \{S_0, \ldots, S_M\}$, where $T_N = S_M > T_0 = S_0$, the fixed swap rate K_{IRS} is obtained by the following equation which is a generalisation of equation (9) (by changing the risk-free discount factor by P_{OIS}).

$$K_{IRS} = \frac{\sum_{j=1}^{M} K_{FRA} P_{OIS}(t, S_j) \delta(S_{j-1}, S_j)}{\sum_{j=1}^{N} P_{OIS}(t, T_j) \delta(T_{j-1}, T_j)}. \tag{11}$$

2.3. Capturing counterparty default risk with CVA

The Basel II accord defines counterpart credit risk as the likelihood that a counterpart to a transaction may default before the maturity of the contract. If the defaulting party is the payer to the other party when default occurs, then this would be an economic loss for the non-defaulting party. Situations where only the default of one party is taken into account is referred to as unilateral counterparty risk. In such a case, only the default of one party has an impact on the valuation. The adjustment to the default-free price of the contract is calculated by the party not defaulting and is called the unilateral credit value adjustment (CVA) (e.g., Sorensen and Bollier (1994), Bielecki and Rutkowski (2001), Brigo and Masetti (2005), Lai and Soumaré (2010) and Soumaré and Tafolong (2016), among many others).

The general procedure to value a cash-flow in the presence of counterparty default risk consists of adding an additional term to the premium to account for default risk. We assume unilateral counterparty default risk, i.e., only one party to the deal can default and the other party is default-free. From Brigo and Masetti (2005), the payoff of the risky contract is given by:

$$\mathbb{E}_t[\Pi^D(t)] = \mathbb{E}_t[\Pi(\mathrm{t})] - \underbrace{LGD\ \mathbb{E}_t\left[1_{\{t<\tau<T\}}P(t,\tau)\max(\mathbb{E}_\tau[\Pi(\tau,T)],0)\right]}_{CVA}, \quad (12)$$

where $\Pi^D(t)$ is the expected payoff of a standard risky reclamation; $\Pi(\mathrm{t})$ is the expected payoff of an equivalent reclamation without counterparty default risk; LGD is the loss given default (i.e., $LGD = 1 - Recovery\ Rate$) assumed constant; τ is the counterparty default time; $P(t,\tau)$ is the stochastic discount factor at time *t* with maturity τ; and $\mathbb{E}_\tau[\Pi(\tau,T)]$ is the net present value of the residual value until contract maturity time T.

It is clear from this formula that the value of a risky claim is equal to the value of a risk-free claim minus an option value. Counterparty credit risk therefore adds an optionality to the original payoff. Denoting $t = T_0$ with a time discretization $T_0, \dots, T_N = T$, the previous formula can be approximated by the following discrete time formula assuming default occurs at the first time T_j following τ (e.g., Brigo and Pallavicini; 2007):

$$\mathbb{E}[\Pi^D(T_0,T)] = \mathbb{E}[\Pi(T_0,\mathrm{T})] -$$

$$\underbrace{LGD \sum_{j=1}^{N} \mathbb{E}\left[1_{\{T_{j-1}<\tau<T_j\}}P\left(T_0,T_j\right)\max\left(\mathbb{E}_{T_j}\left[\Pi\left(T_j,T\right)\right],0\right)\right]}_{CVA}. \quad (13)$$

Assuming that independence between the default time (τ) and the exposure level (Π) simplifies the last expectation term to the simple product of the expectation of the terms in the equation above. However, in real life, independence is not always the case, so we then have to consider cases where there is non-zero correlation between τ and Π. In credit risk valuation, the hypothesis that the underlying asset and the default of the counterparty are independent is often made to simplify the calculations, e.g., Brigo and Masetti (2005) and Sorensen and Bollier (1994), among many others. This can lead to valuation errors when there is a non-zero significant correlation between the two. This is called *wrong-way risk* (WWR, hereafter), when the risk of default of the counterparty and the exposure level underlying the contract increase together (Pykhtin and Zhu; 2007), i.e., there is a positive significant correlation between the two phenomena. This is risky in the wrong direction because when the value of the underlying increases, the counterparty becomes more and more vulnerable, therefore representing a potential for big losses as evidenced during the 2007-2009 subprime crisis.

In this paper, we examine the problem from the viewpoint of the non-vulnerable counterparty entering into an IRS contract with a risky counterparty. We combine this feature with the use of the OIS discount. Hence, in the following sub-sections, we first present the case when there is independence between the exposure level and the default risk of the counterparty. Second, we develop the framework for the case with wrong-way risk using the empirical approach suggested by Ruiz et al. (2013).

2.4. Valuation without wrong-way risk

Let us suppose a counterparty A with no default risk entering into a swap receiver contract with a risky counterparty B, exchanging floating payments for fixed payments at times $T_0, \ldots, T_N$.[5] The contract requires counterparty A to pay a floating rate L and receive a fixed rate K determined at the initiation of the swap contract at time T_0. The payments will be made until the first default time τ of counterparty B or until maturity time T if no default occurs, i.e., $\tau > T$. The fair swap rate, K at time t given in a default-free market is the rate that renders the value of the swap contract null at t. The forward swap rate that gives a fair contract is obtained using the equation (11) above. Obviously, if one supposes that counterparty B can default, then the exact spread to be received on the fixed payment side should be higher to compensate for potential default of the payer counterparty.

[5] Note that, as evidenced in the empirical section, we have chosen a receiver swap to study an appropriate case for wrong-way risk in a unilateral counterparty risk swap. Under this circumstance, the other way around (a payer swap) is not interesting and nor worth exploring.

We propose below a model that allows us to analyse the impact of counterparty default risk on the fair swap rate. In the first place, we assume that the intensity of default τ and interest rates are uncorrelated, leading to a much simpler model to start with. Later we will relax this assumption and study the effect of wrong-way risk with a more realistic model.

***Default intensity as a step function*:**

The default intensity is defined as the probability of defaulting within a given short (infinitesimal) time period Δt (under the filtration $\mathcal{F}_t$) and a default time $\tau > t$, and following the stochastic process X_t:

$$\lim_{\Delta t \to 0} P(\tau \in [t, t+\Delta t)|\mathcal{F}_t) = \lambda(X_t)dt, \tag{14}$$

where $\lambda(X_t) \in [0, \infty)$ is the default intensity. Note that this intensity measures the default probability at time *t* conditional on no previous default. Hence, $\lambda(t)\Delta\mathrm{t}$ is the default probability between t and $t + \Delta\mathrm{t}$, conditional on no previously-occurring default. In that case, the survival probability is:

$$P(t > \tau) = E\left[\exp\left(-\int_0^t \lambda(X_s)ds\right)\right]. \tag{15}$$

In practice, one way to operationalise λ is to use a piecewise constant function where the function takes constant values over defined intervals, since it is much easier to code programs and is less time-consuming compared to stochastic intensity functions. The step function is also easy to calibrate with market data. However its weakness is its discontinuity. For a set of *N* intervals of times with *N* intensity values for each time period, we define the step function for the default intensity as follows:

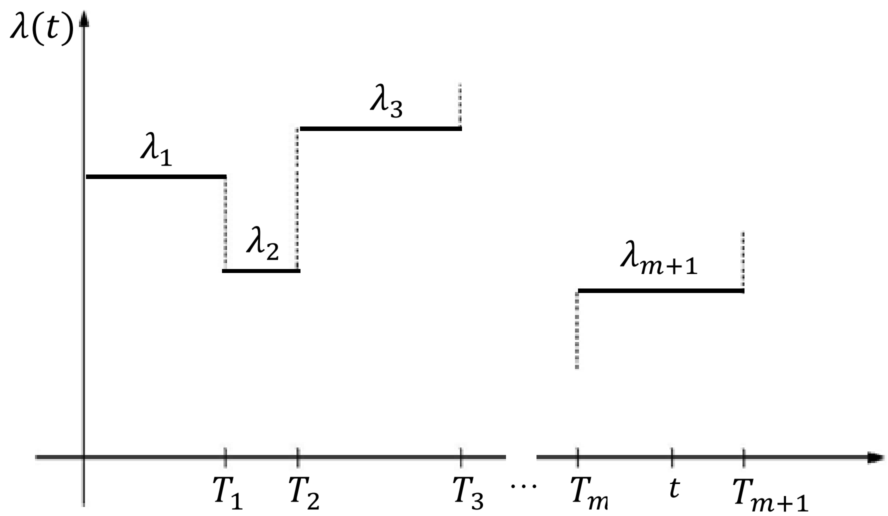
$$\lambda(t) = \sum_{j=1}^{N} \lambda_j \, 1_{\{T_{j-1} < t < T_j\}} = \sum_{j=1}^{N} \lambda_j \left(1_{\{t - T_{j-1} > 0\}} - 1_{\{t - T_j > 0\}}\right). \tag{16}$$

As an illustration, let $m = m(t)$ the highest interval where $t \notin [0, T_m]$, i.e. $m(t) = \max_j\{t \notin [0, T_j]\}$, for all $t \in (0, \infty)$ equation (15) becomes:

$$P(t > \tau) = \exp\left(-\sum_{j=1}^{m(t)} \lambda_j (T_j - T_{j-1}) - \lambda_{m(t)+1}(t - T_{m(t)})\right). \tag{17}$$

Figure 2 exhibits an illustration of this step function for the default intensity.

Figure 2: Illustrative default intensity function $\lambda(t)$



It is common practice in the financial sector to use CDS spreads for different maturities to obtain default probabilities. Hence, if default occurs at time τ within the interval $]T_{n-1}, T_n]$, we assume that loss payment is made at time T_n, i.e., at the end of the period instead of immediately at default time τ. We therefore use the following formula to link CDS spreads and the piecewise default probabilities (see Hull and White (2003) for more details):

$$R(T_N) = \frac{(1-\varphi)\sum_{n=1}^{N} D(T_n)\big(F_\tau(T_n) - F_\tau(T_{n-1})\big)}{\sum_{n=1}^{N}\frac{1}{\delta_n} D(T_n)\big(1 - F_\tau(T_n)\big)}, \tag{18}$$

where φ is the recovery rate in case of default, $D(T_n)$ is the discount factor, and $F_\tau(T_n)$ is the cumulative distribution function of the default probability, i.e., $F_\tau(T_n) = P(T_n \leq \tau)$. We can estimate the parameters of the default intensity $\{\lambda_j\}_{j=1}^{N}$ recursively using the market observed CDS spreads $R(T_N)$.

CVA calculation without wrong-way risk:

We denote by $IRS(t)$ the value at time t of the default-free interest rate swap. Using equation (13) above, we can express the value of the risky interest rate swap $IRS^D(t)$ as follows:

$$IRS^D(t) = IRS(t) - LGD \times EL(t), \tag{19}$$

where the adjustment term, the expected loss $EL(t)$, is defined as follows by Brigo and Masetti (2005):

$$\begin{aligned} EL(t) = {} & \mathbb{E}_t\left[1_{\{t<\tau<T\}} P(t,\tau)\max(\mathbb{E}_\tau[\Pi(\tau,T)],0)\right] \\ & = \int_{T_0}^{T_N} Swaption_{s,T_N}\left(t,K,S(t;s,T_N),\sigma_{s,T_N}\right) d_s\mathbb{Q}(\tau \leq s), \end{aligned} \tag{20}$$

where $Swaption_{s,T_N}\left(t,K,S(t;s,T_N),\sigma_{s,T_N}\right)$ denotes the price of a swaption with maturity s, exercise rate K, underlying forward swap rate $S(t;s,T_N)$, and volatility σ_{s,T_N} for the underlying swap with maturity T_N. The volatility σ_{s,T_N} is obtained from market at-the-money swaption volatility matrix. A swaption being an OTC option on an interest rate swap, a simple explanation for equation (20) is that $\max(\mathbb{E}_\tau[\Pi(\tau,T)],0)$ is an option on the cash flow of a swap.

Assuming that the default intensity and the cash flows are independent, i.e., no wrong-way risk, the calculations become simpler. Additionally, without loss of generality, we can assume, for simplicity, that default occurs at the payment date T_j. Under those circumstances, we can either assume that default occurs before the last payment (postponed payoff) or after the last payment (anticipated payoff). Under these hypotheses, we can express *EL*, for the *postponed* payoff (denoted by P) and the *anticipated* payoff (denoted by A), respectively, as follows:

$$EL^P(t) = \sum_{j=l+1}^{k}\left(\mathbb{Q}_t(\tau > T_{j-1}) - \mathbb{Q}_t(\tau > T_j)\right) Swaption_{j,k}\left(t;K,S_{j,k}(t),\sigma_{j,k}\right)$$

and

$$EL^A(t) = \sum_{j=l+1}^{k}\left(\mathbb{Q}_t(\tau > T_{j-1}) - \mathbb{Q}_t(\tau > T_j)\right) Swaption_{j-1,k}\left(t;K,S_{j-1,k}(t),\sigma_{j-1,k}\right),$$

where $\mathbb{Q}_t$ is the risk-neutral conditional probability measure of the counterparty default. These probabilities can be calculated from the step intensity function. Swaptions values are obtained using Black's (1976) formula.

Although CVA is a dollar value of credit adjustment, in some cases it is better to express it in terms of basis points spread. To convert the dollar value into spread, we use the Vrins and Gregory (2011) formula, where CVA spread, π_β, is calculated as follows:

$$\pi_\beta = \frac{CVA(K)}{DV01 - \beta\, CVA01}, \tag{21}$$

where the parameter $\beta \in [0,1]$, $DV01 = \sum_{i=1}^{N} P(t,T_i)\delta_i$ represents the present value of one dollar payments and $CVA01 = LGD\left(DV01 - \widetilde{DV01}\right)$, with $\widetilde{DV01} = \sum_{i=1}^{N} \mathbb{Q}_t(\tau > T_i) P(t,T_i)\delta_i$ the present value of defaultable one dollar payments.

Ideally, the value of the parameter β should be estimated iteratively, but that requires too many calculations. For this reason, as suggested by Vrins and Gregory (2011), we use $\beta = \frac{1}{2}$ instead for simplicity, without loss of generality, as this gives acceptable results when compared to results obtained with the proper value of β.

2.5. Valuation with wrong-way risk

It is possible to have a dependency between the default intensity of the counterparty and the credit exposure level. This effect is called *wrong-way risk* if the exposure level tends to increase when the default probability of the counterparty increases. Several approaches to model wrong-way risk have been suggested in the literature (see Ruiz et al. (2013) for a review of existing models). We will use the empirical approach proposed by Ruiz et al. (2013) for its parsimony in terms of model simplicity and goodness of fit with market data.

Modeling the interest rate:

As in Brigo and Pallavicini (2007), we use the short-rate Gaussian shifted two-factor process (hereafter G2++) for the interest rate. The sign «++» indicates that the function is calibrated using a deterministic function $\varphi(t;\alpha)$. Under the risk-neutral measure, the interest rate process is given as follows:

$$r(t) = x(t) + y(t) + \varphi(t;\alpha), \tag{22}$$

where $r(0) = r_0$ and the processes x and y, adapted to the filtration $\mathcal{F}_t$, satisfy the following differential equations:

$$dx(t) = -ax(t)dt + \sigma dZ_t^1, \tag{23}$$

$$dy(t) = -by(t)dt + \eta dZ_t^2, \tag{24}$$

with $x(0) = y(0) = 0$ and (Z^1, Z^2) is a bi-dimensional Brownian motion with instantaneous correlation $\rho_{12} \in [-1,1]$. The parameters r_0, a, b, σ and η are positive constants, which jointly with the correlation coefficient ρ_{12} constitute the set of parameters $\alpha = \{r_0, a, b, \sigma, \eta, \rho_{12}\}$ to be determined. The function $\varphi(t;\alpha)$ is a deterministic function defined over the interval $[0, T]$, with $\varphi(0;\alpha) = r_0$. It is obtained by calibrating the market-observed OIS rates.

We suppose that the term structure of the discount factors observed on the market are given by the smoothed curve: $P^M(0,T)$.[6] Denoting by $f^M(0,T)$ the instantaneous forward rate at time 0 with maturity *T* implicit to the term structure $P^M(0,T)$, we have:

$$f^M(0,T) = -\frac{\partial \ln(P^M(0,T))}{\partial T}.$$

In this case (see Brigo and Mercurio (2006), p. 146), the function $\varphi(t;\alpha)$ in equation (22) corresponds to the observed term structure of the discount factors if and only if, for each *T*, we have:

$$\varphi(T;\alpha) = f^M(0,T) + \frac{\sigma^2}{2a^2}(1 - e^{-aT})^2 + \frac{\eta^2}{2b^2}(1 - e^{-bT})^2$$
$$+ \rho_{12}\frac{\sigma\eta}{ab}(1 - e^{-aT})(1 - e^{-bT}) .$$

Modeling the intensity of default:

We use Ruiz et al.'s empirical approach (2013) to capture the dependency between the market factor and the default intensity. We assume the following functional form between the market factor represented by the interest rate r and the default intensity λ:

$$\lambda = g(r) + \sigma_\varepsilon \varepsilon, \tag{25}$$

[6] Note that, even if a complete term structure smooth curve is not available, on may infer it using bootstrating and splines interpolation techniques (see for instance: Ametrano and Bianchetti (2009), Andersen (2007), Ron (2000) and Wolberg (1999)).

where ε is a normalized random variable that can follow any distribution and we assume σ_ε is constant. The ultimate goal is to find the best solution for the function g. For that, we test four different functional forms:

(Linear):	$g_1(r) = A_1 + B_1 r,$	(26a)
(Power):	$g_2(r) = A_2 r^{B_2},$	(26b)
(Exponential):	$g_3(r) = A_3 e^{B_3 r},$	(26c)
(Logarithmic):	$g_4(r) = A_4 + B_4 \ln(r).$	(26d)

The parameters A and B are estimated using the least squares regression method for each function g and the quality of the estimations are compared using the regression coefficient R^2 and the size of the residual error volatility σ_ε. We seek to identify the best function that provides the highest R^2 and the lowest σ_ε. It is usually very challenging to examine historical default events as they are rare events, and it is hence difficult to obtain statistically significant estimates. We will instead use the implied default probabilities embedded in traded CDS spreads using the following approximation proposed by Hull and White (2012):

$$\lambda = \frac{s}{1-RR}, \tag{27}$$

where s is the credit spread and RR is the expected recovery rate given default. With historical available data on OIS rates (as a proxy for the risk-free rate r) and default probabilities, we find the best function g satisfying equation (25). The estimated best fitted functional form is used to simulate the values of λ.

Implicit correlation:

From equation (25), a change in the default intensity is approximated by $\Delta\lambda = g'(r)\Delta r + \sigma_\varepsilon\, \Delta\varepsilon$, hence the correlation between the default intensity λ and the interest rate r is given by:

$$\rho(r) = \frac{g'(r)\sigma_r^2}{\sqrt{g'(r)^2\sigma_r^2+\sigma_\varepsilon^2}\sqrt{\sigma_r^2}} = \frac{g'(r)\sigma_r}{\sqrt{g'(r)^2\sigma_r^2+\sigma_\varepsilon^2}}, \tag{28}$$

where σ_λ and σ_r are the standard deviation of $\Delta\lambda$ and Δr, respectively.

3. Empirical results

3.1. Data

We consider the following three cases of counterparty risks: Wells Fargo (low risk, rated A by S&P), Bank of America (medium risk, rated BBB+ by S&P) and Ally Financial, formerly General Motors (high risk, rated BB by S&P). Default intensities for the three counterparts are implied from their CDS spreads yield curves up to 30 years. All market data were extracted from Bloomberg on November 30th, 2015 and cover the period from June 30th, 2007 to November 30th, 2015, unless otherwise stated. The data starts in 2007 because CDS spreads are available starting that year for all the firms, but not necessarily for earlier. We then have a total of 360 weekly observations for each counterpart. Table 1 provides the market volatility matrix for swaptions.

Tableau 1: At-the-money swaption volatility matrix as of November 30th, 2015

A. OIS rates (in %)

	Tenor								
Maturity	1	2	5	7	10	15	20	25	30
1	52.3	47.32	42.28	40.32	34.83	31.54	29.16	29.26	28.68
2	47.78	44.46	38.34	37.55	33.33	30.39	28.17	27.13	26.07
3	43.7	44.39	36.52	34.63	31.99	29.08	27.06	26.17	25.1
4	43.97	39.49	34.77	33.21	30.99	28.54	26.21	25.53	24.27
5	38.1	36.3	33.61	31.61	29.92	27.12	25.37	24.46	23.55
7	34.38	32.86	30.47	29.22	27.72	25.16	23.6	22.55	21.99
10	27.45	26.56	26.14	25.63	24.32	22.23	20.73	19.95	19.34
15	23.89	22.86	21.82	21.14	20.73	18.82	17.67	17.5	16.66
20	18.53	18.11	18.23	18.07	17.51	17.57	16.61	16.2	14.39
25	17.86	17.43	17.06	17.06	16.87	15.56	15.51	14.69	14.88
30	16.83	16.68	16.23	15.95	15.49	14.95	14.43	14.41	14.04

B. Libor rates (in %)

	Tenor								
Maturity	1	2	5	7	10	15	20	25	30
1	52.61	47.7	42.88	41.07	35.66	32.91	31.1	31.93	32.01
2	48.26	44.98	39.04	38.38	34.3	31.95	30.33	29.93	29.45
3	44.28	45.06	37.34	35.51	33.12	30.83	29.42	29.17	28.68
4	44.74	40.28	35.68	34.21	32.31	30.52	28.76	28.76	28.02
5	38.96	37.2	34.6	32.75	31.44	29.27	28.12	27.84	27.5
7	35.46	33.93	31.72	30.74	29.67	27.73	26.76	26.27	26.31
10	28.69	27.97	28.1	27.91	27	25.42	24.4	24.14	24.02
15	26.89	25.89	25.23	24.75	24.76	23.16	22.39	22.81	22.28
20	22.39	22.03	22.66	22.77	22.51	23.34	22.7	22.75	20.61
25	23.36	22.93	22.93	23.26	23.48	22.28	22.89	22.23	22.96
30	23.87	23.82	23.67	23.55	23.31	23.18	22.99	23.38	23.07

3.2.Valuation results without wrong-way risk

We first present the results for the case without wrong-way risk. We suppose an interest rate swap (IRS) where the party conducting the valuation receives the fixed rate from a risky counterparty and pays the Libor rate. Payments are made semi-annually. We consider three different types of counterparty risk level: low risk (Wells Fargo), medium risk (Bank of America) and high risk (Ally Financial). We assume a constant recovery rate of 40% as in Brigo and Masetti (2005) and Brigo and Pallavicini (2007).

Each counterparty risk case has its own set of default intensities for different maturities obtained from the CDS spreads of the counterparty. Table 2 gives the default intensities and survival probabilities of the three risky counterparties. As expected, default probabilities increase with the risk level of the counterparty and with the time to maturity.

Table 2: Default intensities and survival probabilities Q(τ> T) for the 3 risky counterparties

Maturity (years)	Low risk		Medium risk		High risk	
	Default intensity	Survival prob. (%)	Default intensity	Survival prob. (%)	Default intensity	Survival prob. (%)
1	0.0024	99.76	0.0045	99.54	0.0152	98.47
2	0.0044	99.31	0.0076	98.78	0.0235	96.15
3	0.0087	98.44	0.0129	97.50	0.0310	93.17
4	0.0118	97.27	0.0158	95.95	0.0369	89.75
5	0.0182	95.49	0.0225	93.78	0.0466	85.60
7	0.0197	91.74	0.0272	88.74	0.0610	75.64
10	0.0233	85.46	0.0274	81.65	0.0633	62.38
15	0.0147	79.32	0.0190	74.14	0.0432	50.10
20	0.0147	73.62	0.0190	67.32	0.0432	40.24
25	0.0147	68.32	0.0190	61.12	0.0432	32.32
30	0.0147	63.41	0.0190	55.50	0.0432	25.95

Table 3 reports the (postponed and anticipated) CVA values for the counterparty risk levels. Our risk-free discount factors are derived from OIS rates (results in Panel A), but we perform similar calculations with Libor discount factors (results in Panel B) for comparison. As expected, the credit adjustment spreads increase with both the risk level of the counterparty and the maturity of the swap. We observe a small difference between the anticipated and postponed values of the spreads (less than 0.1 basis points in most cases).

For more accuracy, one can use the average of the two values to obtain a better proxy for the credit adjustment to be made for the swap rate. We also note that the default-free swap rates and the CVA spreads obtained with the OIS discounting curve are higher than those with the Libor discount factors.

Table 3: Implicit "default-free" swap rates and CVA spreads (in basis points)

A. Values with OIS discount rates

Maturity (years)	"Default-free" swap rates	Low risk		Medium risk		High risk	
		Anticipated	Postponed	Anticipated	Postponed	Anticipated	Postponed
5	1.59%	0.04	0.03	0.06	0.05	0.15	0.13
10	2.11%	0.52	0.47	0.67	0.62	1.37	1.28
15	2.38%	1.48	1.42	1.84	1.77	3.47	3.39
20	2.51%	2.63	2.57	3.23	3.17	5.75	5.69
25	2.58%	3.85	3.79	4.67	4.61	7.90	7.88
30	2.61%	5.05	5.00	6.06	6.02	9.83	9.84

B. Values with Libor discount rates

Maturity (years)	"Default-free" swap rates	Low risk		Medium risk		High risk	
		Anticipated	Postponed	Anticipated	Postponed	Anticipated	Postponed
5	1.55%	0.03	0.03	0.05	0.04	0.12	0.10
10	2.05%	0.42	0.39	0.55	0.51	1.10	1.04
15	2.29%	1.20	1.16	1.49	1.45	2.79	2.75
20	2.41%	2.16	2.12	2.63	2.60	4.65	4.64
25	2.47%	3.18	3.14	3.84	3.81	6.44	6.47
30	2.50%	4.19	4.17	5.02	5.00	8.08	8.14

3.3. Valuation results with wrong-way risk

As discussed above, unlike the case of no wrong-way risk, when there is dependency between the default intensity of the counterparty and the exposure level, there is no simple way to calculate the CVA as no closed-form solution exists. We therefore resort to Monte Carlo simulations for our results. Below, we first present the algorithm implementation, and then we present the simulation results.

3.3.1. Algorithm implementation

The implementation proceeds in three steps described as follows.

Step 1 - Simulation of the interest rates

This first step in the implementation process consists of estimating the stochastic interest rate model parameters using market-observed interest rates. Future interest rates are then simulated using the estimated model.

- *Calibration of the interest rate model G2++.* The calibration of the model consists of finding the values of the interest rate model parameters $\alpha = \{r_0, a, b, \sigma, \eta, \rho_{12}\}$ that better fit the market interest rates at the valuation date.
- *Simulation of the scenarios.* Once the parameters values are estimated, we simulate the term structure of interest rates using the calibrated interest rate model of equation (22). More precisely, we simulate the two processes *x* and *y* of equations (23-24) for $M = 100000$ scenarios and $N = 2 * T$ time steps (since payments are done semi-annually for the two legs of the swap). The dynamics of *x* and *y* processes can be expressed in terms of two independent Brownian processes W_1 and W_2 as follows:

$$dx(t) = -ax(t)dt + \sigma dW_1(t),$$

$$dy(t) = -by(t)dt + \eta\, \rho_{12} dW_1(t) + \eta\sqrt{1 - \rho_{12}{}^2}dW_2(t),$$

where $dZ_t^1 = dW_1(t)$ and $dZ_t^2 = \rho_{12} dW_1(t) + \sqrt{1 - \rho_{12}{}^2}dW_2(t)$.

We simulate the two independent Brownian processes W_1 and W_2 for M scenarios and N time steps to generate *x* and *y*. The zero yield curve is generated for all scenarios using the following formula for the zero coupon interest rate, denoted r_{zc} (Brigo and Mercurio; 2006):

$$r_{zc}(t,T) = -\ln\big(A(t,T)\big) + B(a,t,T)x(t,T) + B(b,t,T)y(t,T),$$

with $A(t,T) = \frac{P^M(t,T)}{P^M(0,t)} \exp\left\{\frac{1}{2}[V(t,T) - V(0,T) + V(0,t)]\right\},$

$$B(w,t,T) = \frac{1-e^{-w(T-t)}}{w}, \text{ and}$$

$$V(t,T) = \frac{\sigma^2}{a^2}\left[T - t + \frac{2}{a}e^{-a(T-t)} - \frac{1}{2a}e^{-2a(T-t)} - \frac{3}{2a}\right]$$

$$+\frac{\eta^2}{b^2}\left[T - t + \frac{2}{b}e^{-b(T-t)} - \frac{1}{2b}e^{-2b(T-t)} - \frac{3}{2b}\right]$$

$$+2\rho_{12}\frac{\sigma\eta}{ab}\left[T - t + \frac{e^{-a(T-t)}-1}{a} + \frac{e^{-b(T-t)}-1}{b} - \frac{e^{-(a+b)(T-t)}-1}{a+b}\right].$$

Step 2 - Simulation of the intensity of default

Once interest rates are simulated in the first step, at the second stage, we estimate the default intensity function *g*(*r*) of equation (25) using market observed CDS spreads. For each scenario and time step T_j, the default intensity λ_j is calculated using the functional form *g*(*r*) and the simulated interest rates. Counterparty default is supposed to occur only at contract payment dates T_j. If default occurs in the time interval $]T_{j-1}, T_j]$, we assume that it happens at time T_j.

To determine the default times, we need to approximate the integral of the default intensity by numerical interpolation with a piecewise function as follows:

$$\Lambda(\tau) = \int_t^\tau \lambda(s)ds \approx \sum_{i=0}^{j-1} \lambda_i \, (T_{i+1} - T_i).$$

The first passage time τ^1 being a Poisson process, $\Lambda(\tau^1)$ is distributed exponentially with parameter 1: $\Lambda(\tau^1) \coloneqq \xi \sim Exponential(1)$. Inverting this distribution function yields $\tau^1 = \Lambda^{-1}(\xi)$ for the first passage time.

To simulate the paths of this exponential distribution, we proceed as follows. From the cumulative distribution function of the exponential distribution, we know that a standard uniform random variable U can be defined as $U = 1 - e^{-\xi}$, hence the random exponential variable is obtained with the transformation: $\xi = -\ln(1 - U)$. We then generate M values of the uniform random variable U and transform it to obtain the exponential distribution ξ. We compare these simulated random numbers with the default intensities obtained with equation (16) for each T_j. The default event occurs at time T_j when the value generated from variable ξ is higher or equal to $\Lambda(T_j)$ and T_j is the first time when that occurs.

Step 3 - Calculation of the CVA

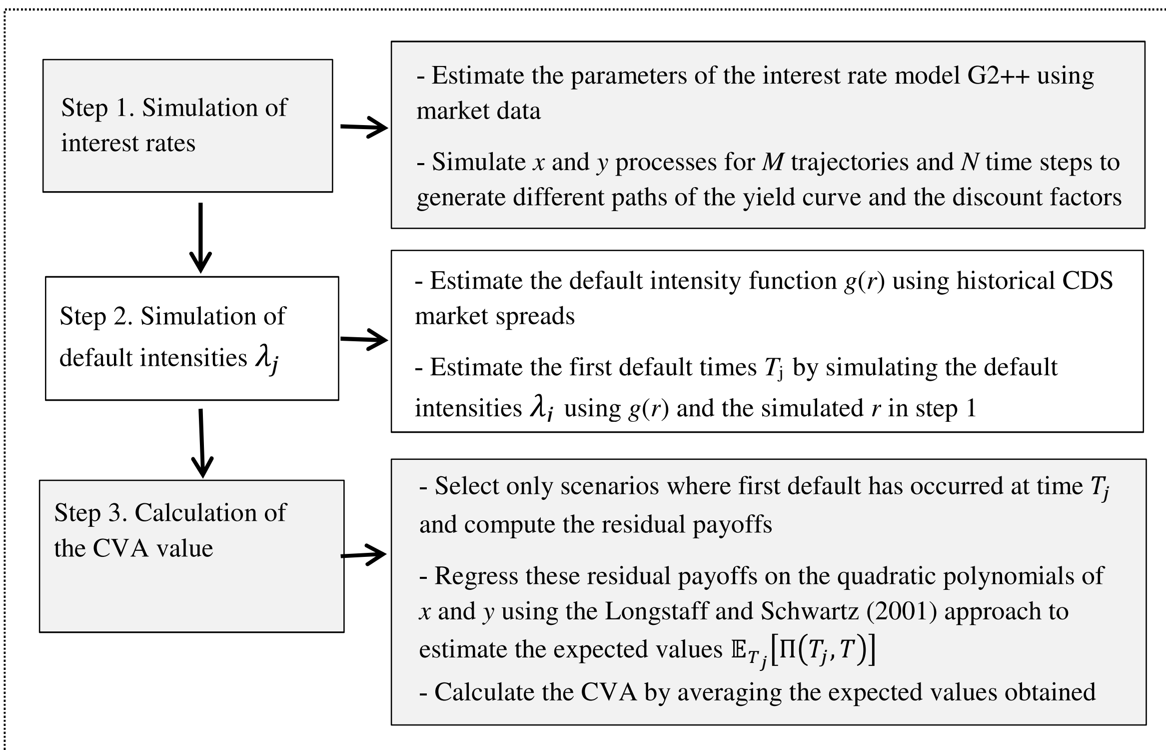
The future expected value $\mathbb{E}_{T_j}\left[\Pi\left(T_j, T\right)\right]$ at the default time T_j in equation (13) is estimated using a polynomial series on the interest rate underlying processes *x* and *y*. The coefficients of the polynomial series expansion are obtained by applying the OLS regression method of Longstaff and Schwarz (2001). Hence, at each time T_j, we only use the set of scenarios where default has occurred at that time and we calculate the residual payoff. We then regress the vector of those residual payoffs on the values of x, y, xy, x^2 and y^2:

$$\mathbb{E}_{T_j}\left[\Pi\left(T_j, T\right)\right] = e_1 + e_2 x + e_3 y + e_4 xy + e_5 x^2 + e_6 y^2.$$

Once the regression coefficients are estimated and the expected value calculated at time T_j, we apply the same procedure to all time steps in order to obtain all expected values. Finally, the value of the CVA is obtained as the present value of the expected losses from the contract given counterparty default as given in equation (13). We obtain this expected loss by averaging the different expected losses. As we stressed above, we will express the CVA in spread basis points (obtained by applying equation (21)).

Figure 3 below summarizes the three implementation steps to calculate the CVA.

Figure 3: Summary of the steps for the implementation of the model



3.3.2. Simulation results

Here we present the simulation results with wrong-way risk. We make the same assumptions as we did in the case without wrong-way risk, with the exception that we allow for negative correlation between market interest rates and default probabilities of the counterparty, so-called wrong-way risk. The calibration of the interest rate model yields the following values for parameters α:

$a = 0.5603,\ b = 0.0164,\ \sigma = 0.0112,\ \eta = 0.0089,\ \rho_{12} = -0.8479$ (with OIS rates);

$a = 0.0563,\ b = 0.0016,\ \sigma = 0.0006,\ \eta = 0.0091,\ \rho_{12} = -0.9705$ (with Libor rates).

We next calibrate the default intensity function using market-observed one-year CDS spreads. Table 4 provides the estimation results of the four functional forms, equations (26a-d), for each counterparty risk level. We also report the R^2 and the variance of the error term σ_ε^2. We can use these two latter indicators to decide which of these functions better fit the data. The best model will more likely be the one with the biggest R^2 and the smallest error term volatility σ_ε. With these criteria, the best models are the power and the exponential functions, with a marginal preference for the exponential function that has the biggest R^2 and the smallest σ_ε.

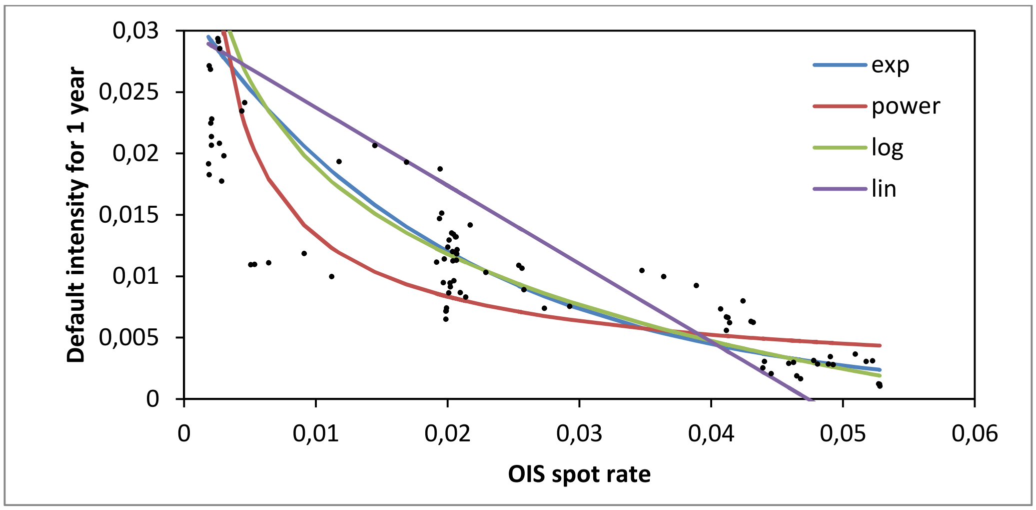
Table 4: Estimates of the intensity function parameters for the 3 counterparty risk levels

Function *g*(*r*)	***A***	***B***	**R^2**	**σ_ε^2**
Panel A: Low risk (Wells Fargo)				
Power	-7.421	-0.6706	72%	$9.2 .10^{-6}$
Exponential	-3.4333	-49.4	81%	$2.1 .10^{-6}$
Logarithmic	-0.0281	-0.0102	66%	$7.8 .10^{-5}$
Linear	0.0301	-0.6358	53%	$9.4 .10^{-4}$
Panel B: Medium risk (Bank of America)				
Power	-8.6538	-0.8195	78%	$6.9 .10^{-5}$
Exponential	-3.1819	-59.08	84%	$6.6 .10^{-6}$
Logarithmic	-0.0465	-0.0139	63%	$1.2 .10^{-4}$
Linear	0.0364	-0.8405	47%	$8.3 .10^{-4}$
Panel C: High risk (Ally Financial)				
Power	-3.4725	-0.4525	40%	$6.4 .10^{-3}$
Exponential	-0.6651	-38.6251	61%	$2.4 .10^{-3}$
Logarithmic	-0.2023	-0.1197	21%	$7.2 .10^{-2}$
Linear	0.5355	-9.9947	31%	$3.6 .10^{-2}$

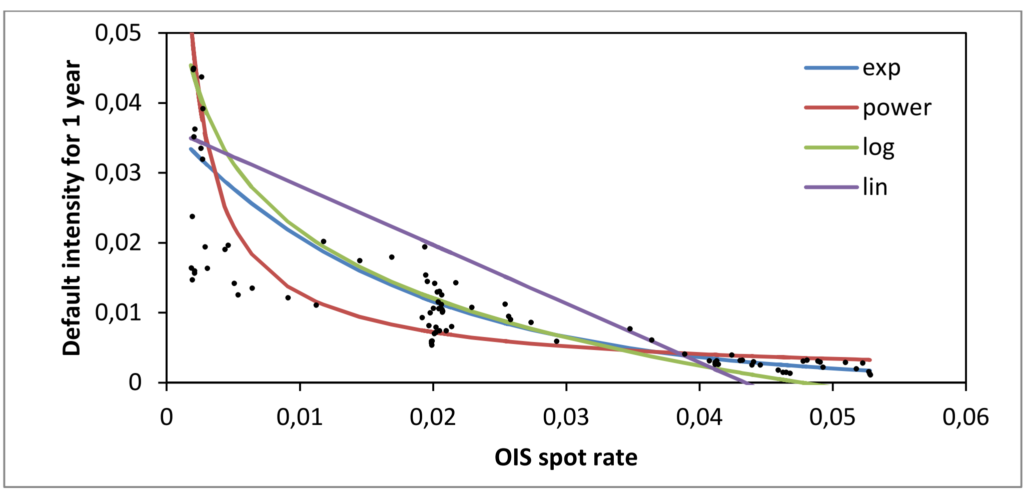
Figure 4 plots the default intensities as a function of the interest rates for the three counterparty risk levels. We can clearly see a dependence structure between the interest rate and the probability of default of the counterparty. The lower the interest rate, the higher the probability of default, and vice-versa. There is then a negative correlation between the two variables, which materialises through a negative value for the parameter B as shown in Table 4. Graphically, the exponential function appears to provide the best fit.

Figure 4: Default intensity function for the 3 counterparty risk levels

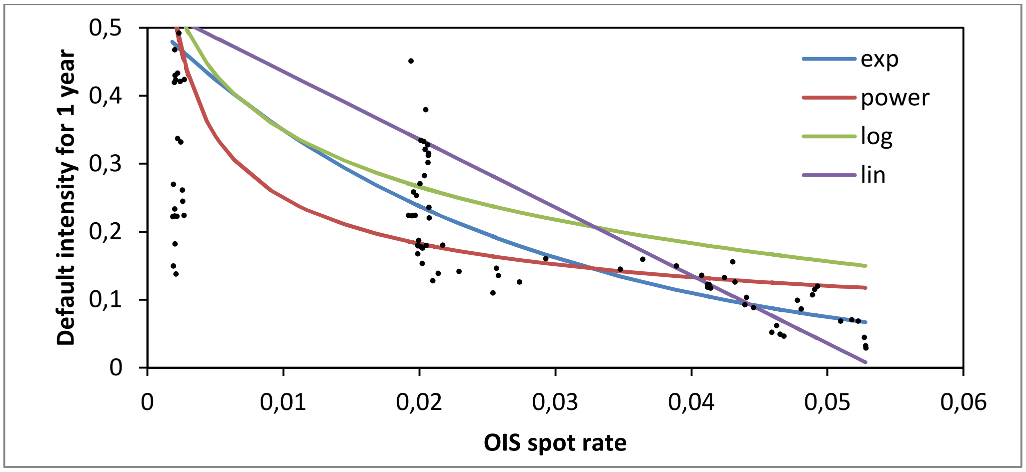
Panel A. Low risk (Wells Fargo)



Panel B. Medium risk (Bank of America)



Panel C. Medium risk (Ally Financial)



Unfortunately, high values of R^2 are not enough to guarantee that the model better fits the data, because such high values can be due to model misspecification or the existence of outliers. We therefore performed additional tests on the residual of the model retained with the R^2 criteria. If the residuals are random, it means that the model fits the data well. Otherwise, if a non-random residuals structure prevails, this is an indication of poor fit. To that end, we performed normality tests on the residuals based on the skewness and kurtosis coefficients, and combined the two tests in a single aggregate statistic. Table 5 gives the tests results for the exponential function for the three counterparty risk levels. We cannot conclude that the skewness and kurtosis of the residual distribution are different from that of a normal distribution at a 5% significance level. This is confirmed by the joint normality test, with which we cannot reject the null hypothesis of normality for the residuals at the 5% significance level. We therefore confirm the choice of the exponential function as the best fit functional form.

Table 5: Normality tests for the error terms

			Joint test	
Counterparty risk	**Skewness test (p-value)**	**Kurtosis test (p-value)**	**Adjusted $\chi_2{}^2$**	**Prob.$> \chi_2{}^2$**
Low risk (Wells Fargo)	0.87	0.06	3.76	0.15
Medium risk (Bank of America)	0.53	0.22	1.93	0.38
High risk (Ally Financial)	0.09	0.18	4.65	0.10

In summary, the data calibration exercises support a dependency structure between the OIS rates and the default probability of the counterparty over the study period, and the exponential function turns out to be the best fit for the default intensity functional form $g(r)$. Therefore, ignoring this non-negligible correlation can lead to inappropriate swap rates.

Table 6 provides the CVA spreads with wrong-way risk and the calculation error value. As in the case without wrong-way risk, the CVA spread increases with the level of risk of the counterparty and with the maturity of the swap. With wrong-way risk, when interest rates decrease, the value of the receiver swap increases. This has some implications for the credit adjustment value in the way that high negative correlation pushes up the CVA significantly, and even more so for more risky counterparties. We also find that the CVA value is higher with OIS discount rates than with Libor discount rates.

Table 6: CVA spreads (in basis points) with WWR

A. OIS discount rates

Maturity (years)	"Default-free" swap rates	Low risk		Medium risk		High risk	
		CVA spr.	error	CVA spr.	error	CVA spr.	error
5	1.59%	0.27	$4.9 .10^{-2}$	0.39	$6.1 .10^{-2}$	2.32	$5.7 .10^{-2}$
10	2.11%	0.77	$1.5 .10^{-2}$	1.18	$1.4 .10^{-2}$	6.65	$1.4 .10^{-2}$
15	2.38%	1.84	$8.4 .10^{-3}$	2.24	$9.5 .10^{-3}$	10.04	$6.3 .10^{-3}$
20	2.51%	3.09	$6.9 .10^{-3}$	3.39	$7.7 .10^{-3}$	11.15	$4.2 .10^{-3}$
25	2.58%	4.27	$6.5 .10^{-3}$	5.13	$6.7 .10^{-3}$	13.61	$2.8 .10^{-3}$
30	2.61%	5.79	$6.1 .10^{-3}$	8.25	$5.8 .10^{-3}$	17.71	$2.5 .10^{-3}$

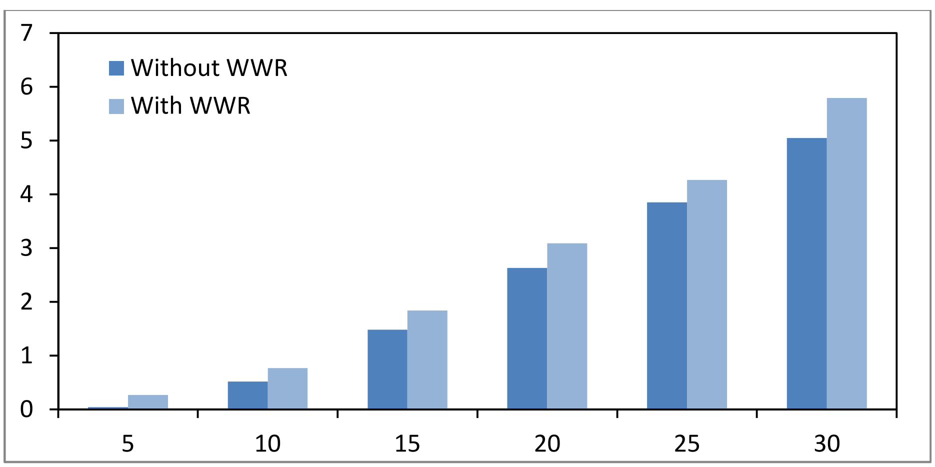
B. Libor discount rates

Maturity (years)	"Default-free" swap rates	Low risk		Medium risk		High risk	
		CVA spr.	error	CVA spr.	error	CVA spr.	error
5	1.55%	0.31	$5.1 .10^{-2}$	0.51	$4.5 .10^{-2}$	2.12	$3.5 .10^{-2}$
10	2.05%	0.90	$1.7 .10^{-2}$	1.34	$1.9 .10^{-2}$	5.58	$1.2 .10^{-2}$
15	2.29%	1.59	$1.2 .10^{-2}$	2.18	$1.5 .10^{-2}$	6.14	$6.5 .10^{-3}$
20	2.41%	2.38	$1.1 .10^{-2}$	3.48	$1.3 .10^{-2}$	8.82	$5.8 .10^{-3}$
25	2.47%	3.47	$1.3 .10^{-2}$	4.95	$1.2 .10^{-2}$	9.44	$7.2 .10^{-3}$
30	2.50%	5.75	$1.5 .10^{-2}$	7.74	$1.4 .10^{-2}$	13.95	$1.0 .10^{-2}$

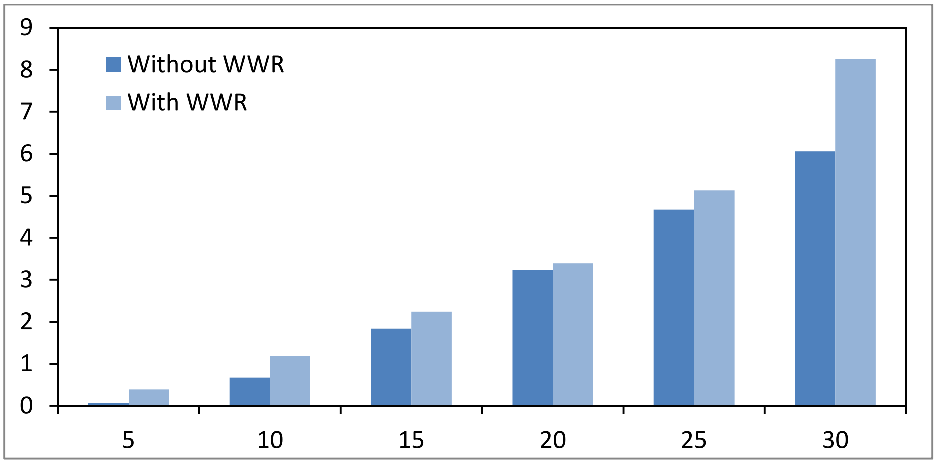
When we compare the CVA values obtained with and without wrong-way risk, we observe that spreads are higher when wrong-way risk is accounted for. This is illustrated by Figure 5, which presents a comparison of CVA spreads with and without wrong-way risk for the three counterparty risk levels. We observe that the impact of wrong-way risk on CVA is more important when the probability of default of the counterparty is high and for higher maturity. A simple explanation is that the probability of default increases with the risk level of the counterparty, with the time to maturity of the contract leading to a greater exposure of the portfolio to default of the counterparty when there is wrong-way risk. For example, if we consider the particular case of Ally Financial (counterparty with the highest likelihood to default), the CVA spread for a 30-year maturity contract moves from 9.83 bps (without wrong-way risk) to 17.71 bps (with wrong-way risk), an almost 50% increase, which means that ignoring wrong-way risk in the swap valuation will imply lower levels of credit adjustment values.

Figure 5: Comparison of CVA spreads with and without wrong-way risk

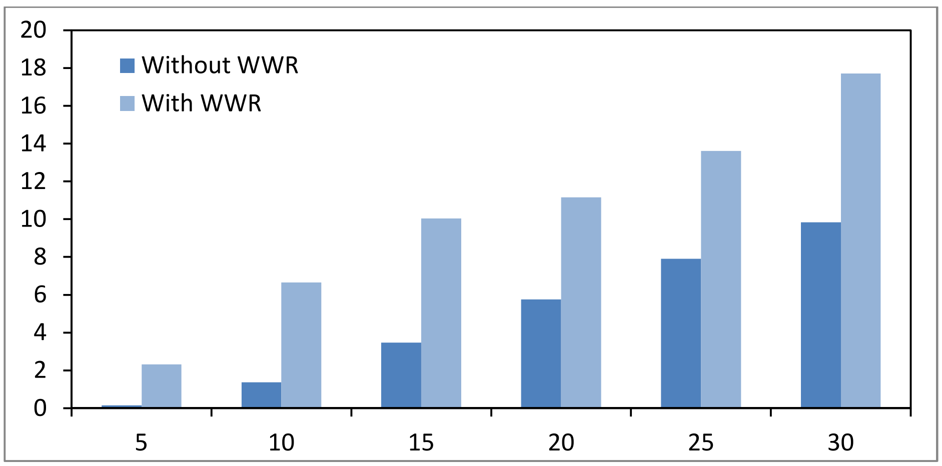
Panel A. Low risk (Wells Fargo)



Panel B. Medium risk (Bank of America)



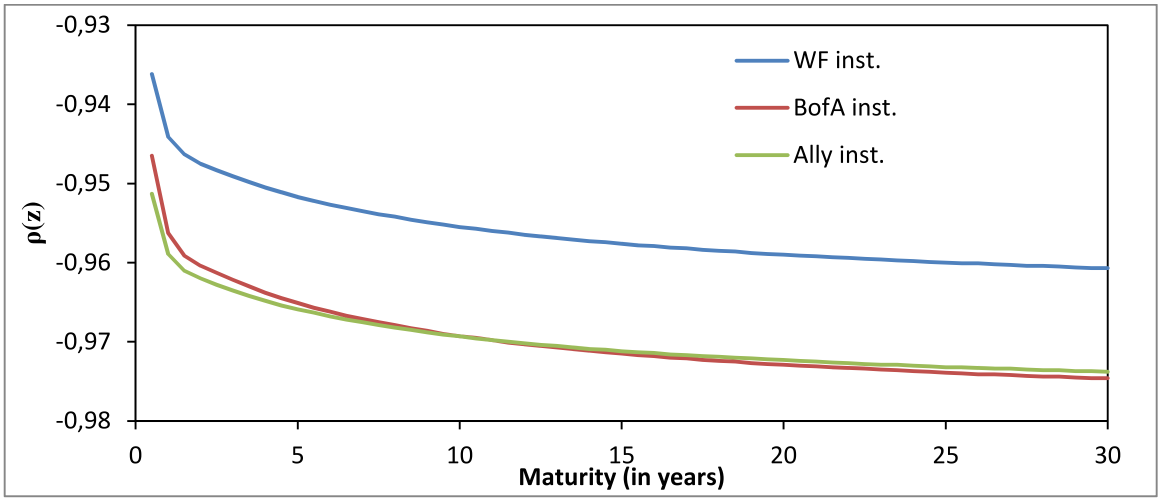
Panel C. High risk (Ally Financial)



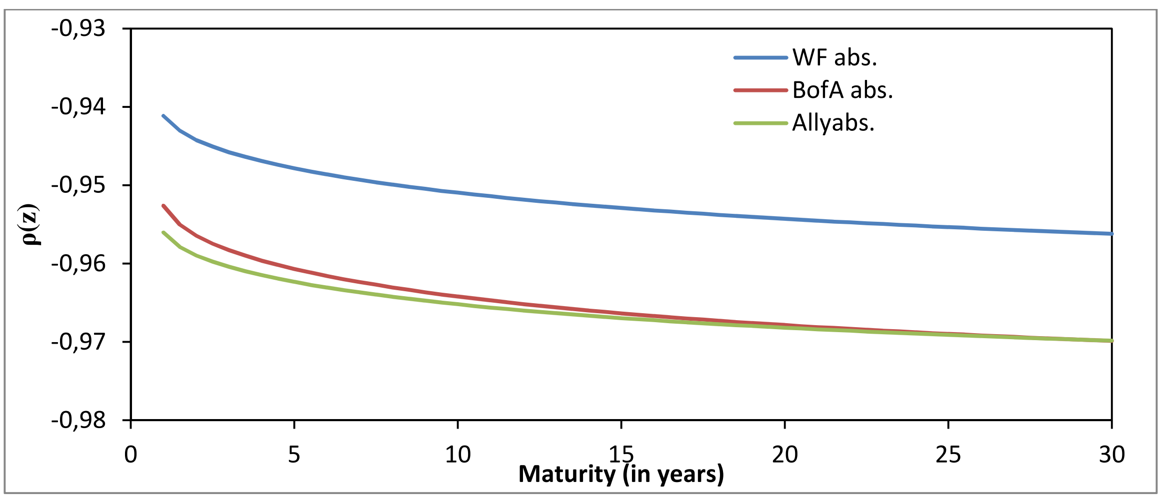
The results therefore clearly support the importance of considering wrong-way risk in the credit valuation of the counterparty, and hence, support our empirical approach to model wrong-way risk with the dependency structure proxied by the correlation $\rho(r)$ between the default probabilities and the interest rates. Figure 6 plots the relation of the correlation $\rho(r)$ (cumulative and instantaneous) as a function of contract maturity for the three counterparty risk cases. We use the instantaneous correlation and the cumulative correlation over the maturity period. The cumulative correlation is better as it represents the overall level of dependency over the maturity period of the contract to be priced. We observe that the more risky the counterparty, the more negatively correlated are its default probability and the interest rate, hence, the swap contract has a much higher exposure to counterparty default and subsequently requires higher CVA spreads. In addition, correlations between default intensities and interest rates increase in absolute term with the length of the time to maturity.

Figure 6: Evolution of the correlations as a function of maturity for the 3 counterparty risk levels

Panel A. Instantaneous correlations



Panel B. Cumulative correlations



4. Conclusion

In this paper, we revisited the valuation framework of an interest rate swap following changes in financial markets after the 2007-2009 subprime credit crisis. Indeed, since the crisis, OIS rates have become the preferred risk-free discount rates for market practitioners. In addition, the calculation of credit value adjustment (CVA) to account for counterparty risk has become common practice in the financial industry. Given these changes, we propose an extended valuation framework which jointly accounts for wrong-way risk and uses OIS rates to discount the cash flows. We build on the CVA calculation formula of Brigo and Pallavicini (2007) and the empirical approach of Ruiz et al. (2013) to capture wrong-way risk.

Our findings are consistent with previous results found in the existing literature. In particular, we found that the risk of the counterparty has a significant impact on interest rate swap contracts, with the impact being amplified when wrong-way risk is taken into account. The use of OIS discount risk-free rates leads to relatively higher CVA valuation than the ones obtained with the traditional Libor rates, hence avoiding undervaluation of swap contracts.

Therefore, we first need to account for counterparty risk as well as wrong-way risk when valuing interest rate swaps. Second, the empirical approach of Ruiz et al. (2013) adopted here to capture wrong-way risk through the correlation measure is parsimonious and easy to implement with market data. And finally, it is important to use OIS discount rates as a proxy for the risk-free discounts.

We limit ourselves to the valuation of vanilla interest rate swaps by assuming unilateral counterparty default. Future extension of our work could include the following extended features: (i) possible default of the two counterparts in the contract, so-called bilateral counterparty default, (ii) consideration of a portfolio of interest rate swap contracts instead of only one in order to benefit from the netting feature imbedded in swap master agreements.

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